K. L. E. Society's

G. I. Bagewadi Arts, Science and Commerce College, Nipani - 591237

Accredited at 'A' level by NAAC with CGPA 3.35

Affiliated to Rani Channamma University, Belagavi, Karnataka, India

Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

Ref. No. Date :

2.2.1: Advance learners and enrichment Programs Organized for them

- *i. List of advance learners subject wise.*
- *ii. Topper as a teacher*
- *iii. Project assigned to advance learners*
- *iv. Advance Learners participated and Presented papers in Conference*
- *v. Question Papers and Question banks solved by Advance learners*
- *vi. Distributed books to toppers*
- *vii. Felicitation to toppers, Rank holders*
- *viii. Out come: Ranks, Centum scorers*

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Ref. No. *Date :*

i. List of Advance Learners Subject Wise

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Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116

Number of advanced Learners Subject wise and Class wise in the Year 2019-20

Subject	Class	No. of advanced learners
Physics	B.Sc I	13
	B.Sc II	26
	B.Sc III	12
	Total	**51**
Chemistry	B.Sc I	08
	B.Sc II	90
	B.Sc III	58
	Total	**156**
Mathematics	B.Sc I	22
	B.Sc II	17
	B.Sc III	32
	Total	**71**
Botany	B.Sc I	08
	B.Sc II	07
	B.Sc III	13
	Total	**28**
Zoology	B.Sc I	08
	B.Sc II	07
	B.Sc III	13
	Total	**28**
Commerce	B.Com I	45
	B.Com II	19
	B.Com III	37
	Total	**101**
Economics	BA I	12
	BA II	09
	BA III	10
	Total	**31**
	Grand Total	**466**

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

K.L.E Society's

G.I. Bagewadi College, NIPANI

DEPARTMENT OF PHYSICS

List of Advanced Learner 2019-20

Class : B.Sc Ist & IInd Sem

Reg. No.	Student Name	Marks
S1919405	Adinath Camble	76
S1919421	Amruta Khanai	85
S1919427	Akshata Yaranal	78
S1919429	Ashwini Dodabangi	81
S1919430	Ashwini Rangapure	80
S1919440	Gayatri Gurav	78
S1919512	Priya Basannavar	79
S1919531	Roopali Vairat	84
S1919532	Rutika Kamate	79
S1919535	Rutuja Shendure	89
S1919584	Sourabh Madale	88
S1919590	Sujata Utagi	88
S1919591	Sukshay Padre	76

Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

PRINCIPAL
G.I. Bagewadi Arts, Science &
Commerce College, NIPANI

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE * NIPANI

K.L.E Society's

G.I. Bagewadi College, NIPANI

DEPARTMENT OF PHYSICS

List of Advanced Learner 2019-20

Class : B.Sc IIIrd & IVth Sem

Reg. No.	Student Name	Marks
S1819407	Aishwarya Udale	85
S1819424	Anurada Rayagondanavvar	90
S1819430	Asma Multani	85
S1819461	Lakshmi Mantur	91
S1819462	Lakshmi Vadagole	86
S1819477	Megha Kumbar	90
S1819479	Misaba Jamadar	95
S1819482	Neha More	86
S1819502	Pranali Karape	94
S1819560	Shivaling Goture	89
S1819562	Shivani Nimbalkar	92
S1819564	Shivaprasad Toli	97
S1819569	Shrikant mali	99
S1819570	Shrikant Vaddar	94
S1819571	Shreya Desai	93
S1819573	Shruti Kumbar	96
S1819574	Shruti Patil	86
S1819577	Shrutika Jadav	92
S1819586	Shubhangi Shyndage	86
S1819597	Sonali Madiwal	96
S1819604	Sukanya Chougala	93
S1819606	Sumit Muragude	95
S1819609	Suraj Khot	92
S1819620	Vijay Chougale	85
S1819624	Vishwanath R	89
S1819628	Yashoda Kajave	93

Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

K.L.E's G.I. BAGEWADI ARTS, SCI & COM. COLLEGE NIPANI

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Commerce College, NIPANI

K.L.E Society's

G.I. Bagewadi College, NIPANI

DEPARTMENT OF PHYSICS

List of Advance Learner 2019-20

Class : B.Sc V^{th} & VI^{th} Sem

Reg. No.	Student Name	Marks
S1717609	Aishwarya Padre	85
S1717621	Aruna Hegade	80
S1717633	Deepa Kedarshetti	79
S1717636	Dilshad Mulla	82
S1717644	Jyoti Bagade	80
S1717672	Muskan Shekhaji	81
S1717686	Pooja Jadhav	83
S1717692	Pradnya Bhivashe	94
S1717753	Sonali Bharade	81
S1717757	Soundarya Patil	88
S1717770	Tejashwini Patil	80
S1717772	Ummesalma Mulla	80

Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

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Commerce College, NIPANI

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K.L.E. Society's

G.I. Bagewadi Arts, Science and Commerce College, Nipani-591237
'College with Potential for Excellence'

DEPARTMENT OF CHEMISTRY

"IQAC Initiative"

[Re-accredited at 'A' level by NAAC with CGPA 3.35]

Ph: 08338-220116, 220119
Website: www.klegibnpn.org
E-mail: klegib_npn@yahoo.co.in

ENRICHMENT PROGRAMME FOR ADVANCE LEARNERS

KEY STEPS:

1. Students who have scored more than 80% in previous year are considered as advance learners.
2. They are asked to give seminars on selected topics in chemistry.
3. Asked them to solve IA and previous year question papers and evaluated by the concerned staff and H.O.D.
4. Asked them to refer more number of advanced text books from library as well as department.
5. They will be provided with some videos regarding the chemistry for better understandings.

CONVENOR

H.O.D.

Head
Department of Chemistry
K.L.E's G. I. B. College, Nipani.

PRINCIPAL

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K.L.E. Society's
G.I. Bagewadi College, Nipani.

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

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G.I.BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE,

NIPANI- 591237 Dist:- Belgavi

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e-mail- kle_gibnpn.yahoo.in.

IQAC INITIATIVE

Advance Learner FOR I & II SEMESTER B.SC. - NOVEMBER 2019-20

si.no	Register no	Name of the student	% at entry (1st sem)	% at exit (2nd sem)	growth
1	S1919430	ASHWINI RANGAPURE	87%	Promoted	--
2	S1919448	KAVITA UTAGI	86%	Promoted	--
3	S1919459	MAHALAXMI MANE	85%	Promoted	--
4	S1919467	MAYURI KHANDEKAR	85%	Promoted	--
5	S1919510	PREETI BORAGALLE	86%	Promoted	--
6	S1919511	PRITAM CHOUGULE	45%	Promoted	--
7	S1919590	SUJATA UTAGI	86%	Promoted	--
8	S1919591	SUKSHAY PADRE	88%	Promoted	--

As per RCU direction B.Sc - 2nd Sem students are promoted to higher class

CONVENOR

H.O.D

Head
Department of Chemistry
K.L.E's G. I. B. College, Nipani.

PRINCIPAL

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K.L.E. Society's
G. I. Bagewadi College, Nipani.

K.L.E.'s G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

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G.I.BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE,
NIPANI- 591237 Dist:- Belgavi
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e-mail- kle_gibnpn.yahoo.in.
IQAC INITIATIVE

Advance Learner FOR III & IV SEMESTER B.SC. - NOVEMBER 2019-20

si.no	Register no	Name of the student	% at entry (III sem)	% at exit (IV sem)	growth
1	S1819406	AISHWARYA KURUNDWADE	94%	promoted	--
2	S1819407	AISHWARYA MUDHALE	86%	promoted	--
3	S1819409	AKSHATA CHINCHANE	94%	promoted	--
4	S1819410	AKSHATA KOLI	90%	promoted	--
5	S1819414	AKSHAY SHIRGAVE	94%	promoted	--
6	S1819420	ANJALI PATIL	96%	promoted	--
7	S1819421	ANKITA BABANNAVAR	86%	promoted	--
8	S1819422	ANKITA KUMBAR	87%	promoted	--
9	S1819423	ANURADHA MUDDAPPAGOL	88%	promoted	--
10	S1819424	ANURADHA RAYAGONNAVAR	97%	promoted	--
11	S1819428	ASHWINI JUGALE	92%	promoted	--
12	S1819430	ASMA MULTANI	96%	promoted	--
13	S1819434	CHAITALI KHOT	89%	promoted	--
14	S1819435	DANESHWARI PATIL	90%	promoted	--
15	S1819436	DEVADAS MALABA	85%	promoted	--
16	S1819438	DHANASHRI KUMBHAR	88%	promoted	--
17	S1819440	DIVYA KAMIRE	86%	promoted	--
18	S1819441	DIVYA YELLURE	94%	promoted	--
19	S1819446	HARSHALA PATIL	86%	promoted	--
20	S1819451	KAJAL RAKTADE	93%	promoted	--
21	S1819453	KAPIL NAVANALE	89%	promoted	--
22	S1819454	KASHISH AWATI	86%	promoted	--
23	S1819461	LAXMI MANTUR	92%	promoted	--
24	S1819462	LAXMI VADAGOLE	93%	promoted	--
25	S1819468	MANASI GURAV	86%	promoted	--
26	S1819469	MANISHA KHOT	89%	promoted	--
27	S1819470	MANJULA BHAVE	95%	promoted	--
28	S1819473	MANJUSHREE SATTIGERI	89%	promoted	--
29	S1819474	S1819474	95%	promoted	--
30	S1819477	MEGHA KUMBHAR	90%	promoted	--
31	S1819478	MEGHA SABALE	90%	promoted	--
32	S1819479	MISABA JAMADAR	92%	promoted	--
33	S1819482	NEHA MORE	97%	promoted	--
34	S1819495	POOJA GALATAGE	90%	promoted	--
35	S1819502	PRANALI KHARAPE	93%	promoted	--
36	S1819504	PRATIBHA ALURI	85%	promoted	--
37	S1819508	PRATIKSHA BHADARGADE	90%	promoted	--

K.L.E.'s G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

38	S1819511	PREETI MATIWADE	86%	promoted	--
39	S1819512	PRERANA NANDANI	86%	promoted	--
40	S1819521	RESHMA ARJUNWADE	89%	promoted	--
41	S1819527	ROHAN PATIL	89%	promoted	--
42	S1819530	RUDRAGOUDA PATIL	91%	promoted	--
43	S1819533	RUTUJA ARJUNWADE	91%	promoted	--
44	S1819535	RUTUJA DESAI	86%	promoted	--
45	S1819536	RUTUJA MALABA	90%	promoted	--
46	S1819538	RUTUJA PAWAR	86%	promoted	--
47	S1819544	SAKSHATA JATRATE	88%	promoted	--
48	S1819548	SANA MULLA	88%	promoted	--
49	S1819550	SANMATI IRAGAR	86%	promoted	--
50	S1819552	SATAGOUDA RAMANAKATTI	88%	promoted	--
51	S1819554	SEEMA KULKARNI	92%	promoted	--
52	S1819558	SHEFALI MAGDUM	86%	promoted	--
53	S1819560	SHIVALING GOTURE	87%	promoted	--
54	S1819562	SHIVANI NARAYANRAO NIMBAL	90%	promoted	--
55	S1819563	SHIVANI SUTAR	87%	promoted	--
56	S1819564	SHIVAPRASAD TOLI	92%	promoted	--
57	S1819569	SHRIKANT MALI	100%	promoted	--
58	S1819570	SHRINATH WADDAR	89%	promoted	--
59	S1819571	SHRIYA DESAI	90%	promoted	--
60	S1819572	SHRUTI KAMATE	89%	promoted	--
61	S1819573	SHRUTI KUMBAR	96%	promoted	--
62	S1819574	SHRUTI PATIL	90%	promoted	--
63	S1819575	SHRUTI PATIL	90%	promoted	--
64	S1819576	SHRUTI YALAGOUDANAVAR	94%	promoted	--
65	S1819577	SHRUTIKA JADHAV	95%	promoted	--
66	S1819579	SHRUTIKA TELE	93%	promoted	--
67	S1819581	SHUBHAM KAMATE	85%	promoted	--
68	S1819583	SHUBHAM ROTE	88%	promoted	--
69	S1819586	SHUBHANGI SHANDAGE	90%	promoted	--
70	S1819589	SIDDHI PATIL	87%	promoted	--
71	S1819593	SNEHA PATIL	85%	promoted	--
72	S1819595	SNEHAL PATIL	91%	promoted	--
73	S1819597	SONALI MADIWAL	92%	promoted	--
74	S1819603	SUCHITA SAPAGALE	87%	promoted	--
75	S1819604	SUKANYA CHOUGALA	94%	promoted	--
76	S1819605	SUKANYA SINGANAVAR	86%	promoted	--
77	S1819606	SUMIT MURGUDE	93%	promoted	--
78	S1819607	SUPRIYA KHOT	88%	promoted	--
79	S1819609	SURAJ KHOT	92%	promoted	--
80	S1819613	VAIBHAVI PARIT	87%	promoted	--
81	S1819614	VAISHANAVI MORE	86%	promoted	--
82	S1819616	VARSHA JADHAV	88%	promoted	--
83	S1819617	VARSHA KORE	87%	promoted	--
84	S1819619	VIDYA JAYAKAR	86%	promoted	--

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85	S1819620	VIJAY CHOUGALA	88%	promoted	--
86	S1819622	VINAYAK KORI	91%	promoted	--
87	S1819624	VISHWANATH RUDRAGOUDAR	92%	promoted	--
88	S1819625	VIVEK DHAMANEMATH	92%	promoted	--
89	S1819628	YASHODA KAJAVE	100%	promoted	--
90	S1836489	UMMESALMA M. TAMBOLI	93%	promoted	--

As per RCU direction B.Sc - 4th Sem students are promoted to higher class

CONVENOR

H.O.D

Head
Department of Chemistry
K.L.E's G. I. B. College, Nipani.

PRINCIPAL

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K.L.E. Society's
G. I. Bagewadi College, Nipani.

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE
NIPANI

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NIPANI- 591237 Dist:- Belgavi

[Reaccredited at 'A' level by NAAC with CGPA-3.35] Ph-08338220116

e-mail- kle_gibnpn.yahoo.in.

IQAC INITIATIVE

Advance Learner FOR V & VI SEMESTER B.SC. - NOVEMBER 2018-19

si.no	Register no	Name of the student	% at entry (5th sem)	% at exit (6th sem)	growth
1	S1717609	AISHWARYA PADRE	92%	92%	YES
2	S1717613	AISHWARYA Awade	76%	fail	NO
3	S1717621	ARUNA HEGADE	84%	87%	YES
4	S1717622	ASHWINI PATIL	84%	92.00%	YES
5	S1717625	BHAGYASHRI BEDKIHALE	79%	86.00%	YES
6	S1717627	CHAITALI SADALAGE	91%	87.00%	NO
7	S1717630	DAKSHA PATEL	92%	88.00%	NO
8	S1717633	DEEPA KEDARSHETTI	73%	89.00%	YES
9	S1717636	DILSHAD MULLA	84%	88%	YES
10	S1717644	JYOTI BAGADE	84%	82%	NO
11	S1717645	JYOTI PATIL	76%	**85%**	YES
12	S1717650	KAVYA MANE	82%	**84%**	YES
14	S1717657	LAXMI KHOT	77%	**81%**	YES
15	S1717658	LAXMI SANSUDDI	85%	90%	YES
16	S1717663	MANASI BAVADEKAR	82%	83%	YES
17	S1717665	MANSOORA MOMIN	92%	86%	NO
18	S1717667	MAYURI BABAR	84%	85%	YES
19	S1717668	MAYURI SADALAGE	80%	85%	YES
20	S1717672	MUSKAN SHEKHAJI	100%	90%	NO
21	S1717677	NIKHITA HAVALE	82%	86%	YES
22	S1717680	NUTAN SALUNKHE	82%	80%	NO
23	S1717682	PARVATI CHOUGULE	84%	84%	YES
24	S1717686	POOJA JADHAV	90%	84%	NO
25	S1717688	POOJA MAGADUM	84%	87%	YES
26	S1717690	POOJA PATIL	84%	82%	NO
27	S1717691	POONAM KHOT	87%	90%	YES
28	S1717692	PRADNYA BHIVASHE	96%	90%	NO
29	S1717695	PRAMEELA SHETTY	86%	76%	NO
30	S1717702	PRATIKSHA SURYAVANSHI	94%	88%	NO
31	S1717707	PRIYANKA MAHAJAN	81%	89%	YES
32	S1717710	PRUTHVIRAJ PATIL	84%	88%	YES
33	S1717712	PUSHPADANT UPADHYE	93%	87%	NO
34	S1717713	RACHANA TANDALE	90%	88%	NO
35	S1717714	RAHUL HOSURI	92%	81%	NO
36	S1717722	RUSHIKESH GHATAGE	94%	78%	NO
37	S1717723	RUTUJA PATIL	94%	83%	NO
38	S1717729	SAMIKSHA GEBISE	84%	86%	YES
40	S1717733	SANJEEVINI HASURE	86%	85%	NO

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

41	S1717736	SARIKA SWAMI	84%	87%	YES
42	S1717737	SAVITA PATHADE	92%	93%	YES
43	S1717741	SHAMBALA KUMBHAR	89%	90%	YES
44	S1717743	SHITALHUJARE	83%	86%	YES
46	S1717749	SHUBHANGI KESARKAR	79%	81%	YES
47	S1717751	SIDDHANT SHINGADI	80%	54%	NO
48	S1717753	SONALI BHARADE	91%	89%	NO
49	S1717754	SONALI JAIN	75%	75%	YES
50	S1717756	SOUJANYA KAMATE	85%	90%	YES
51	S1717757	SOUNDARYA PATIL	94%	96%	YES
52	S1717765	SUSHMA ANKAL	86%	90%	YES
53	S1717766	SWAPNA GORAWADE	68%	68%	YES
54	S1717769	TANUJA A ADISERI	78%	91%	YES
55	S1717770	TEJASWINI PATIL	80%	88%	YES
56	S1717771	UMESH PUJARI	81%	89%	YES
57	S1717772	UMMESALMA MULLA	90%	91%	YES
58	S1717778	ZAINABI LANGOTI	76%	79%	YES

CONVENOR

H.O.D.
Head
Department of Chemistry
K.L.E's G. I. B. College, Nipani.

PRINCIPAL
PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE
NIPANI

K. L.E. Society's
G.I. Bagewadi Arts, Science & Commerce College, Nipani
DEPARTMENT OF MATHAMETICS
Programmes conducted for Advance learners

At the beginning of the academic year we prepare the list of *Advance learners* by considering their previous semester result, usually more than 80% we consider as *advance learners*. For those students we conduct following activities.

- Topper as a teacher
- Extra book facility
- Helping slow learners in solving old question papers
- Encourage them to solve question banks and old question papers
- Encourage them to participate and present papers in National and International seminars/conferences
- PG toppers used to engage one or two subtopics to UG students
- Encourage them to give seminars on PPT
- Felicitating rank holders
- Awarding cash prizes to centum scores and highest scorers in Mathematics

Outcome: Above activities are helpful to students to expose and increase their knowledge towards academic growth.

G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI K.L.E.S

Head
Department of Mathematics
K.L.E's G. I. B. College. Nipani.

PRINCIPAL
K. L. E. Society's
G. I. Bagewadi College, Nipani.

K. L.E. Society's

G.I. Bagewadi Arts, Science & Commerce College, Nipani

DEPARTMENT OF MATHEMATICS

Students List of Advance learners for the year 2019-20

B.Sc. I & II Semester Semester

S.No.	R. No	REG.NO.	Name of the student	% at entry level	% at exit level
1	129	S1919603	Miss Swati Murkunde	70	75.5
2	195	S1919590	Sujata Utagi	96.1	88.5
3	127	S1919591	Sushay Sukumar padre	92.5	99
4	35	S1919401	A I Nafeesabanu	91.5	84
5	194	S1919564	Shubham Dundappa Vatagude	90.5	79
6	185	S1919441	Girija Suresh Kulkarni	90.5	75
7	120	S1919584	Mr.Sourabh P Madhale	89.66	74
8	83	S1919503	Mr Prathamesh Sanjay Dongare	87	47
9	49	S1919429	Ashwini S Dodabhangi	86.5	75.5
10	104	S1919540	Sahana A Mathapati	85.66	77
11	61	S1919459	Mahalaxmi R Mane	85.5	94.5
12	94	S1919527	Miss. Rohini Ramesh Sankpal	85.33	59.5
13	127	S1919594	Mr. Sunil S Gaded	84	63
14	111	S1919558	Shreya Sanadi	83.8	86.5
15	68	S1919472	Namrata J Yamakanmarde	83	63.5
16	64	S1919465	Mr Manjunath Khot	82.66	63
17	56	S1919448	Kavita Utagi	81.5	78
18	40	S1919414	Akshata Kolekar	81.5	F
19	216	S1919581	Sourabh Chougule	80.66	F
20	91	S1919520	Mr Rakesh D Kudale	80.66	58.5
21	112	S1919562	Miss Shruti Rajakumar Thane	80	86
22	126	S1919596	Mr Suraj Pattanakude	80	57.5

Maintenance of growth = $\frac{5}{22}$ x 100 = 22.72%

HOD

Head

Department of Mathematics

K.L.E's G. I. B. College, Nipani.

G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI K.L.E's

PRINCIPAL

K.L.E. Society's

G. I. Bagewadi College, Nipani.

K. L.E. Society's

G.I. Bagewadi Arts, Science & Commerce College, Nipani

DEPARTMENT OF MATHEMATICS

Students List of Advance learners for the year 2019-20

B.Sc. III & IV Semester

Sr. No.	R.No.	REG.NO.	Name of the student	% at entry level	% at exit level
1	68	S1819408	Akanksha Hajare	85.00	78.5
2	72	S1819422	Ankita Kumbar	90.45	92.5
3	75	S1819424	Ashika Shivapure	86.00	93.5
4	102	S1819469	Manisha Khot	85.5	86
5	106	S1819477	Megha Kumbhar	90.00	88.5
6	108	S1819479	Misaba Jamadar	93.00	93
7	144	S1819554	Seema Kulkarni	88.00	84.5
8	150	S1819564	Shivaprasad Toli	89.5	89.5
9	151	S1819569	Shrikant Mali	89.00	87
10	153	S1819571	Shreya Desai	93.5	91
11	154	S1819573	Shruti Kumbar	99.00	87.5
12	161	S1819586	Shubhangi Shandge	98.00	95.5
13	166	S1819597	Sonali Madiwal	85.00	84.5
14	169	S1819604	Sukanya Chougala	96.00	97
15	170	S1819606	Sumit Murgude	89.00	96.5
16	172	S1819609	Suraj Khot	89.00	79
17	181	S1819628	Yashoda Kajave	98.00	95.5

Maintenance of growth = $\frac{7}{17}$ x100 = 41.17%

HOD

Head

Department of Mathematics

K.L.E's G. I. B. College, Nipani.

G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI K.L.E's

PRINCIPAL

PRINCIPAL

K.L.E. Society's

G. I. Bagewadi College, Nipani.

K. L.E. Society's

G.I. Bagewadi Arts, Science & Commerce College, Nipani

DEPARTMENT OF MATHEMATICS

Students List of Advance learners for the year 2019-20

B.Sc. V & VI Semester

Sr. No.	R.No.	REG.NO.	Name of the student	% at entry level	% at exit level
1	3	S1717606	Aishwarya Mali	87.00	65
2	6	S1717609	Aishwarya Padare	96.00	95.33
3	15	S1717621	Aruna Hegade	89.00	68
4	203	S1717622	Ashwini Patil	88.5	74.66
5	204	S1717625	Bhagyashri Bedakihale	86.5	77.66
6	21	S1717627	Chaitali Sadalage	92.00	79.66
7	23	S1717633	Deepa Kedarshetti	91.5	85.66
8	25	S1717635	Deepali Patil	85.5	71.66
9	26	S1717636	Dilshad Mulla	96.00	93.00
10	30	S1717644	Jyoti Bagade	88.00	72.66
11	31	S1717645	Jyoti Patil	88.00	80.00
12	38	S1717657	Laxmi Khot	94.00	91.00
13	39	S1717658	Laxmi Sansudi	96.00	92.33
14	43	S1717663	Manasi Bavadekar	93.00	71.33
15	45	S1717665	Mansoora Momin	94.5	80.33
16	46	S1717667	Mayuri Babar	89.00	82.00
17	47	S1717668	Mayari Sadalage	90.00	74.66
18	49	S1717672	Muskan Shekhaji	96.00	91.00
19	51	S1717680	Nutan Salunke	88.5	84.33
20	52	S1717682	Parvati Chougule	93.5	78.66
21	55	S1717686	Pooja Jadhav	89.00	84.66
22	56	S1717688	Pooja Magadum	90.5	83.33
23	58	S1717690	Pooja Patil	91.5	87.00
24	59	S1717691	Poonam Khot	89.00	83.00
25	60	S1717692	Pradnya Bhivase	98.00	95.66
26	64	S1717704	Preana Potajale	85.00	87.33
27	87	S1717753	Sonali Bharade	93.5	97.66
28	90	S1717757	Soundarya Patil	94.00	93.66
29	97	S1717766	Swapna Gorawade	92.5	83.00

30	99	S1717769	Tanuja Adiseri	94.5	79.66
31	100	S1717770	Tejashwini Patil	97.00	80.66
32	101	S1717772	Ummesalam Mulla	100	87.66

Maintenance of growth = $\frac{2}{32} \times 100 = 6.25\%$

HOD
Head
Department of Mathematics
K.L.E's G. I. B. College, Nipani.

KLE's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

K. L.E. Society's
G.I. Bagewadi Arts, Science & Commerce College, Nipani
DEPARTMENT OF BOTANY
2019-20
Programs arranged for slow learners

1. Special attension is given to improve performance of slow learners .
2. Slow learners are identified on the basis of result of the semester.
3. Class Tests:
   In each semester two class tests are conducted to check the students understanding.This practice imparts a habit of regular studies. A result analysis is performed and on that basis and retest is conducted for students who either fail or remain absent.
4. Continuous Assessment:
   The term work is assessed continuously. The total term work generally consist of 10 marks. The total term work conducted on a day is assessed by the faculty on the same day and marks are assigned right in front of the student thereby maintaining transparency.
5. Providing question banks to the students
6. repeating the learning points to the students.
7. Guide students to the main points and tests,
8. Teaching reading skills to the students.
9. Study schedule given to the students and giving home work for the students

Programs arranged for Advance learners

1. High performing students are identified on the basis of internal assessment, university Examination,involment in classroom activities .
2. Advising students to participate in seminar,quizzes to develop their stage courage & improve their presentation skills.
3. Students are also provided opportunites to develop their creativity by participating National level conference.
4. Bright students are motivated & inspired to get university ranks.
5. Students are encouraged to take part in projects.
6. Students are encouraged to take up competitive Exams. They are provided with question papers of previous year various competitive exams.

NIPANI

HOD
HEAD
Department of Botany
G. I. Bagewadi College, Nipani.

IQAC-Coordinator
IQAC Co-ordinator
K.L.E's G. I. B. College, Nipani.

Principal
Principal,
G. I. Bagewadi Arts, Science & Commerce College, NIPANI.

K. L.E. Society's
G.I. Bagewadi Arts, Science & Commerce College, Nipani
DEPARTMENT OF BOTANY
Students List of Advance learners for the year 2019-20

B. Sc. I & II Semester

Sl.No	Register No.	Name of the Student	% at Entry	% at Exit
1	S1919424	Ankita Patil	83.00	NO Exams Due to covid-19
2	S1919487	Padmaja Dodannavar	82.00	
3	S1919507	Pratiksha Kumbhar	82.00	
4	S1919553	Shital Patil	82.00	
5	S1919575	Somesh Mali	84.00	
6	S1919613	Vinashree Kole	90.00	
7	S1919583	Sourabh Kenawade	84.00	
8	S1919619	Zeba Bagalkoti	82.00	

B. Sc. III& IV Semester

Sl.No	Register No.	Name of the Student	% at Entry	% at Exit
1	S1819468	Mansi Gurav	80.00	NO Exams Due to covid-19
2	S1819470	Manjula Bhave	92.00	
3	S1819531	Rukmini Talwar	90.00	
4	S1819547	Sana Gadampalli	88.00	
5	S1819599	Soumya Magdum	84.00	
7	S1819617	Varsha Kore	80.00	

B.Sc.V &VI Semester

Sl.No	Register No.	Name of the Student	% at Entry	% at Exit
1	S1717687	Pooja Kesarkar	83.00	81.00
2	S1717713	Rachana Tandale	80.00	84.00
3	S1717712	Pushpadant Upadhye	86.00	71.00
4	S1717723	Rutuja Patil	86.00	87.00
5	S1717724	Sabil Makandar	82.00	76.00
6	S1717725	Sachin Badakar	82.00	74.00
7	S1717729	Samiksha Gibise	82.00	81.00
8	S1717733	Sanjeevini Hasure	95.00	87.00
9	S1717740	Shahida Desai	82.00	89.00
10	S1717739	Seema Datawade	81.00	80.00
11	S1717756	Soujanya Kamate	82.00	84.00
12	S1717768	Swati Tawadare	81.00	85.00
13	S1717771	Umesh Pujari	81.00	82.00

Growth maintained =53%

K.L.E.'s G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

HOD
Head
Department of Botany
K.L.E's G. I. B. College, Nipani.

IQAC-Coordinator
IQAC Co-ordinator
K.L.E's G. I. B. College, Nipani.

Principal
Principal,
G. I. Bagewadi Arts, Science &
Commerce College, NIPANI.

K. L.E. Society's
G.I. Bagewadi Arts, Science & Commerce College, Nipani
DEPARTMENT OF ZOOLOGY
Students List of Advance learners for the year 2019-20
B. Sc. I & II Semester

Sl.No	Register No.	Name of the Student	% at Entry	% at Exit
1	S1919424	Ankita Patil	83.00	NO Exams
2	S1919487	Padmaja R Dodannavar	82.00	Due to covid-19
3	S1919507	Pratiksha B Kumbhar	82.00	
4	S1919553	Shital S Patil	82.00	
5	S1919575	Somesh Appasab Mali	84.00	
6	S1919613	Vinashree Kole	90.00	
7	S1919583	Sourabh Arun Kenawade	84.00	
8	S1919619	Zeba Bagalkoti	82.00	

B. Sc. III& IVsemester

Sl.No	Register No.	Name of the Student	% at Entry	% at Exit
1	S1819468	Mansi S. Gurav	80.00	NO Exams
2	S1819470	Manjula Bhave	92.00	Due to covid-19
3	S1819531	Rukmini B Talwar	90.00	
4	S1819547	Sana Gadampalli	88.00	
5	S1819599	Soumya Magdum	84.00	
7	S1819617	Varsha R. Kore	80.00	

G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI K.L.E.'s

B.SC V&VI

Sl.No	Register No.	Name of the Student	% at Entry	% at Exit
1	S1717687	Pooja Kesarkar	83.00	81.00
2	S1717713	Rachana Tandale	80.00	84.00
3	S1717712	Pushpadant Upadhye	86.00	71.00
4	S1717723	Rutuja Patil	86.00	87.00
5	S1717724	Sabil Makandar	82.00	76.00
6	S1717725	Sachin Badakar	82.00	74.00
7	S1717729	Samiksha Gibise	82.00	81.00
8	S1717733	Sanjeevini Hasure	95.00	87.00
9	S1717740	Shahida Desai	82.00	89.00
10	S1717739	Seema Datawade	81.00	80.00
11	S1717756	Soujanya Kamate	82.00	84.00
12	S1717768	Swati Tawadare	81.00	85.00
13	S1717771	Umesh Pujari	81.00	82.00

HOD
Head
Department of Zoology
K.L.E's G. I. B. College, Nipani

PRINCIPAL
G.I. Bagewadi Arts, Science & Commerce College, NIPANI.

K.L.E.'s G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE
NIPANI

K. L.E. Society's
G.I. Bagewadi Arts, Science & Commerce College, Nipani

DEPARTMENT OF COMMERCE

Programmes conducted for Advance learners

At the beginning of the academic year we prepare the list of *Advance learners* by considering their previous semester result, usually more than 80% we consider as *advance learners.* For those students we conduct following activities.

- Topper as a teacher
- Extra book facility
- Helping slow learners in solving old question papers
- Encourage them to solve question banks and old question papers
- Encourage them to participate and present papers in National and International seminars/conferences
- PG toppers used to engage one or two subtopics to UG students
- Encourage them to give seminars on PPT
- Felicitating rank holders

Outcome: Above activities are helpful to students to expose and increase their knowledge towards academic growth.

HOD Head
Department of Commerce
K.L.E's G. I. B. College, Nipani.

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

PRINCIPAL PRINCIPAL
G.I. Bagewadi Arts, Science & Commerce College, NIPANI.

K.L.E. Society's
G.I.Bagewadi Arts, Science and Commerce College, Nipani-591237
Accredited at 'A' level by NAAC with CGPA 3.35
(Affiliated to Rani Channamma University, Belagavi, Karnataka, India)
Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116

DEPARTMENT OF COMMERCE

Students List of Advance learners for the year 2019-20

B.Com. I & II Semester

Sr. No.	Reg. No.	Name of the student	% at Entry level(PUC II)	% Exit level	Growth
01	C1930617	Arati Kate	85.00	91.00	Yes
02	C1930601	Aarushi Rangole	87.00	90.00	Yes
03	C1930602	Abhilasha Koot	86.66	91.00	Yes
04	C1930603	Adarsha Joke	82.71	91.57	Yes
05	C1930605	Akshata Bhivashe	81.29	91.00	Yes
06	C1930614	Anjali A Muradande	86.16	90.00	Yes
07	C1930619	Archana Patil	85.16	79.71	No
08	C1930620	Ashish Vhanawade	85.50	84.71	No
09	C1930622	Asmita Mengane	93.16	96.00	Yes
10	C1930611	Anagha Mohite	85.29	93.00	Yes
11	C1930617	Aarati Kote	80.29	91.00	Yes
12	C1930629	Ganesh Deshinge	86.83	87.00	Yes
13	C1930636	Jyoti R Puthane	87.66	90.00	Yes
14	C1930637	Jyoti Subedar	89.33	93.57	Yes
15	C1930645	Komal Gadakari	95.83	95.00	No
16	C1930646	Komal Kallimani	89.00	93.00	Yes
17	C1930648	Mahadev Gorade	83.00	85.71	Yes
18	C1930652	Manjunath Gavade	89.83	86.86	No
19	C1930624	Kaveri Boragalli	85.29	93.00	Yes
20	C1930657	Nidhi Chauhan	94.00	82.14	No
21	C1930660	Nishya P Firagannavar	82.30	80.71	No

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

22	C1930664	Padmavati Shillepatil	80.00	90.00	Yes
23	C1930669	Prajwal Patil	83.83	85.00	Yes
24	C1930676	Priya Karambale	82.00	92.00	Yes
25	C1930679	Revati Naik	81.83	88.00	Yes
26	C1930680	Ritika Palase	84.00	87.00	Yes
27	C1930682	Rushikesh Patil	85.33	92.00	Yes
28	C1930685	Rutuja Powar	85.83	92.71	Yes
29	C1930695	Sabiya A Mujavar	88.16	91.00	Yes
30	C1930687	Sahil Shrikhande	92.50	93.00	Yes
31	C1930691	Sandeep Masti	85.16	84.00	No
32	C1930692	Sandhya S Patil	86.83	86.00	No
33	C1930694	Saraswati S Kamagouda	85.33	80.57	No
34	C1930699	Sheetal Badiger	90.33	Result Withheld	No
35	C1930697	Shraddha Kale	85.16	90.00	Yes
36	C1930702	Shreya Khot	95.5	93.00	No
37	C1930705	Shruti S Kote	89.16	89.00	No
38	C1930611	Anagha Mohite	89.66	93.00	Yes
39	C1930709	Siddharth Chandagade	89.16	88.86	No
40	C1930713	Sourabh Mane	81.00	87.57	Yes
41	C1930722	Suraj Magadum	81.33	78.57	No
42	C1930731	Vinayak R Mangure	87.16	88.00	Yes
43	C1930733	Vinod P Hipparagi	85.00	82.57	No
44	C1930735	Vrushali R Akiwate	88.00	89.00	Yes
45	C1930736	Yallaling D Khot	89.83	86.00	No

Maintenance of Growth = 30/45 * 100 = 66.67%

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

H.O.D
Head
Department of Commerce
K.L.E's G. I. B. College, Nipani.

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DEPARTMENT OF COMMERCE

Students List of Advance learners for the year 2019-20

B.Com. III & IV Semester

Sr. No.	Reg. No.	Name of the student	% at Entry level	%Exit level	Growth
01	C1830207	Arihant Havale	87.63	91.62	Yes
02	C1830210	Ashwini Halagadagi	84.38	93.00	Yes
03	C1830216	Deepali Chilayi	82.63	91.25	Yes
04	C1830217	Gayatri Kharade	81.25	89.5	Yes
05	C1830221	Kaveri Divate	85.00	93.00	Yes
06	C1830222	Kiran Naik	88.00	95.00	Yes
07	C1830225	Laxmi Patil	85.25	92.65	Yes
08	C1830228	Mahesh Pratap	81.50	89.62	Yes
09	C1830229	Prajakta Malaganve	88.50	95.00	Yes
10	C1830243	Prajyoti Khot	81.38	91.12	Yes
11	C1830257	Rohini Karade	81.00	90.00	Yes
12	C1830261	Rutuja S Walake	88.75	95.00	Yes
13	C1830273	Shubham Pachandi	86.25	94.00	Yes
14	C1830274	Shubham Thane	93.13	97.00	Yes
15	C1830275	Soujanya Patil	88.88	94.62	Yes
16	C1830278	Sujata Kamble	82.75	92.37	Yes
17	C1830284	Vidya Kesti	80.50	90.62	Yes
18	C1830286	Vijaya Jadhav	81.13	90.25	Yes
19	C1823082	Siddhagonda Patil	84.25	91.38	Yes

Maintenance of Growth = 19/19 * 100 = 100%

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

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DEPARTMENT OF COMMERCE

Students List of Advance learners for the year 2019-20

B.Com. V & VI Semester

Sr. No.	Reg. No.	Name of the student	% at Entry level	%Exit level	Growth
01	C1730201	Aishwarya Vhanawade	90.86	87.62	No
02	C1730205	Ambuja Jadhav	81.29	78.62	No
03	C1730208	Amruta Kulkarni	90.57	85.11	No
04	C1730209	Ankita Koot	88.29	84.43	No
05	C1730210	Ankita Patil	92.71	89.22	No
06	C1730211	Annapurna Kadam	84.29	79.14	No
07	C1730212	Apoorva Kamate	95.00	92.73	No
08	C1730214	Ashwini Malage	80.29	74.43	No
09	C1730215	Ashwini Nasalapure	90.08	90. 8	Yes
10	C1730216	Basavaraj Kabadagi	83.14	80.65	No
11	C1730217	Darshan Dandage	90.00	87.43	No
12	C1730218	Nikita Dhadake	87.41	87.71	Yes
13	C1730223	Jooli Havale	88.00	85.30	No
14	C1730224	Jyoti Chavan	88.86	82.76	No
15	C1730228	Kiran Mamadapure	85.57	78.35	No
16	C1730230	Kirti Patil	87.14	84.86	No
17	C1730238	Netra Hegade	87.57	84.32	No
18	C1730242	Omkar Koot	83.14	83.86	Yes
19	C1730243	Padmashri Chinchawade	88.86	84.81	No
20	C1730244	Padmini Khot	83.71	80.16	No
21	C1730248	Pooja R Patil	88.29	87.41	No

K.L.E.S's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

22	C1730249	Pooja S Patil	83.14	79.76	No
23	C1730252	Pranita Bharmal	85.00	80.89	No
24	C1730254	Prarthana Rodd	81.71	78.76	No
25	C1730257	Rajvardhan Patil	80.43	79.65	No
26	C1730259	Rohini Kokane	87.71	85.41	No
27	C1730260	Rohini Miraje	86.29	87.03	Yes
28	C1730265	Rutuja Chougule	83.14	85.46	Yes
29	C1730266	Saba Bagban	80.29	78.86	No
30	C1730285	Snehal Patil	93.29	92.08	No
31	C1730286	Soniya Bhosale	86.14	85.24	No
32	C1730291	Sukanya Malloli	81.29	82.81	Yes
33	C1730292	Suraksha Aswale	93.14	85.57	No
34	C1730296	Swati Hokale	85.00	85.11	Yes
35	C1730297	Sweta Bachate	83.57	76.62	No
36	C1730302	Veena Alatagi	91.71	90.46	No
37	C1529411	Asmita S Gumthannavar	87.43	82.84	No

Maintenance of Growth = 7/37 * 100 = 18.92%

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

H.O.D
Head
Department of Commerce
K.L.E's G. I. B. College, Nipani.

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

K. L. E Society's

G. I. Bagewadi Arts, Science & Commerce College, Nipani

Department of Economics

Students List of Advance Learners for the year 2019-20

B.A I & II semester

Sl. No	Name of the Student	% at Entry PU	% at Exit II Sem
1	Vishal Khot	87.00	Waiting
2	Beera Gavade	80.00	Do
3	Laxmi Karegar	84.00	Do
4	Mahadev Pujeri	85.00	Do
5	Prathami Kamble	80.00	Do
6	Priyanka Gandagudi	86.00	Do
7	Rahul Roge	91.00	Do
8	Sachin Malage	81.00	Do
9	Sagar Biku	80.00	Do
10	Shweta Kamble	86.00	Do
11	Sonali Khot	85.00	Do
12	Apeksha Mahajan	94.00	Do

Maintenance of Growth =

Students List of Advance Learners for the year 2019-20

B.A III & IV Semester

Sl. No	Name of the Student	Score in % III Semester	Score in % IV Semester
1	Prem Sanadi	91.00	Waiting.
2	Saraswati Ranage	92.00	Do
3	Ashwini Mane	93.00	Do
4	Arifa Gidde	83.00	Do
5	Gangadhar Kadapure	88.00	Do
6	Jyoti Mane	80.00	Do
7	Laxmi Chauhan	87.00	Do
8	Pooja Patil	83.00	Do
9	Seema Goture	93.00	Do

Maintenance of Growth =

H.O.D.
Dept. of Economics
G.I Bagewadi College, Nipani

G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI K.L.E.S

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

Students List of Advance Learners for the year 2019-20

B.A V & VI Semester

Sl. No	Name of the Students	% at Entry V Sem	% at Exit VI Sem
1	Deepali Shingale	93.00	93.00
2	Maruti Gavade	90.00	87.5
3	Annapurneshwari Mudhale	83.00	81.5
4	Firojkhan Khanu	83.00	84.0
5	Gayatri PAtil	81.00	88.5
6	Gajala Gavandi	92.00	92.0
7	Komal Patil	92.00	91.0
8	Pallavi Kurade	85.00	83.0
9	Sangita Mali	84.00	86.5
10	Snehal Mangasule	83.00	82.0

Maintenance of Growth = 4/6 x 100 =66.66

H.O.D.
Dept. of Economics
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Ref. No. *Date :*

2.2.1: Topper as a Teacher

2019-20

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Department of Mathematics

Topper as a Teacher 2019-20

S. No.	Name of the student	Class	Class Engaged	Topic	Date	Guide
1	Jyoti A. Chavan	M.Sc. III sem	B.Sc. V Sem.	R-K Second Order Method	12/09/2019	JNM
2	Jyoti A. Chavan	M.Sc. III sem	B.Sc. V Sem.	Calculus of Variation	24/09/2019	KBS
3	Jyoti A. Chavan	M.Sc. III sem	B.Sc. V Sem.	Calculus of Variation	25/09/2019	KBS
4	Jyoti A. Chavan	M.Sc. III sem	B.Sc. V Sem.	Calculus of Variation	26/09/2019	KBS
5	Pooja H. Yadhav	M.Sc. III sem	B.Sc. V Sem.	Able's Theorem	03/09/2019	JNM
6	Jyoti A. Chavan	M.Sc. III sem	M.Sc. I Sem.	Able's Formulae & Examples	04/09/2019	JNM
7	Pooja H. Yadav	M.Sc. III sem	B.Sc. V Sem.	Geodesics Calculus of Variation	23/09/2019	KBS
8	Priya Patil	M.Sc. III sem	B.Sc. V Sem.	Gamma Beta function	20/10/2019	GLK
9	Jyoti A. Chavan	M.Sc. IV sem	B.Sc. II Sem.	Pedal Equation in Polar Form	29/02/2020	KBS
10	Jyoti A. Chavan	M.Sc. IV sem	B.Sc. II Sem.	Pedal Equation in Polar Form	02/03/2020	KBS
11	Jyoti A. Chavan	M.Sc. IV sem	B.Sc. II Sem.	Pedal Equation in Polar Form	03/03/2020	KBS

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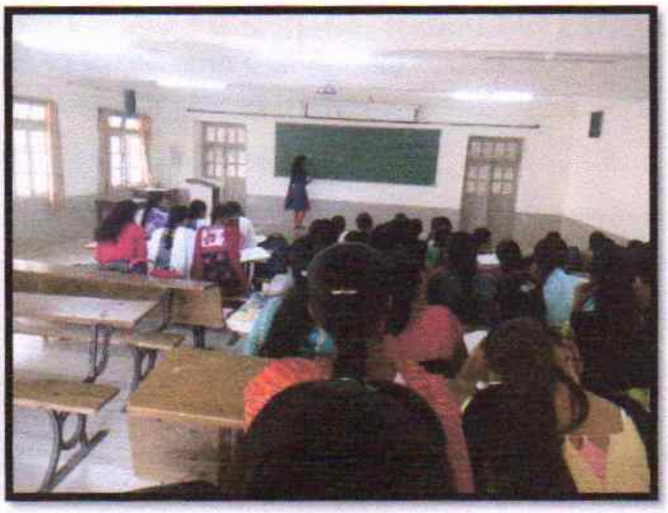
TOPPER AS A TEACHER

Topic: "Properties of beta, gamma functions" Class: B.ScVI Sem

Name: Priya Patil
Class: M.Sc IV Sem

Date:05/10/2019
Time: 11.30 am to 12.30 pm

Topic: Differentiation under Integral sign Class: B.Sc V Sem


Name: Pooja Patil
Class: B.Sc VI Sem

Date:23/07/2019
Time: 4.00 pm to 5.00 pm

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DEPARTMENT OF MATHEMATICS

Topper As Teacher.

SEMINAR

TOPIC: R-K Method

PAPER: II. (Paper – II). (BSc V sem)

NAME: Jyoti. A. Chavan.

CLASS: M.Sc III rd sem

DATE: 12/9/19

GUIDE: Smt. J. N. Magdum.

Runge - Kutta Method. 12/9/19

Euler's method is less efficient in practical method to be small to obtain the reasonable accuracy. Thus R-K Method where designed to give the greater accuracy & they possess the advantage of remaining only the value at the selected point on the sub interval.

2nd order R-K Method:-

letus consider the initial value problem

$$y' = \frac{dy}{dx} = f(x, y) \quad -(1) \quad \text{with } y(x_0) = y_0$$

The first approximation solution of eqn (1) y. at $x = x_1 = x_0 + nh$ is given by

$$y_1 = y_0 + \frac{h}{2} \{ f(x_0, y_0) + f(x_1, y_1^{(0)}) \} \quad -(2)$$

where $y_1^{(0)} = y_0 + h f(x_0, y_0) \quad -(3)$

using (2) & (3) becomes

$$y_1 = y_0 + \frac{h}{2} \{ f(x_0, y_0) + f(x_0 + h, y_0 + h f(x_0, y_0)) \} \quad -(4)$$

put $hf_0 = h f(x_0, y_0) = k_1 \quad -(4)$

put this value in (4)

$$y_1 = y_0 + \frac{h}{2} \{ f(x_0, y_0) + f(x_0 + h, y_0 + k_1) \}$$

$$y_1 = y_0 + \frac{1}{2} \{ k_1 + h f(x_0 + h, y_0 + k_1) \}$$

$$y_1 = y_0 + \frac{1}{2} \{ k_1 + k_2 \}$$

where $k_2 = hf(x_0 + h, y_0 + k_1)$

III'ly second app. of y at $x = x_2$ is given by

$$y_2 = y_1 + \frac{1}{2}\{k_1 + k_2\}$$

where $k_1 = hf(x_1, y_1) = hf_1$

$k_2 = hf(x_1 + h, y_1 + k_1)$

III'ly third app. of y at $x = x_3$ is given by

$$y_3 = y_2 + \frac{1}{2}\{k_1 + k_2\}$$

$k_1 = hf(x_2, y_2) = hf_2$

$k_2 = hf(x_2 + h, y_2 + k_1)$

On continuing this process now y value at $x = x_n = x_0 + nh$

$$y_n = y_{n-1} + \frac{1}{2}\{k_1 + k_2\}$$

where $k_1 = hf_{n-1} = hf(x_{n-1}, y_{n-1})$

$k_2 = hf(x_{n-1} + h, y_{n-1} + k_1)$

$\forall n = 1, 2, 3 \ldots\ldots$

R-K 4th order Method:-

let $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$ — ①

the first app. value of y at $x = x_1 = x_0 + h$ is given by

$$y_1 = y_0 + \frac{1}{6}\{k_1 + 2(k_2 + k_3) + k_4\}$$

where $k_1 = hf_0 = hf(x_0, y_0)$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$$k_3 = h f(x_0 + \frac{h}{2}, \quad y_0 + \frac{k_2}{2})$$

$$k_4 = h f(x_0 + h, \quad y_0 + k_3)$$

Now $\underline{II}^{nd}$ approximation value at $x = x_2$

$$y_2 = y_1 + \frac{1}{6}\{k_1 + 2(k_2 + k_3) + k_4\}$$

$$k_1 = h f_1 = h f(x_1, y_1)$$

$$k_2 = h f(x_1 + \frac{h}{2}, \quad y_1 + \frac{k_1}{2})$$

$$k_3 = h f(x_1 + \frac{h}{2} \quad y_1 + \frac{k_2}{2})$$

$$k_4 = h f(x_1 + h, \quad y_1 + k_3)$$

Continuing this process at $x = x_n$ is given by

$$y_n = y_{n-1} + \frac{1}{6}\{k_1 + 2(k_2 + k_3) + k_4\}$$

$$k_1 = h f_n = h f(x_{n-1}, y_{n-1})$$

$$k_2 = h f(x_{n-1} + \frac{h}{2} \quad y_{n-1} + \frac{k_1}{2})$$

$$k_3 = h f(x_{n-1} + \frac{h}{2} \quad y_{n-1} + \frac{k_2}{2})$$

$$k_4 = h f(x_{n-1} + h, \quad y_{n-1} + k_3)$$

$$\forall n = 1, 2, 3 \ldots$$

Ex: solve $y' = x + y^2$ with $y(0) = 1$ by R-K 2nd order method.

Sol:- let given $y' = x + y^2$ with $y(0) = 1$

$$x_0 = 0, \quad y_0 = 1$$

let $y_n = y_{n-1} + \frac{1}{2}\{k_1 + k_2\}$

for $n = 1$

$$y_1 = y_0 + \frac{1}{2}\{k_1 + k_2\}$$

$k_1 = h f_0 = h f(x_0, y_0) = h f(0, 1)$

Here $h = 0.2$

$k_1 = 0.2 \times 1 = 0.2$

$k_2 = h f(x_0 + h, y_0 + k_1)$

$= 0.2 f(0 + 0.2, 1 + 0.2)$

$= 0.2 f(0.2, 1.2)$

$= 0.328$

$\therefore y_1 = y_0 + \frac{1}{2}\{k_1 + k_2\}$

$= 1 + \frac{1}{2}\{0.2 + 0.328\}$

$= 1.264$

$y_1 = y(x_1) = y(x_0 + h) = y(0 + 0.2)$

$\Rightarrow y(0.2) = \underline{1.264}$

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DEPARTMENT OF PHYSICS

List of Topper As a Teacher

2019-20

Sl.No	Name of the Student	Class	Topoic
1	Aishwarya Padre	Bsc Vst Sem	Geometrical Optics
2	Asma Multani	Bsc II Sem	Diesel Engine
3	Deepali Patil	Bsc IIIrd Sem	Rigid Body
4	Shruti Kumbar	Bsc IIIrd Sem	Elastic Collision
5	Shruti Yalagoudanavar	Bsc IIIrd Sem	Moment of Inertia of a Flywheel
6	Kapali Navanale	Bsc IVthSem	Forced Vibrations

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B.Sc II sem

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NIPANI

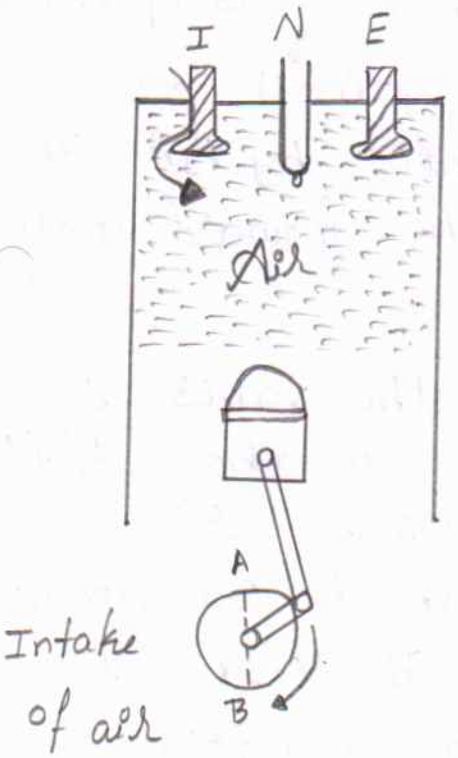
NAME : ASMA MULTANI

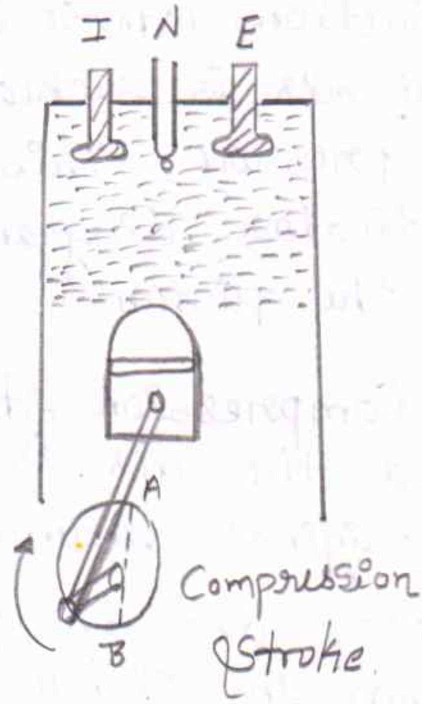
TOPIC : DIESEL ENGINE

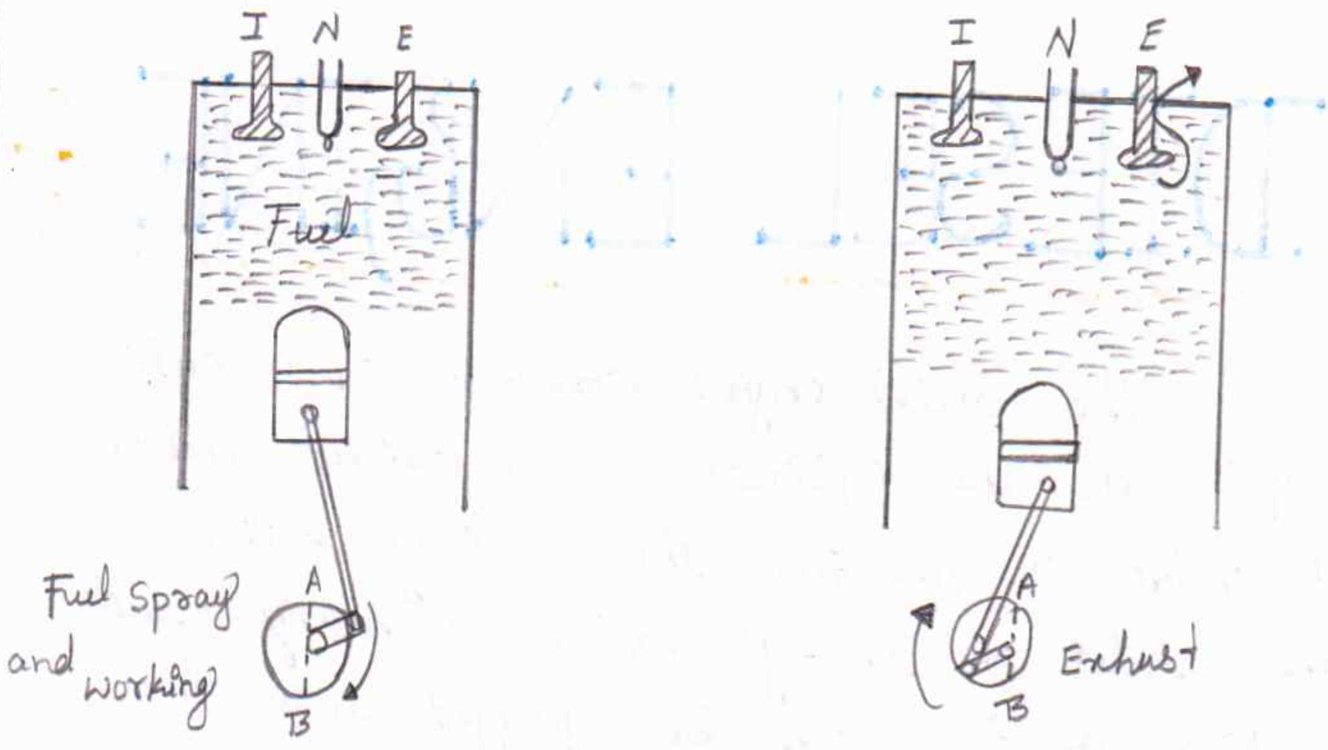
ROLL NO : 78

DIESEL ENGINE :-

The diesel engine consists of a cylinder and piston. The cylinder is provided with inlet valve I for fas the working substance another inlet N through which a heavy crude of acting as fuel can be sprayed into the cylinder, E be the exhaust valve. The operations of these inlets and outlets are controlled by the motions of the piston.





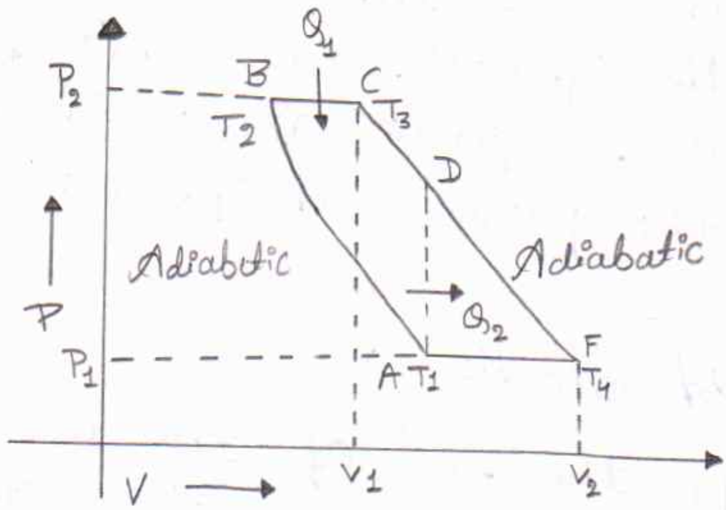


Principle of Working :

The working of the diesel engine is shown in figure. The diesel cycle consist of five operations

1. The Suctions Stroke : The air valve I is opened and pure air is sucked into the cyclinder at constant pressure. This is represented by P_1A on the indicator diagram during the forward moti-on of the piston.

2. The Compression stroke ; All the valves are closed and the air is compressed to about $1/17^{th}$ of its original volume. The temperature rises to about 1000°c and the pressure to about 35 atmosphere. when the piston move from B to A. The curve takes the path AB on the indicator diagram.



3. The injection of oil : The supply value is opened and oil under pressure is sprayed into the cylinder. The oil burns Spontaneously as the tempeature in the cylinder is much above its ignition point. The Pressure during this process is maintained constant by regulating the oil supply. This is respented by BC on the indicator diagram during the second forward motion of the pistion. due to Combustion the tempearture rises to about 2000°c at this Stage the oil supply is cut off.

4. The working Stroke →

All the valve are closed, the gas mixture is allowed to expand adiabatically, the piston moving forward untill the curve CD is described. The mixture must be allowed to expand freely until the pressure drops to the original value respresented by

F. But this would involve a large represented volume for the cylinder. In actual practice the exhaust valve is opened at a stage represented by D and the pressure drops to A.

5. The exhaust Stroke →

At the end of the working stroke cylinder is filled with useless gas which emerge out through the exhaust valve. This is represented by AP_1 during the back -ward motion of the piston. For the next forward motion, the machine start working for the next cycle.

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DEPARTMENT OF COMMERCE

Year : 2019-20

Topper as a Teacher

Sl. No	Date	Name of Student	Class Taken	Topic	Guide
01	10/07/2019	Aarushi Rangole	B.Com I Sem	Departmental Accounts	Smt. P R Kamate
02	13/07/2019	Prajakta Malagave	B.Com III Sem	Solved Problem on Valuation of Shares	Smt. P R Kamate
03	20/07/2019	Jooli Havale	B.Com III Sem	Value of Goodwill	Smt. P R Kamate
Link	https://drive.google.com/file/d/1gF5Oc_cEQy7Zx1JYrN8tFLUKY84YI-aI/view?usp=sharing				
04	21/08/2019	Kaveri Borgalli	B.Com I Sem	Single Entry System	Smt. P R Kamate
05	06/09/2019	Asmita Mengane	B.Com ISem	Royalty Accounts	Shri C V Koppad
Link	https://drive.google.com/file/d/1DzM7fX6reAdbLR-n-BAOOctp2i-hCNui/view?usp=drivesdk				
06	21/09/2019	Ankita Koot	B.Com V Sem	Solved Problem on LIFO	Shri B M Hiremath
07	18/01/2020	Shubham Thane	B.Com IV Sem	Solved Problem on Internal Reconstruction	Smt. P R Kamate

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08	03/02/2020	Komal Gadkari, B.Com II Sem	B.Com II Sem	Branch Accounts	Shri C V Koppad
Link	**https://drive.google.com/file/d/1wGicNzEoNbm0GIUJU1AOXOSvqNEwNHqo/view?usp=drivesdk**				
09	02/03/2020	Veena Alatagi	B.Com VI Sem	Solved Problem on Operating Costing	Shri B M Hiremath

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TOPPER AS A TEACHER

Name: Miss. Veena Alatagi **Date: 02/03/2020**

Topic: Solved Problem on Operating Costing **Class: B.Com VI Sem**

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TOPPER AS A TEACHER

Name: Miss. Prajakta Malagave **Date: 13/07/2019**

Topic: Problem on Valuation of Shares **Class: B.Com III Sem**

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2019-20

KLE's G I Bagewadi College, Nipani

Topper as a Teacher

Seminar Topic	:Departmental Accounts
Name	: Aarushi Rangole
Class Taken	: B.Com I Sem
Guide	: Smt P R Kamate
Date	: 10/07/2019

DEPARTMENTAL ACCOUNTS

Meaning:

A big business concerns dealing in different kinds of goods or services is usually divided into a number of departments.
A business having a number of departments each specializing in a particular line of activity is called departmental undertaking.
Under one management and under one roof the different goods and services are rendered is called departmental undertaking.
The accounts relating to different goods or services are called departmental accounting

NEED FOR DEPARTMETNAL UNDERTAKING (OBJECTIVES OR ADVANTAGES):

1. To ascertain the result of each department
2. To compare the trading result of one department with the another department
3. To take necessary steps either to improve the department which is under loss or to close down all together the department which is under loss.
4. To evaluate the performance of each department
5. To reward the departmental managerial staff on the basis of trading results.
6. To have effective managerial control over the working of each department

APPORTIONMENT OR ALLOCATION OF COMMON EXPENSES:

Apportionment of expenses means allocation of common expenses among the different departments on suitable basis

APPORTIONMENT OR ALLOCATION OF COMMON EXPENSES:

Sl. No	BASIS OF ALLOCATION	COMMON EXPENSES
01	Net purchase ratio (Total purchases minus Purchase returns or returns outwards)	Carriage inwards, fright, octroi, duty etc
02	Net sales ratio. (Total sales minus sales returns or returns inwards)	Commission on sales, discount allowed, carriage outwards, bad debts, RDD, RFDD, advertisement, sales tax etc
03	Staff appointed ratio (No of employees)	Salary, wages, labour welfare, canteen expenses etc.
04	Space or area occupied	Rent, rates, taxies, insurance on building, repairs of building, depreciation on building etc.
05	Closing stock ratio	Insurance on goods, godown rent etc
06	Time spent or time devoted ratio	Salary of works manager
07	Value of assets ratio	Depreciation, repairs, maintenance of assets
08	Units consumed ratio	Lighting and heating, power, motive power, electricity etc.

IMPORTANT NOTES:

1. If the basis for allocation in not given in the problem, then those expenses should be apportioned on the basis of sales ratio (turnover ratio)
2. However, there are certain common indirect expenses which cannot be apportioned on any one of the above basis. Such expenses should be directly recorded in the General Profit and Loss Account, which is meant for the entire organization. Such expenses are bank interest, accountancy charges, audit fees, income tax, and insurance on comprehensive policy.

APPORTIONMENT OR ALLOCATION OF COMMON Incomes:

[illegible]	[illegible]	[illegible]
01	Net purchase ratio	Discount received, commission earned
02	According to given ratio or allocated equally	Interest from bank, interest on investments etc.

INTERDEPARTMENTAL TRANSFER OR TRANSACTIONS:

Transfer of goods and services from one department to another department is called interdepartmental transactions.

Treatment:

1. Interdepartmental transfer of goods from one department to another department is appearing in the trading account. The entry is:
   Receiving Department A/c ....Dr
   To Giving Department A/c
2. Interdepartmental transfer of service from one department to another department is appearing in the profit and loss account. The entry is:
   Receiving Department A/c ....Dr
   To Giving Department A/c

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ii. Project assigned to advance learners

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List of Projects/Field Visits by Advanced Learners in the Year 2019-20

S.No	Name of the students	Class	Department	Name of the project/Field work
01	Jyoti Bagade Muskhan Shekhaji Pradnya Bhivase Sonali Bharade Soundarya Patil Tejashwini Patil Ummesalma Mulla	B.Sc VI Sem	Physics	Synthesis of Nickel nano particles
02	Parvate Chougale Aruna Hegade Dilshad Mulla Laxmi Sansudi	B.Sc VI Sem	Mathematics	Crowd Control
03	Pradnya Bhivase Aishwarya Padre Muskan Shekhaji Sonali Bharade	B.Sc VI Sem	Mathematics	Multi and simply connected regions in complex analysis
04	Rachana Tandale Pushpadant Upadhye	B.Sc VI Sem	Botany	Biodiversity
05	Manasi Gurav Manjula Bhave Rukmini Talawar Sana Gadampalli Soumya Magadum	B.Sc IV Sem	Zoology	Organic Farming VS Chemical Farming

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DEPARTMENT OF PHYSICS

PROJECT

YEAR 2019-20

SYNTHESIS OF NICKEL NANOPARTICLES

FOR

B.Sc VI SEMESTER STUDENTS

DEPARTMENT OF PHYSICS

PROJECT

YEAR 2019-20

SYNTHESIS OF NICKEL NANOPARTICLES

FOR

B.Sc VI SEMESTER STUDENTS

GUIDE: Mr S.B.VAIRAT **DATE: 14.10.2019**

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CERTIFICATE

This is to certify that the B.Sc VIth Semester students of our department have satisfactorily completed the Project on " Synthesis of Nickel Nanoparticles" under the guidance of staff members of Physics Department, for the academic year 2019-20.

HOD
Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

Principal
PRINCIPAL
G.I. Bagewadi Arts, Science & Commerce College, NIPANI

List of Participants

Sl. NO .	Name of the students	class
1	Aayushi Kadam	BSc III year
2	Parvati Chougule	BSc III year
3	Jyoti Bagade	BSc III year
4	Savita Pathade	BSc III year
5	Soundarya Patil	BSc III year
6	Pooja Patil	BSc III year
7	Poonam Khot	BSc III year
8	Bhagyashri Bedkihale	BSc III year
9	Jyoti Patil	BSc III year
10	Nutan Salunkhe	BSc III year
11	Mrunali Salunkhe	BSc III year
12	Nikita Nadage	BSc III year
13	Pradnya Bhivase	BSc III year
14	Deepali Patil	BSc III year
15	Ashwinin Patil	BSc III year
16	UmmeSalma Mulla	BSc III year
17	Mayuri Sadalage	BSc III year
18	Sushma Patil	BSc III year
19	Deepali Chougule	BSc III year
20	Pooja Chougule	BSc III year
21	Muskan Shekhaji	BSc III year
22	Mansoora Momin	BSc III year
23	Aniket Jadhav	BSc III year
24	Hemant Sasane	BSc III year
25	Rahul Hasuri	BSc III year
26	Akash Shinde	BSc III year
27	Vishal Mokashi	BSc III year
28	Sheevaleela Hirekodi	BSc III year
29	Shivani Patil	BSc III year
30	Shweta Patil	BSc III year
31	Vidya Jangade	BSc III year
32	Tanuja A	BSc III year
33	Pruthviraj Patil	BSc III year

Head
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KLE SOCIETY'S G I BAGEWADI ARTS, SCIENCE & COMMERCE

COLLEGE, NIPANI-591237

Project on crowd control

Submitted by : Parvati Chougule
Aayushi Kadam
Chaitali Sadalage
Pooja Mahajan
Sushma Ankali

B.Sc VI Sem

Submitted to:
K.L.E G.I.Bagewadi college, Nipani
Department of mathematics.

Guided by: Dr.M.M.Shankrikopp ma'am

FOR THE YEAR 2019-20

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PROJECT

On

SIMPLY AND MULTI-CONNECTED REGION

By

B.Sc VI Semester Students

1. Miss Pradnya Bhivase
2. Miss Sonali Bharade
3. Miss Aiswarya Padre
4. Miss Muskana Shekaji
5. Miss Aiswarya Mali
6. Mansoora Momin
7. Miss Shubhangi Kesarkar

Guide: Dr.(Smt.)M. M. Shankrikopp

YEAR 2019-20

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K.L.E. Society's

G.I.BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPANI.

DEPARTMENT OF BOTANY

PROJECT REPORT

ON

"BIODIVERSITY OF K.L.E G.I.BAGEWADI CAMPUS"

Submitted by B.Sc. VI semester Botany Students of Academic Year

2019–2020

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

Resource Person

Smt. S B Patil Mam, Dr. Smt.. S P Shirgave & Smt. S S Sunnal Mam

Reg. No. :- S1717681, S1717708, S1717712, S1717713.

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G.I.BAGEWADI ARTS, SCIENCE & COMMERCE NIPANI

CERTIFICATE

DEPARTMENT OF BOTANY

This is to certify that Mr. /Miss. S1717681, S1717708, S1717712, S1717713

of B.Sc. VI semester has satisfactorily completed the Project work

beyond curriculum in Botany during the year 2019-2020

Staff Member in-charge

Head of the Department

Head
Department of Botany
K.L.E's G. I. B. College, Nipani.

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

K.L.E Society's

G I BAGEWADI ARTS SCIENCE AND COMMERCE COLLEGE

NIPPANI

DEPARTMENT OF ZOOLOGY

2019-20

Beyond curriculum Project

on

Organic farming Vs Chemical farming

Class IV sem

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

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NIPPANI

DEPARTMENT OF ZOOLOGY

2019-20

Beyond curriculum Project

on

Organic farming Vs Chemical farming

Class IV sem

Roll No: S1819599
S1819468
S1819470
S1819531
S1819547

Exam No.

Ly

Staff incharge	**Examiner**	**HOD**
1 Dr.V.R.Naik	1 Dr.smt.V.R.Naik	Ly
2 Prof.Smt.K.I.Pattan	2 Smt.K.I.Pattan	KIpattan

KLE SOCIETY'S G I BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPANI-591237

A PROJECT

ON

"Application Of Derivatives in Real Life"

SUBMITED TO

DEPARTMENT OF MATHEMATICS

BY

Anuja Patil, Prajecta Bhore, Trupti Magadum and Shweta Patil

Of

M.Sc. II SEMMESER

KLE's G. I. Bagewadi Arts, Science and Commerce College, Nipani

Under Guidance of

Miss Girija L. Karahuppi

Academic Year 2019-2020

Head
Department of Mathematics
K.L.E's G. I. B. College, Nipani

PRINCIPAL
K. L. E. Society's
G. I. Bagewadi College, Nipani.

KLE SOCIETY'S G I BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPANI-591237

A PROJECT

ON

"NUMBER THEORY AND CRYPTOGRAPHY"

SUBMITED TO

DEPARTMENT OF MATHEMATICS

BY

[1]Miss. Jyoti A.Chavan and [2]Miss.Pooja H.Yadav

[3]Miss. Priya V. Patil and [4]Mr. Mahadev G.Nalawade

KLE's G. I. Bagewadi Arts, Science and Commerce College, Nipani

E-mail: [1]jyotichavansmile@gmail.com and [2]poojahyadav1997@gmail.com

[3]priyapatil2818@gmail.com and [4]mahadev222222@gmail.com

Of

M.Sc. IV SEMMESER

Under Guidance of

DR.(SMT). M. M. SHANKRIKOPP

M.Sc, PhD

Academic Year 2019-2020

NUMBER THEORY AND CRYPTOGRAPHY

Head
Department of Mathematics
K.L.E's G. I. B. College, Nipani

PRINCIPAL
K. L. E. Society's
G. I. Bagewadi College, Nipani.

KLE SOCIETY'S G I BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPANI-591237

A PROJECT

ON

"Application Of Mathematics In Air Traffic"

SUBMITED TO

DEPARTMENT OF MATHEMATICS

BY

Sangeeta More, Aishwarya Zele, Anuradha Hindalkar, Anita Hamidwade and Snehal Jadhav

Of

M.Sc. II SEMMESER

Under Guidance of

Miss Vinaya Khot

Academic Year 2019-2020

Head
Department of Mathematics
K.L.E's G. I. B. College, Nipani

PRINCIPAL
K. L. E. Society's
G. I. Bagewadi College, Nipani.

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Ref. No. *Date :*

iv. Advance Learners participated and Presented papers in conference

SEMINAR / CONFERENCE/ WORKSHOP: ATTENDED / PAPER PRESENTED BY ADVANCED LEARNERS 2019-20

Sr. No.	Name of the student	Name of the seminar/ conference with ISBN/ ISSN	Paper Presented / attended	Topics	Class	Subject	Date & Place
1	Miss. Sarasvati Ranage	National level workshop			BA II		10-01-2020 Devchand College Arjunnaga, Maharashtra .
2	Miss Parvati Chougale,	State level Online Student Seminar On Applications of Mathematics	Presented	Crowd Control	B.Sc III	Mathematics	16/05/2020, K.L.Society's S.K. Arts College and H. S. K. Science Institute, Hubballi
3	Miss Parvati Chougala 1st Place, with Cash Prize Rs. 5000	Inter Collegiate Mathematics PPT Presentation Competition	Presented	Use of technology in teaching of Mathematics; Sequence & Series	B.Sc III	Mathematics	28/02/2020,SKE Society's GSS College, Belagavi
4	Miss Pradnya Bivase 1st Place, with Cash Prize Rs. 5000	Inter Collegiate Mathematics PPT Presentation Competition	Presented	Use of technology in teaching of Mathematics; Sequence & Series	B.Sc III	Mathematics	28/02/2020,SKE Society's GSS College, Belagavi
5	Miss Sukannya Chougale	Inter Collegiate Mathematics PPT Presentation Competition	Presented	Use of Mathematics in Daily Life	B.Sc II	Mathematics	28/02/2020,SKE Society's GSS College, Belagavi
6	Pratiksha Suryavanshi	One Day National Seminar on Career		English Communication	B.Sc III	English	21 January, 2020, Devachand College,

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		Prospects and competencies through English Studies					Arjunnagar
7	Annapurneshwari Mudhale	One Day National Seminar on Career Prospects and competencies through English Studies	Presented	English as a Global Language	BA II	English	21 January, 2020, Devachand College, Arjunnagar
8	Pratiksha Suryavanshi	One day national level Students seminar on Teaching and Learning English in Technological Era	Presentd	Trends and Issues of Use of ICT in English Teaching	B.Sc III	English	17.2.2020, SCP Arts, Science and DDS Commerce College, Mahalingpur
9	Daksha Patel , Secured I Rank	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Poster presentation on Green Economy	B.Sc III	Chemistry	One day National Conference at KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
10	Chaitali Sadalage	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Poster presentation on Green Economy	B.Sc III	Chemistry	One day National Conference at KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
11	Jyoti Bagade , Secured II Rank	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Poster presentation on Green Chemistry	B.Sc III	Chemistry	One day National Conference at KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
12	Miss.Sushma Ankali	Concepts of Green Chemistry and Applications of Spectroscopic	Attended		B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

		methods in Chemistry					
13	Miss.SoundaryaA.Patil	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
14	Miss.Swapna J. Ghorwade	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
15	Miss.Snehal D.Patil	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
16	Miss.YashodhaP.Kajave	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
17	Miss.Shivani Y. Sutar	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
18	Miss.Kavya Mane	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
19	Miss.Deepa Kedarshetti	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020

20	Miss.Sonali Bharade	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
21	Miss.Dilshad.M.Mulla	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
22	Miss.Shubhangi Kesarkar	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
23	Miss.Pradnya Bhivase	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
24	Miss.Aishwarya Padre	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
25	Mr.Rushikesh.S.Ghatage	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
26	Miss.Shambala Kumbar	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
27	Miss.MuskanShekhaji	Concepts of Green	Attended	---	B.Sc III	Chemistry	National Conference KLE's

		Chemistry and Applications of Spectroscopic methods in Chemistry					G.I.B Nipani. 14th FEB 2020
28	Mr.Umesh Pujari	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani: 14th FEB 2020
29	Mr.Sumith Murgude	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Attended	---	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020
30	Miss.Pratiksha M. Suryavanshi	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Recent Trends in Green Technology	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
31	Miss. Manasi.A.Bavadekar	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Applications of Green Chemistry in minimization of Pollution	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
32	Miss. Mansura.T.Momin	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Applications of Green Chemistry in minimization of Pollution	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
33	Miss .Parvati Chougale	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Role of Green Chemistry minimization of pollution	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**

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34	Mr.ShubhamA.Rote	Concepts of Green Chemistry and Applicatons of Spectroscopic methods in Chemistry	Presented	Principles, Uses and importance f Green Chemistry	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
35	Miss Sanjeevini Hasure	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Green technology	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
36	Mr. Rahul Hosuri	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Green Economy	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
37	Miss.Ashwini Patil	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Green Chemistry in Pharmaceutical Industry	B.Sc III	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
38	Mr.Shrikant Mali	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Green Chemistry Potential For Past, Present and Future	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
39	Mr.Sumit Murgude	Concepts of Green Chemistry and Applications of	Presented	Green Chemistry Potential For	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**

		Spectroscopic methods in Chemistry		Past, Present and Future,			
40	Miss.Shruti kamate	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Green Energy and Renewable Sources	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**
41	Miss.Pratibha Aluri	Concepts of Green Chemistry and Applications of Spectroscopic methods in Chemistry	Presented	Green Energy and Renewable Sources	B.Sc II	Chemistry	National Conference KLE's G.I.B Nipani. 14th FEB 2020 **ISBN 978-81-930758-7-6**

Co-ordinator IQAC
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जनता शिक्षण मंडल का

देवचंद कॉलेज, अर्जुननगर

तहसिल-कागल, जिल्हा-कोल्हापुर

हिंदी विभाग द्वारा आयोजित

राष्ट्रीय कार्यशाला

प्रमाणपत्र

श्री./श्रीमती/प्रा./डॉ. कुमारी . सरस्वती . बी. शानगे . बी. ए. IV semको

के. एल. ई. जी. आय. बागेवाडी कॉलेज निपानी .

शुक्रवार दि. १० जनवरी, २०२० को आयोजित "निवेदक, फिल्म एवं नाट्य अभिनय" इस विषय पर

राष्ट्रीय कार्यशाला में सहभाग हेतु प्रमाणपत्र दिया जा रहा है।

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प्रा. अनिता चिखलीकर
अध्यक्ष,
हिंदी विभाग

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani.

डॉ. पी. एम. हेरेकर
प्रधानाचार्य,
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Department of Mathematics

State Level Online Students Seminar on

"Applications of Mathematics"

16^{th} May 2020

Certificate

This is to certify that Mr/Ms **Parvati Chougule** *of* K.L.E's G.I. Bagewadi College, Nipani *has* ~~*participated*~~*/presented a paper entitled* 'Crowd Control' *at the State Level Online Students Seminar on "Applications of Mathematics" organized by the Department of Mathematics, S.K. Arts College and H.S.K. Science Institute, Hubballi held on* 16^{th} *May 2020.*

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

Dr. Prabhavati S. Mandalageri
Organizing Secretary

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani.

Dr. Lingaraj D Horakeri
Principal

PRINCIPAL
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G. I. Bagewadi College, Nipani.

South Konkan Education Society's

GOVINDRAM SEKSARIA SCIENCE COLLEGE, BELAGAVI

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Department Of Mathematics

This is to certify that ~~Mr.~~/Miss Pradnya M. Bhivase of K.L.E's G.I.B College, Nippani College, has secured First place in the Inter Collegiate/~~Class~~ Power Point competition conducted by the Department of Mathematics, Govindram Seksaria Science College, Belagavi, held on 28 February 2020

Prof. M.S, Nagasuresh
Head, Department of mathematics
G.S.S. College

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani.

Dr. Nagaraja D. Hegde
Principal
G. S. S. College

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National Seminar

on

CAREER PROSPECTS AND COMPETENCIES THROUGH ENGLISH STUDIES

Organised by

DEPARTMENT OF ENGLISH

Certificate

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE * NIPANI

This is to certify that Mr./Miss/Mrs./Dr. Annapurneshwari Mudhale of G.I.B. College, Nipani has participated/presented a paper entitled English as a global language in the One Day National Seminar for UG-PG Students held on Tuesday, 21 January 2020.

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani

Dr.Smt.G.D.Ingale
Convener

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani

Dr.P.M.Herekar
Principal

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KLE SOCIETY'S

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IQAC INITIATIVE

DEPARTMENT OF ENGLISH

(Skill development Programmee)

ONE DAY NATIONAL LEVEL STUDENT SEMINAR

ON

TEACHING & LEARNING ENGLISH IN THE TECHNOLOGICAL ERA

CERTIFICATE

K.L.E.S G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

This is to certify that ~~Mr.~~/Miss Pratiksha. Suryavanshi

of KLE's G. I. Bagewadi College Nippani has

participated in the One Day National Level Student Seminar held

on 17th February, 2020

He / She has Presented paper entitled Trends & Issues of use of ICT in English Teaching.

Co-ordinator IQAC
K.L.E. Society's
G. I. Bagewadi College, Nipani.

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

Shri. Virupaksha A. Adahalli	Dr. Chandrakant A. Langare	Dr. B. M. Patil
Organizing Secretary	Chief Guest	Grade - 1 Principal

K. L. E. Society's

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Ref. No. *Date :*

v. Question Papers and Question banks solved by Advance learners

Department of Mathematics

B.Sc I st sem Mathematics Paper I Question Bank 2019-20

Unit I: Properties of real numbers

Two marks Questions:

1. Write (i) commutative laws (ii) Associative laws of real numbers w.r.t addition and multiplication.
2. If a+b =a+c then prove that b = c and what is this law is called?**(2013 Dec.,2015)**
3. If ac = bc and $c\neq0$ then prove that b=c.**(2009)**
4. (i) Prove that a.0 =0.a =0 **(2009)** (ii). Prove that zero has no reciprocal
5. (i) If ab = 0 then prove that either a=0 or b=0. (ii) P.T - 0 = 0.
6. Prove that(i) –(-a)=a and (ii) $(a^{-1})^{-1} = a$ for $a\neq0$.
7. For $a,b\in R$, prove that (i) a(-b) = (-a)(b) = -ab. (ii) (-a)(-b) = ab and hence $(-a)(-a) = a^2$
8. For $a,b\in R$ and $a\neq0$, $b\neq0$ prove that $\frac{a}{b} = \frac{c}{d}$ iff ad = bc.
9. (i) Prove that –(a+b) = (-a) + (- b)
   (ii) If a, b are non zero real numbers then p.t. $(ab)^{-1} = a^{-1}\ b^{-1}$
10. Prove that (i) a(b-c) = ab – ac (ii) –b +(a+b) = b (iii) $(a+b)^2 = a^2+2ab+b^2$
11. (i) Prove that $\frac{a}{b} + (-\frac{1}{b})a = 0$. (ii)
12. State (i) Law of Trichotomy (ii) Law of transitive (iii) Law of addition (iv) Law of multiplication.
13. If a and b are real numbers then prove that (i) a>b if and only if –a<-b (ii) a<b iff –a>-b
14. (i) Prove that a>0 => -a<0, (ii) a<0 => -a>0.
15. (i)Prove that $\forall a\in R$, $a^2 \geq 0$**(2011)**. (ii) Prove that 1>0
16. If a>b and c<0 then prove that ac < bc.
17. If a>b and c>d then prove that (i) a+c > b+d **(2011)**
    (ii) a–d> b – c for all real numbers a, b, c and d.
18. If a, b, c and d are +ve real numbers such that a>b and c>d then prove that ac>bd.
19. If a>b then prove that $a>\frac{a+b}{2}>b$ i.e A.M of a and b is between a and b.**(2003)**
20. (i) If a>band ab>0 then prove that $\frac{1}{a} < \frac{1}{b}$ (ii) If a>b>0 and c>d>0 then P.T $\frac{a}{d} > \frac{b}{c}$.**(2007)**
21. (i)If a>1 then P.T $a^2>a$ (ii) If 0<a<1 then P.T.$a^2<a<1$ and also $a^n<1$.
22. Prove that sum of two positive real numbers is positive.
23. Product of two +ve real numbers is +ve and also product of two –ve real nos. is +ve.
24. For any two real numbers a and b>0, prove that $(a-b)^2\geq0$ and hence prove that $a^2+b^2\geq2ab$.
25. For +ve real numbers x and y , prove that $\frac{x+y}{2} > \sqrt{xy}$ **(2011)**
26. For any +ve real number x, prove that $x + \frac{1}{x} \geq 2$.**(2007, 2013, 2015)**
27. If a, b>0 and $a\neq b$, then prove that $\frac{a}{b} + \frac{b}{a} >2$.
28. If a+b>0, then prove that $(a+b)>\frac{2ab}{a+b}$
29. Prove that (i) |x| =|-x| (ii) $-|x| \leq x \leq |x|$ for $x\in R$. (iii) $\sqrt{x^2} = |x|$
30. Prove that |xy| = |x| |y| and $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$
31. Solve the inequality (i) 3x-10 <7x+2 (ii) $2x-1 \leq \frac{8-x}{2}$ (iii) $2x+\frac{1}{3}>2-3x$ (iv) $4x^2\leq 9$
    (v) $\frac{x+5}{2} > \frac{x-2}{4}$.
32. Find the solution set for (i) $x^2 +x -12>0$ (ii) (x-3)(x-4)>0 (iii) (x-2)(x+3)<0

(iv) $x^2-5x-84\geq 0$ (v) $(x+1)(x-3)(x-4)<0$.

33. Prove that (i) $\sqrt{3}+\sqrt{17}>\sqrt{7}+\sqrt{10}$ (ii) $2+\sqrt{7}<5$ (iii) $(1+\sqrt[3]{7})<3$.
34. Define upper and lower bounds of a set A.
35. Define supermum and infimum of a set A.
36. Prove that set of natural numbers is not bounded above or unbounded above.
37. State completeness property (or the least upper bound property)of real numbers.
38. State density property of set of rational numbers.
39. Which is greater than $\sqrt{2}+\sqrt{6}$ or $\sqrt{3}+\sqrt{5}$
40. Find the sup. and inf of set (i) S = {-1, -2, -3,-4} (ii) S = $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \text{---------------}\}$.
41. Find the lub and glb of the set S = [-1, 1], are they members of S?
42. Find the lub and glb of the set S = (3, 8) , are they members of S?
43. Give an example of a set which is (i) bounded above but not below (ii) bounded below but not above (iii) neither bounded above nor bounded below.
44. If $a<b$ and $c>0$ then prove that $a<b+c$
45. Prove that $n^2\leq 2^n$

Five Marks questions:

1. Write all field axioms of real numbers.
2. For $a,b\in R$ and $a\neq 0$, $b\neq 0$ prove that $\frac{a}{b}=\frac{c}{d}$ iff $ad = bc$.
3. If $a\neq b$ and $a>b$ then prove that $a^3+3ab^2>b^3+3a^2b$.
4. If $a\neq b$, $a+b>0$ then prove that $a^3+b^3>a^2b+ab^2$.**(2006)**
5. If $\frac{a}{b}\leq\frac{c}{d}$ then prove that $\frac{a+b}{b}\leq\frac{c+d}{d}$.
6. For all real numbers a, b, c prove that $a^2+b^2+c^2\geq ab+bc+ca$.**(many times asked)**
7. If a,b,c are positive real numbers then prove that $a^2b+ab^2+b^2c+bc^2+a^2c+ac^2\geq 6abc$. **(2011, 2015)**
8. Prove that $(a^2+b^2)(c^2+d^2)\geq(ac+bd)^2$ (Cauchy's Inequality)**(2015).**
9. State and prove Archimedean property of real numbers.
10. Prove that $\frac{a+b}{2}\leq\left(\frac{a^2+b^2}{2}\right)^{\frac{1}{2}}$ for all $a, b\in R$.
11. Prove that $\frac{x^3+y^3+z^3}{3}\geq xyz$ or $x^3+y^3+z^3\geq 3xyz$ if x,y,z are +ve real numbers.
12. Prove that (i) $\frac{a+b+c}{3}\geq\sqrt[3]{abc}$ (ii) similarly a_1, a_2, a_3 and a_4 are four real numbers then prove that $\frac{a_1+a_2+a_3+a_4}{4}\geq\sqrt[4]{a_1a_2a_3a_4}$
13. Prove that for $x, y\in R$, (i) $|x\pm y|\leq|x|+|y|$ (ii) $|x+y|\geq|x|-|y|$, when does equality holds.
14. Prove that $||x|+|y||\leq|x|+|y|$.
15. If $d>0$, then $|c|\leq d$ if and only if $-d\leq c\leq d$.
16. If $d>0$, then $|x-a|\leq d$ if and only if $a-d\leq x\leq a+d$.
17. $\forall\, u, v, w\in R$, prove that $\frac{u}{v+w}+\frac{v}{u+w}+\frac{w}{u+v}\geq\frac{3}{2}$.
18. For all +ve real numbers a,b,c prove that $(a+b)(b+c)(c+a)\geq 8abc$.
19. Show that $a^2b^2+b^2c^2+a^2c^2>abc(a+b+c)$ if a,b,c are positive reals.
20. Prove that $\frac{1}{|x+y|}\leq\frac{1}{|x|-|y|}$
21. Write the functions (i) $f(x)=|x+2|-|x-1|$ (ii) $f(x)=|x|+|x-3|$ in terms of x.

Unit II: Limits and Continuity

Two Mark questions

1. Define continuity of a function at a point and in an interval
(Define continuity of a function f(x) at x = a)
2. Define right hand and left hand limits of a function.
3. State the conditions under which the function f(x) is continuous at x=a.
4. State theorems included in Algebra of limits.
5. Define differentiability of a function at a point.
6. Prove that f(x) = sinx for all x∈R is continuous.
7. Prove that f(x) = |x| for all x∈R is continuous at x = 0.
8. If $f(x) = (x^2 - 4)/(x-2)$ if $x \neq 2$
   f(x) = k at x=2 , then find k if f(x) is continuous at x =2.
9. If $f(x) = [(1-2^x)/x]$ if $x \neq 0$
   f(0) = - log2 then show that f(x) is continuous at x = 0
10. Show that the function $f(x) = (\tan^{-1}x)/x$ for $x \neq 0$
   = 1 for x =0 is continuous at x=0.
11. If f(x) = log(1+x) for $x \neq 0$
   = 1 for x=0 , show that f(x) is continuous at x= 0.
12. If $f(x) = (1+2x)^{1/x}$ for $x \neq 0$
   $= e^2$ for x =0 , then show that f(x) is continuous at x=0
13. Show that f(x) = 4x+3 when x<4
   = 3x+7 when $x \geq 4$ is continuous at x=4.
14. If $f(x) = [2(1-e^x)/x]$ if $x \neq 0$
   & f(0) = -2 then show that f(x) is continuous at x = 0
15. Show that the function $f(x) = x^2$ when $x \neq 1$
   = 0 when x=1 is discontinuous at x=1.
16. Show that $f(x) = [(x^2 -16)/x-4)]$ when $x \neq 4$
   = 6 when x = 4 is not continuous at x= 4.
17. Show that the function f(x) = (sinx)/x when $x \neq 0$
   = 1 when x=0 is continuous at x=0.
18. Show that f(x) = 3x-2 if $x \leq 0$
   = x+1, x>0 is discontinuous at x=0. **(2014, 2016)**
19. Show that the function $f(x) = (\sin^{-1}x)/2x$ when $x \neq 0$
   = 1 when x=0 is discontinuous at x=0.
20. Show that the function $f(x) = (1+3x)^{1/x}$ when $x \neq 0$
   $= e^3$ when x=0 is continuous at x=0.
21. Give an example of a bounded function which is not continuous
22. Discuss the continuity of the function f(x) at x=1/2
   if f(x) = x when $0 < x \leq 1/2$
   = 1-x when $1/2 < x \leq 1$
24. If f(x) = x-2 if $x \leq 1$
   = x+2 if x>1 then f(x) is discontinuous at x=1.
25. If f(x) = 4x when $0 < x \leq 4$
   = 3x+7 when x>4 then f(x) is not continuous at x=4

26. Discuss the continuity of the function

$f(x) = 2/(5-x)$ if $x<3$

$= 5-x$ if $x \geq 3$

27. Define uniform continuity of f(x).

28. Discuss the continuity of the function

$f(x) = \frac{x}{|x|}$ when $x \neq 0$

$= 0$ when $x = 0$ at $x = 0$.

29. Examine the continuity of the function at x = 0 if

$f(x) = \frac{xe^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ when $x \neq 0$ and

$f(0) = 0$ at $x = 0$.

30. If $f(x) = 1+x$ if $x \leq 2$

$= 5-x$ if $x>2$ then f(x) is continuous at x=2.**(2011)**

31. Discuss the continuity of the function

$f(x) = 3x-2$ if $x \leq 1$

$= 2x-1$ if $x>1$ at $x=1$.

32. Give an example of a bounded function in a closed interval [a,b] which is not continuous.

33. Does the limit exist for $f(x) = \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ at x=0.

II. Five Mark Questions.

1.If f(x) and g(x) are continuous are continuous at x=a then prove that i) f(x) + g(x) ii) f(x) – g(x) iii) f(x).g(x) v)f(x)/g(x) when g(x)≠0 v) lf(x)l are continuous.

2. If $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$ then prove that

i) $\lim_{x\to a} [f(x) + g(x)] = l + m = \lim f(x) + \lim g(x)$ **(2011)**

ii) $\lim_{x\to a} [f(x) - g(x)] = l - m = \lim f(x) + \lim g(x)$

iii) $\lim_{x\to a} [f(x)g(x)] = lm = \lim f(x) \lim g(x)$

iv) $\lim_{x\to a} [f(x)/g(x)] = l/m = \lim f(x)/ \lim g(x)$

3. If $f(x) = \begin{cases} x^2 & \text{for } x<0 \\ x & \text{for } 0 \leq x<1 \\ 2-x & \text{for } 1 \leq x<2, \end{cases}$ show that f(x) is continuous at x=0,1

4. If $f(x) = \begin{cases} x & \text{for } x<1 \\ 2-x & \text{for } 1 \leq x \leq 2 \\ -2+3x-x & \text{for } x>2, \end{cases}$ discuss the continuity of f(x) at x=1,2. **(2016)**

5. If f(x) is define as $f(x) = x-1$ if $x<0$

$= x^2-1$ if $0 \leq x<2$ **(2011)**

$= 3x^2/4(x-1)$ if $x \geq 2$ then prove that f(x) is continuous at x =0 & x=2

6. Discuss the continuity of the function at x=0 if

$f(x) = e^{1/x} / (e^{1/x}+1)$ when $x \neq 0$ and $f(0) = 0$

7. Examine the continuity of the function at x = 0 if

$f(x) = \frac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$ when $x \neq 0$ and

$f(0) = 0$ **(2015)**

8. Find the values of a & b if $f(x) = ax^2 + b$ when $x<2$
$= 2$ when $x = 2$
$= 2ax - b$ when $x > 2$, is continuous at $x = 2$.

9. Is the function $f(x) = e^{\sqrt{x}}/(1+e^{\sqrt{x}})$ for $x \neq 0$ and $f(0) = 0$ at $x = 0$?.

10. Discuss the continuity of $f(x) = x-1$ if $x<0$
$= x^2-1$ if $0 \leq x < 2$
$= \frac{3x^2}{4(x-1)}$ if $x \geq 2$, discontinuous at x=0 and 2.

11. If $f(x) = \frac{x^3}{a^2}$ when $0<x<a$
$= a$ when $x=a$
$= 2a - \frac{a^3}{x^2}$ when $x>a$ then f is continuous.

12. Discuss the continuity of $f(x) = \frac{1-\cos x}{(\tan^{-1}x)^2}$ for $x \neq 0$
$= 1$ for $x = 0$, at x=0.

13. Discuss the continuity of $f(x) = 4x+3$ for $x<4$
$= 3x+7$ for $x \geq 4$ at x=4.

14. Discuss the continuity of $f(x) = x^2+2$ for $x>1$
$= 2x+1$ for $x=1$
$= 3$ for $x<1$ at x=1.

15. Discuss the continuity of $f(x) = (x-a)\sin\frac{1}{(x-a)}$ for $x \neq a$
$= 1$ for $x = a$, at x=a.

16. Discuss the continuity of $f(x) = \frac{e^{\frac{1}{x}}\sin\frac{1}{x}}{1+e^{\frac{1}{x}}}$ for $x \neq 0$
$= 0$ for $x = 0$ at x=0. **(2011)**

17. Is the function $f(x) = \frac{e^{ax}}{1+e^{ax}}$ for $x \neq 0$
$= 0$ for x=0 at x=0?.

18. Discuss the continuity of $f(x) = \frac{1}{1+e^{\frac{1}{x}}}$ for $x \neq 0$
$= 0$ for $x = 0$ at x=0.

19. Discuss the continuity of $f(x) = 5x - 4$ for $x \leq 1$
$= 4x^2 - 3x$ for $x = 0$, at x=0.

20. Discuss the continuity of $f(x) = \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$ for $x \neq 0$
$= 1$ for x=0 at x=0. **(2014, 2017)**

21. If $f(x) = \frac{2^{x+2}-16}{4^x-16}$ for $x \neq 2$
$= A$ for x=2 is continuous at x=2 then find A.

22. If f(x) is continuous at a point a and $f(a) \neq 0$ then prove that f(x) has the same as f(a) in an interval around the point.

23. If f(x) is continuous in [a,b] then it attains its supremum and infimum.

24. Prove that the function which is continuous in a closed interval [a,b] is bounded in that interval.

OR

Prove that every continuous function in a closed interval is always bounded in that interval.

27. State and prove intermediate value theorem

28. If f is continuous in [a,b] then show that f(x) is uniformly continuous in [a,b].

29. Define uniform continuity of a function at a point x = a. If f(x) is uniform continuous then it is continuous also at that point. Give an example to show that the converse is not

B. Sc. II Semester

Mathematics, Paper II

Boolean Algebra Question Bank

Two Mark Questions

1) Define equivalence relation with an example.
2) Define Partially ordered relation.
3) Define Hasse Diagram.
4) Draw Hasse diagram of (Z_{30}^{+} , /) with usual order of relation.
5) Let 'n' be the +ve integers and D_n denotes set of all divisors of n then if n = 15, Draw the Hasse diagram.
6) $\forall$ a, b $\in$ A in a lattice $(A, \leq)$, prove that i) $a \vee b = b \vee a$ & ii) $a \wedge b = b \wedge a$ **[2018]**
7) For any a, b in a lattice $(A, \leq)$ show that i) $a \leq a \vee b$, ii) $a \wedge b \leq a$ **[2015, 2017]**
8) State the adsorption property in a lattice.
9) Define Distributive Lattice.
10) In a distributive lattice if an element has a complement then prove that it is unique.
11) In any lattice $(A,\leq)$ if $a \leq b$ & $c \leq d$ then prove that i) $a \vee c \leq b \vee d$ and ii) $a \wedge c \leq b \wedge d$ **[2017]**
12) In a lattice $(A,\leq)$ with universal upper & lower bounds 1& 0 prove that $a \vee 1 = 1$, $a \wedge 1 = a$, $a \vee 0 = a$, $a \wedge 0 = 0$
13) Prove that a+ 1=1 in a Boolean algebra. **[2016]**
14) State the 'principle of duality'. **[2016]**
15) Define lattice with an example. **[2017]**
16) Define Bounded Lattice. **[2018]**
17) Define complemented lattice.
18) Define i) Boolean Lattice. ii) Boolean algebra **[2015, 2016]**
19) Define i) Boolean Polynomials. ii) Boolean Functions.
20) In a Boolean algebra show that , i) $a\vee(\bar{a} \wedge b) = a \vee b$ & ii) $a \wedge (\bar{a} \vee b) = a \wedge b$.

Five Mark Questions

1) Let $(A,\leq)$ be a lattice then $\forall$ a, b $\in$ A, i) $a \vee b = b$ iff $a \leq b$

ii) $a \wedge b = a$ iff $a \leq b$

iii) $a \wedge b = a$ iff $a \vee b = b$ **[2016]**

2) Prove that join and meet operations are associative.

3) In a lattice $(A, \leq)$ with universal upper & lower bounds 1& 0 prove that $a \vee 1 = 1$, $a \wedge 1 = a$, $a \vee 0 = a$, $a \wedge 0 = 0$

4) In any lattice $(A, \leq)$ if $a \leq b$ & $c \leq d$ then prove that i) $a \vee c \leq b \vee d$ and ii) $a \wedge c \leq b \wedge d$ **[2015]**

5) $\forall$ a, b $\in$ A in a lattice $(A,\leq)$, prove that i) $a \vee (a \wedge b) = a$ & ii) $a \wedge (a \vee b) = a$. **[2018]**

6) Prove that in a complemented lattice $(A,\leq)$ if $a \leq b$ then i) $\bar{a} \vee b = 1$ & ii) $a \wedge \bar{b} = 0$ **[2018]**

7) $\forall$ a, b $\in$ A in a Boolean lattice $(A, \leq)$, prove that i) $\overline{a \vee b} = \bar{a} \wedge \bar{b}$ & ii) $\overline{a \wedge b} = \bar{a} \vee \bar{b}$ **[2018]**

8) In a Boolean algebra show that, i) $a \vee (\bar{a} \wedge b) = a \vee b$ & ii) $a \wedge (\bar{a} \vee b) = a \wedge b$. **[2015, 2017]**

9) Let a,b,c be elements in a lattice $(A,\leq)$ show that i) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$ ii) $(a \wedge b) \vee (a \wedge c) \leq a \wedge (b \vee c)$ **[2015, 2016 2017]**

10) Prove that if the **Join** operation is distributive over the **Meet** operation then the **Meet** operation is distributive over the **Join** operation and conversely.

11) Show that in a Boolean algebra, for any a and b $a = b$ iff $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) = 0$ **[2016]**

12) Consider the Boolean polynomial $E(x_1, x_2, x_3) = x_1 \vee (\overline{x_2} \wedge x_3)$. Construct the Boolean function $f: \{0,1\}^3 \rightarrow \{0,1\}$, determined by this polynomial.

13) Define Boolean function and draw the truth table for the Boolean expression $(\overline{x_1} \vee x_2) \vee (\overline{x_2 \vee x_3})$

Paper I- Differential Calculus

UNIT I: Polar Coordinates

Two Mark Questions

1. Define polar coordinates.
2. Find the angle ϕ and ψ(i.e angle between tangent and radius vector angle between tangent and initial line) for the curves (i) $r=ae^{\theta\cot\alpha}$ **(2009)** (ii) $r= a(1-\cos\theta)$ **(2015, 2016)** (iii) $r= a\cos\theta$ (iv)$r^n = a^n \cos n\theta$ (v) $r^2 = a^2 \sin 2\theta$ (vi) $2r = a$ (vi) $r = 2\sin\theta$ (vii) $r= 2a\cos\theta$ (viii) $r= a(1+\cos\theta)$**(2010)** (ix) $r= a(1-\sin \theta)$ (x) $r = a/\theta$ (xi) $r = a\theta$
3. For the curve $\frac{2a}{r} = 1-\cos\theta$ prove that $\phi = \pi - \frac{\theta}{2}$ **(2013, 2008)**
4. Find $\tan\phi$ for the cardioid $r= a(1+\cos\theta)$.

   OR

   Find the angle between the radius vector and tangent line to the curve $r= a(1+\cos\theta)$
5. Show that in equiangular spiral $r=ae^{b\theta}$ the tangent is inclined at a constant angle with radius vector.
7. Prove that the tangent at any point (r, θ) to the curve $r^2 = a^2 \sin 2\theta$ makes an angle 3θ with initial line.

   OR

   Show that angle between the tangent at any point P and the line joining P to origin is same at all points of the curve $\log(x^2+y^2) = k \tan^{-1}(y/x)$ **(2007)**
8. Find slope of the tangent to the curves i) $r = a \cos\theta$ (ii) $r = a$ (iii) $r^3 = a^3 \cos 3\theta$
9. Show that the curves $r=ae^{\theta}$ and $re^{\theta}=b$ cut orthogonally.**(2013, 2007)**
10. Find the angle of intersection of the curve $r^2 = a^2 \cos 2\theta$ & $r^2 = a^2 \sin 2\theta$ **(2008)**
11. Find the angle of intersection of the curve $r = a\cos\theta$ and $2r = a$**(2010)**
12. Find the angle of intersection of the curve $r = a$ & $r= 2a\cos\theta$
13. Show that $r = a\theta$ and $r\theta = a$ intersect orthogonally.**(2011)**
14. Find the pedal equation of the line (i)$r = a\theta$ (ii) $r =a\sin\theta$ (iii) for $r\theta = a$, prove that $p = \frac{r^2}{r^2+a^2}$ **(2017)** (iv) $a^2 = r^2 \cos 2\theta$ (v) $r = a \cos \theta$ **(2016)** (vi)$r=a(1-\cos\theta)$**(2012, 2013)** (vii) $r= a(1+\cos\theta)$
15. Derive the formula for the length of perpendicular from pole to the tangent.
    Or With usual notation prove that $p=r\sin\phi$
16. Write down the formulas for polar sub tangent and subnormal to **(2015, 2013, 2011, 2010,2007)**
17. For the curve $r^2 = a^2 \sin 2\theta$ prove that polar sub tangent is $a\sqrt{\sec 2\theta}\tan 2\theta$ **(2017)**
18. For the curve $\frac{2a}{r} = 1-\cos\theta$ prove that polar sub tangent is $2a\operatorname{cosec}\theta$ **(2016).**
19. Find length of polar sub tangent and subnormal to the curve $r= a(1+\cos\theta)$ **(2009)**
20. Define pedal equation.
21. Write the relation between polar and Cartesian coordinates.
22. Show that for the curve $r =a\theta$, the polar subtangent is varies as square of the radius vector.(2012)
23. Find Find the angle between the radius vector and tangent line to the curve $r= a(1- \cos\theta)$ at $\theta = \frac{\pi}{3}$ **(2012)**
24. Find slopes of tangents to the curves (i) $r = a e^{\theta}$ at $\theta = \frac{\pi}{4}$ (ii) $r= a(1- \cos\theta)$at $\theta = \frac{\pi}{6}$ (iii) $4r =\sin 4 \theta$

25. For the curve y = a logsec($\frac{x}{a}$) , prove that $\frac{ds}{dx}$ = sec($\frac{x}{a}$) and $\frac{ds}{dy}$ = cosec($\frac{x}{a}$).

26. Find $\frac{ds}{dx}$ and $\frac{ds}{dy}$ for the curves (i) $y^2 = 4ax$, (ii)$ay^2 = x^3$ (iii) y = acosh($\frac{x}{a}$).

27. Find $\frac{ds}{dt}$ and angle ψ for the curves (i) x = a cost, y = bsint (ii) **x= a cos**^{3}t and y = a sin^{3}t (iii) x= a(cost –sint) , y = a(cost + sint). (v) x= a sect , y =btant.

28. Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the curves (i) r = aθ (ii) r = a(1-cosθ) (iii) $r^2 = a^2 \cos 2\theta$

Five Marks Questions

1. With usual notation show that (i) tanϕ = r dθ/dr **(2009, 2011, 2013, 2016,2017),** (ii) $1/p^2 = 1/r^2 + 1/r^4(dr/d\theta)^2$**(2016, 2013, 2012, 2011, 2006, 2008, 2009)**
2. Show that ∅ is $\frac{3\pi}{4}$ for the curve $r^m = a^m$ (cosmθ - sinmθ) at θ = 0,
3. Find the angle of intersection of the curves r = $\frac{a\theta}{1+\theta}$ and r = $\frac{a}{1+\theta^2}$ **(2013)**
4. Prove that spirals $r^n = a^n$cosnθ and $r^n = b^n$sinnθ intersect orthogonally **(2017, 2015, 2009).**
5. Prove that parabolas r =a sec^{2}θ/2 and r = b cosec^{2}θ/2 cut orthogonally.**(2008)**
6. Prove that the curves r = a(1+cosθ) and r = a(1-cosθ) cut orthogonally.
7. Find the angle of intersection of the curves (i) r = acosθ and 2r = a **(2013)** (ii)r = acosθ and r = a(1-cosθ)**(2010)**
8. Find the angle of intersection of the curves r = sinθ + cosθ & r = 2sinθ **(2007,2011, 2015)**
9. Find the angle of x^n of the curves r=a(1+cos θ) & r=b(1-cosθ).**(2016)**
10. Find the angle of intersection of the curves r = 6 cosθ and r= a(1+cosθ) **(2012)**
11. **Find the angle of intersection of the curves** r=a(1+sin θ) & r=b(1-sinθ).**(2009)**
12. Prove that the curves r = a(sinθ + cosθ) and r = a(sinθ - cosθ) intersect orthogonally
13. Show that pedal equation for the curve $x^2 + y^2 = 2ax$ is $r^2 = 2ap$
14. Find the pedal equation for the parabola 2a/r = 1 - cosθ **(2013, 2007)**
15. Find p-r equation for the curve (i) $a^2 = r^2 \cos 2\theta$ **(2007) (ii) r = a cosec**$^2\frac{\theta}{2}$ **(2009)**
16. Find the pedal equation of the curve(i) $r^m = a^m$cosmθ**(2006, 2010).** (ii) $r^n = a^n$sinnθ.**(2012, 2011)**
17. Find the angle of x^n of the parabolas r=a/1+cosθ, r=b/1-cosθ.**(2006, 2011)**
18. Find the pedal equation of the curve r=a(1-sinθ).
19. Prove that for the curve r= a(1+cosθ) ,tangent at θ = $\frac{\pi}{3}$ is parallel to initial line and θ = 2$\frac{\pi}{3}$, tangent is perpendicular to initial line.
20. Find the pedal equation for conic $\frac{l}{r}$ = 1 + ecosθ
21. Find the pedal equation of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ **(2016, 2013)**
22. Prove that pedal equation for ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ =1 is $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2}$ **(2015)**
23. Find the pedal equation of the curve $x^2 + y^2 = 2ax$ **(2017)**
24. Find the pedal equation for asteroid x= **a cos**^{3}θ and y = a sin^{3}θ.
25. Find pedal equation for hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ =1
26. Show that length of perpendicular from the pole to the to tha curve r = a(1-cosθ) is 2a sin^{3}θ.
27. Find pedal equation for x = **a cos**^{2}θ and y = b sin^{2}θ.
28. Find the pedal equation for the $y^2 = 4a(x+a)$. **(2016)**
29. Find pedal equation for x = a e^t(sint –cost) and y = a e^t(sint +cost).
30. Find pedal equation for x = a sect and y = b tant

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DEPARTMENT OF MATHEMATICS

B.Sc III Sem

Question Bank

Paper I: Sequence

Q.No I: Two mark questions.

1. Define convergent sequence and give an example.
2. Define divergent sequence and give an example.
3. Define oscillatory t sequence and give an example.
4. Define bounded sequence and give an example.
5. Define bounded above sequence and give an example of only bounded above .
6. Define bounded below sequence and give an example of only bounded below .
7. Define lub and glb of a sequence or Define supremum and infimum of a seq.
8. Define monotonic increasing sequence and give an example.
9. Define monotonic decreasing sequence and give an example.
10. Define monotonic sequence and give an example.
11. Define range of a sequence and write the range of the seq.{ 1,2,3,1,2,3,1,2,3,--}
12. Define null sequence and give an example.
13. Prove that every convergent seq. is bounded.
14. Give an example of a seq. which is bounded but not convergent.
15. Give an example of a seq. which is monotonic but not convergent.
16. Give an example of a seq. which is monotonic increasing and bounded.
17. Give an example of a seq. which is monotonic decreasing and bounded.
18. Give an example of a seq. which is monotonic increasing and not bounded above.
19. Give an example of a seq. which is monotonic decreasing and not bounded Below.
20. Prove that the sequence { 1, 1/2, $1/2^2$, $1/2^3$, ------------} is convergent.
21. Prove that the sequence $\{a_n\}$ where $a_n = 1+/3+1/3^2 + 1/3^3+$ -------$1/3^n$ is monotonic increasing.
22. Find the limit of $\{a_n\}$ where i) $a_n = n^{1/n}$ ii) $a_n = (n+1)^{1/n}$ iii) $a_n = (n!)^{1/n}$

    iv) $a_n = n^{1/2n}$ v) $a_n = (-1)^n 5$ vi) a_n = 2n+3/3n+4 vii) $a_n = 1+(-1)^n$

    viii) $a_n = n/n^2+1$ ix) $a_n = 1+/5+1/5^2 + 1/5^3+$ -------$1/5^n$ x) $a_n = (1+\frac{b}{n})^{n/b}$ xi) a_n = {1.1, 1.11, 1.111, ------}

xii) $\{a_n\} = \{.8, .88, .888, \text{------------------------}\}$

23. Prove that a sequence $\{a_n\}$ where i) $a_n = 1/n + 1/(n+1) + 1/(n+2) + \text{---} 1/n+(n-1)$ is monotonic decreasing ii) $a_n = 1+1/1! + ½!+1/3! + \text{--------} +1/n!$ is increasing.

24. Give an example of a seq. which is not monotonic.

25. State Cauchy's first theorem on limits.

26. State Cauchy's second theorem on limits.

27. State Cauchy's sequence and give an example.

28. Prove that every Cauchy's Seq. is bounded.

29. Give an example of a seq. which is bounded but not Cauchy's.

30. Show that $\lim_{n\to\infty} \frac{1}{n}[(1+\frac{1}{2}+\frac{1}{3}+ - - - - \frac{1}{n})] = 0.$

31. Show that $\lim_{n\to\infty} \frac{1}{n}[\frac{1}{2}+\frac{2}{3}+\frac{3}{4} - - - - \frac{n+1}{n}] = 1.$

32. Show that $\lim_{n\to\infty} 1/n(1+2^{1/2}+ 3^{1/3}\text{-------}n^{1/n}) = 1.$

35. Show that a seq. $\lim_{n\to\infty}\{1, ½^2, 1/3^2, \text{----------}1/n^2,\text{------}\}$ is a Cauchy's Seq. by definition.

Q.No II: Five mark questions.

36. Prove that every convergent seq. is bounded. Is the converse true? Justify your answer.

37. Prove that every monotonic seq. either converges or diverges.

38. Prove that monotonic increasing bounded above seq. is convergent and converges to its lub.

39. Prove that monotonic decreasing bounded below seq. is convergent and nverges to its lglb.

40. Prove that monotonic increasing not bounded above seq. is divergent and erges to ∞.

41. Prove that monotonic decreasing not bounded beow seq. is divergent and diverges to -∞.

41. Discuss the convergence of the seq. $\{a_n\}$ where i) $a_n=(n+1)/n$ ii) $a_n= 2n-7/3n+2$ iii) $a_n= 3n+4/2n+1$ iv) $a_n= (1+1/n)^n$ v) $a_n= (\sqrt{n+1} - \sqrt{n})$ (in each case prove that seq. is monotonic, bounded and find the limit 0).

42. Show that the seq.$\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2.

43. Show that Show that the seq.$\{S_n\}$ defined by $S_1 = 1$ and $S_{n+1} = \sqrt{3s_n}$ converges to 3.the seq.$\{S_n\}$ defined by $S_1 = 1$ and $S_{n+1} = \sqrt{7s_n}$ converges to .

44. Show that the seq.$\{S_n\}$ defined by $S_1 = 1$ and $S_{n+1} = \sqrt{3s_n}$ converges to 3.

45. Show that the seq.$\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2+a_n}$ converges to +ve root of the eq. +v root of $x^2-x-2=0$

46.Prove that lt $r^n = 0$ if $|r|<1$

47. State and prove Cauchy's first theorem on limits.

48. State and prove Cauchy's second theorem on limits.

49. If a seq $\{a_n\}$ is such that $a_n \rightarrow l$ then prove that $\log_{n\to\infty} \frac{a_1+a_2+---a_n}{n} = l$

50. State and prove Cauchy's criteria for convergence of the seq.

51. Prove that a seq. $\{a_n\}$ where $a_n = 1+1/2+1/3+1/4+---1/n$ is not a convergent seq.

52. Prove that every Cauchy's Sequence is convergent.

53. Prove that limit of a sequence is unique.

54. If $\{x_n\}$is a sequence of positive terms then prove that $\lim_{n\to\infty} x_n^{\frac{1}{n}} = \lim_{n\to\infty} \frac{x_{n+1}}{x_n}$

55. . Show that the seq.$\{x_n\}$ defined by $x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{6+x_n}$ converges to 6.

56. Prove that the sequence $\{\left(1+\frac{1}{n}\right)^n\}_{n\in N}$is convergent and converges to e where $2<e<3$

57. Prove that every convergent seq. is Cauchy's seq.

K.L.E Society's
G. I. Bagewadi Arts, Science & Commerce College, NIPANI
Assignments For the year 2019-20
B.Sc. VI Sem.
Paper III
Assignment

1. Two Mark questions

1. Define topology and give an example.
2. Define discrete topology on nonempty set Y and give an example.
3. Write any two mutually comparable proper topologies on X = {a, b ,c}
4. Write any four non comparable topologies on X = {a, b, c}
5. Write a topology on X = {a, b, c} in which a proper nonempty subset is both open and closed.
6. Give an example to show that union of two topologies is need not be a topology.
7. Let X = {1,2,3,4} and τ = {X, Ø,{1, 2}, {3, 4}} and A= {1,2} then is it open and closed? Why?.
8. Define indiscrete topology on X with example.
9. If A = {1,2,3} be sub set of R then is it open in (R,U) ?
10. Every open interval is open set in (R, U).
11. Every closed interval is closed set in (R, U).
12. Prove that every singleton set is closed in (R, U).
13. Give an example to show that arbitrary intersection of open sets need not be open set.
14. Give an example to show that arbitrary union of closed sets need not be closed set.
15. Show that finite non-empty subset of R is not open.
16. Give an example to show that $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$.
17. Define interior, exterior and boundary of a non-empty subset A of a topological space x.
18. Let X = {1, 2, 3} and τ = {X, Φ, {1}, {2}, {1, 2}, {2, 3}} be a topology on X. If A= {1, 3} then find derived set of A.
19. Let X = {a, b, c} and τ = {X, Ø,{b}, {a,c}} then prove that set {b, c} is a nhd. of the pt. 'b'.
20. Let X = {a, b, c} and τ = {X, Ø, {a}, {a, b}} then prove that the subset {a, c} is dense in X.
21. Prove that A is closed iff A = $\bar{A}$ in a topological space (X, τ).
22. Prove that A is open iff A = A^o in a topological space (X, τ).
23. For any subsets A and B of a topological space X prove that $A \subset B \Longrightarrow A^o \subset B^o$
24. Let X = {a, b, c} and τ = {X, Ø,{a,b}, {a,c}, {a}} then find τ_Y where Y={a}.
25. Define separable space and show that the real space (R, U) separable.
26. Prove that subspace of indiscrete space is indiscrete.
27. Prove that the real space (R, U) is T_2.
28. Prove that subspace of T_1 is T_1.
29. Prove that subspace of T_2 is T_2.
30. Define T_1 and T_2 space.
31. Prove that every T_2 space is T_1 space.
32. Prove that every singleton set in T_2-space is closed.
33. Define base and sub-base for a topology.
34. Define T_1-space and P.T co-finite topology is T_1-space.

2. Five Mark questions

1. Show that the finite intersection of topologies on a set is again a topology on a set.
2. State and prove co-finite topology on X.
3. Give an example to show that arbitrary union of closed sets need not be closed.
4. Give an example to show that arbitrary intersection of open sets need not be open.
5. State and prove usual topology on R.

B. Sc IV SEM Q. B

1. Prove that subset consisting of all even permutations is subgroup of symmetric group S_n.
2. Define generator of a group and give an example.
3. Howmany generators , acyclic group G of 10 has?, if a is generator of G the n what are the other generators?.
4. Define right and left cosets of H in G.
5. Let C be additive group of compex numbers and R be the subgroup of reals then find the coset of R in C.
6. Define cyclic group and normal subgroup.
7. What is cyclic group and give an example.
8. Prove that intersection of two normal subgroups of group G is normal subgroup.
9. Prove that H = {(1), (2,3)} is not a normal subgroup of the symmetric group S_3={(1),(12),(13),(23),(123),(132)}
10. Prove that every subgroup of abelian is normal.
11. Show that H={1,-1} is a normal subgroup of the multiplicative group G={1,-1,i,-i}.
11. Define factor group (quotient group) ii) centre group.
12. Prove that quotient group of abelian group is abelian.
13. If G is a group and H is subgroup of of G of index two then prove that H is normal.
14. Define homomarphism and isomorphism of two groups.
15. Let C be additive group of complex numbers and R be the subgroup of reals and f: C → R is defined f(x+iy)= x, show that f is homomorphism.
16. Prove that homomorphic image of abelian is abelian.
17. If f: G→ G^1 is homomarphism then show that $[f(a^{-1})] = [f(a)]^{-1}$ $\forall a \in G$
18. Define Kernal of homomarphism of a group.
19. If (a,m)=1 then prove that (a,m-a) =1.
20. Define equivalence relation and give an example.
21. If (a,b) =1 and a/p, b/p then show that ab/p.
22. If a ≡ b (modm), c≡d (modm) then prove that a±c ≡ b±d(modm).
23. Prove that product of r consecutive integers is divisible by r!.

24. Prove that n(n+1)(2n+1) is divisible by 6.

25. Prove that $n(n^2-1)$ is divisible by 6.

26. Prove that fifth power of positive integer n has the same unit place digit as n has.

27. If n is any odd integer then prove that n^2-1 is divisible by 8.

28. Find the number of divisors of 510.

29. Define Euler's Φ function and find i) Φ(2) ii) Φ(14) iii) Φ(16) iv) Φ(8) v) Φ(18) vi) Φ(24) viii) $\Phi(2^5)$ ix) Φ(3600).

30. . If m>0 then prove that Φ(m) is even.

31. With usual notayions prove that $\Phi(p^k) = p^k(1-(1/p))$

32. Find the number of divisors of 8064.

33. Find the number and sum of divisors of 600.

34. Find the number of integers less than 1026 and prime to it.

35. Find the number of integers less than 440 and prime to it.

36. Find the number of integers less than 1024 and prime to it.

37. Give counter example to disprove Φ(mn= Φ(m) Φ(n) ∀ integers m,n..

OR

38. Prove or disprove the statement Φ(mn) = Φ(m) Φ(n) ∀ integers m,n.

39. Find the number of divisors of 21,600

40. Prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7.

41. Prove that $3^{2n}+7$ is divisible by 8.

42. . Prove that $2^{4n}-1$ is divisible be 15.

43. Find the highest power of 5 contained in i) 58! ii) 500!

44. Find the highest power of 7 contained in i) 50! ii) 500!

45. Find the highest power of 5 contained in 158!

46. Find the highest power of 3 contained in 100!.

47. State Fermate's Theorem.

48. If p is prime number then prove that $a^p \equiv a \pmod p$

49. Show that $a^{p-1} - b^{p-1} \equiv 0 \pmod p$ if a & are prime to p.

50. Show that n^5-n is divisible by 30.

51. Show that $a^{12} - b^{12}$ is divisible by 13 if a & are prime to 13.

52. State Wilson's Theorem in number theorem.

II Five Mark Questions.

1. Prove that set G(S) or S_n of all permutations on given set S form a group under the composition of mappings.
2. Prove that that every permutation of a finite set can be expressed as product of disjoint cycles.
3. Prove that A subset A_n of all even permutations in S_n (n>2) is subgroup of S_n.
4. Prove that that every permutation of a finite set can be expressed as product of transpositions in ifinitely many ways.
5. Prove that every subgroup of cyclic group is cyclic.
6. Prove that every cyclic group is abelian. But prove that converse is not true.
7. Prove that every group of prime order is cyclic.
8. If G is a cyclic group of order 9 generated by a and H is a subgroup generated by a^3. Write the Caley table of G/H.
9. If H is a subgroup of group G & if a,b ЄG then prove that $H_a = H_b$ iff ab^{-1} ЄH.
10. If G is an infinite cyclic group generated by a then prove that G has exactly two generators a and a^{-1}.
11. State and prove Lagrange's Theorem of groups.

    1. OR

    If G is a group of finite order n and H is a subgroup of G then prove that order of H divides order of G.
12. A subgroup H of a group G is normal iff the product of any two right cosets of H in G is again right coset of H in G.

    1. OR

    A subgroup H of a group G is normal iff Ha Hb = Hab ∀ a,b ЄG.
13. Prove that intersection of two normal subgroups is normal.
14. Prove that a subgroup H of a group G is normal iff $a^{-1}ha \in H$, ∀aЄG and hЄH.

15. Prove that a subgroup H of a group G is normal iff $a^{-1}Ha = H$, $\forall a \in G$ and $h \in H$.

16. Prove that a subgroup H of a group G is normal iff H is a subgroup of index 2.

17. Prove that every quotient group of cyclic group is cyclic.

18. If $f: G \rightarrow G^1$ is homomorphism then prove that i) f(e) is identity in G^1 ii) $f(a^{-1}) = [f(a)]^{-1}$, $\forall a \in G$ iii) If H is a subgroup of G then prove that f(H) is also subgroup of G^1 and what is identity of f(H). iv) If H is a normal subgroup of G then prove that f(H) is also normal subgroup of G^1

19. If H is a normal subgroup of a group G then prove that xH.yH = xyH ii) the set of all cosets of H in G forms a group w.r.t binary operation given in (i).

19. Define factor group. If G is abelian then show that G/H is also abelian. OR

20. Prove that factor group(quotient group) of abelian is abelian.

If $f: G \rightarrow G^1$ is homomorphism then prove that Kerf is normal subgroup of G. OR

21. If $f: G \rightarrow G^1$ is homomorphism of groups with kernel K then show that K is normal subgroup of G.

22. If $f : G \rightarrow G^1$ is homomorphism of group G onto group G^1 then prove that f is isomorphism iff Kerf = {e} when e is identity in G.

23. Prove that every cyclic group of order n is isomorphic to the

24. additive group of integers Z_n modulo n.

25. If G is a group of reals under addition and G^1 is a group of non –zero reals nder multiplication and $f : G \rightarrow G^1$ is defined by

20. $f(x) = 2^x$ $\forall x \in G$, then prove that f is homomorphism and find kerf.

21. If G is additive group of integers and G^1 is the multiplicative group of fourth roots of unity & $f : G \rightarrow G^1$ defined by $f(n) = i^n$ show that f is homomorphism.

22. If Z is the additive group of integers and E is subgroup of even integers. Define a function $f: G \rightarrow E$ be f(x) = 2x then prove that f is isomorphism and find Kerf.

23. Define homomorphism and isomorphism. Show that f: P→R by f(x) = $\log_{10}x$ is an isomorphism where P is the multiplicative group of integers and R is the additive group of reals. Find kerf.
24. Prove that every permutation group is isomorphic to finite group.
25. State and prove Fundamental Theorem of Homomorphism. OR
26. Prove that the map f: G →G/N defined by f(x) = xN is a homomorphism of G onto G/N.
27. If $\alpha : G \to G^1$ is homomorphism & K be the kernel of α in G then show that $\alpha(G)$ is isomorphic to G/K.
28. Prove that every number greater that 1 can be expressed as product of primes. OR
29. State and prove Fundamental Theorem of Arithmetic. OR
30. Show that every composite number has atleast one prime divisor.
31. Derive the formula for number and sum of divisors of any +ve integer n and also find the no. of divisors of 480.
32. Find the highest power of 3 contained in 1000!.
33. Show that $5^{10} - 3^{10}$ is divisible by 11.
34. Prove that $10^n + 3.4^{n+2} + 5$ is divisible by 9.
35. Prove that $3^{2n+2} - 8n - 9$ is divisible by 64.
36. Prove that $9^n - 8^n - 1$ is divisible by 8.
37. Prove that $3^{4n+2} + 5^{2n+1} \equiv 0 \pmod{14}$
38. Prove that i) $3^{2n+1} + 2^{n+2}$ is divisible by 7 ii) $n^5 - n$ is divisible by 30
39. If x,y,z are three consecutive integers then show that $(\sum x)^3 - (3\sum x^3)$is divisible by 108.
40. If n is a prime number greater than 3 then provr that $(n^2 - 1)$ is divisible by 24.
41. State and prove Euler's Theorem.
42. If n is any composite number and p_1, p_2,p_3,------p_n are distinct primes then prove that $\Phi(n) = n\,(1-1/p_1)\,(1-1/p_2)(1-1/p_3)\,(1-1/p_4)\text{-------}(1-1/p_n)$
43. If m and n are relatively prime numbers then prove that $\Phi(mn) = \Phi(m)\,\Phi(n)$.
44. Prove that if p is a prime number then prove that $\Phi(p^r) = p^r(1- 1/p)$

45. Find the number of divisors and sum of divisors of 21600.

46. State and prove Fermate's theorem. OR

47. If p is a prime number then prove that $a^p - a$ is divisible by p.

48. If p is a prime number then show that i) $1^{p-1} + 2^{p-1} + 3^{p-1} + \cdots$

49. $(p-1)^{p-1}+1$ is divisible by p *or* $1^{p-1} + 2^{p-1} + 3^{p-1} + \cdots (p-1)^{p-1}+1 \equiv 0 \pmod{p}$ ii) $[(p-1)!]^{p-1} \equiv 0 \pmod{p}$

50. If a is any number prime to n then prove that $a^{\Phi(n)} \equiv 1 \pmod{n}$.

51. If p and q are distinct prime numbers then prove that $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$ *OR* $p^{q-1} + q^{p-1} - 1$ is divisible by pq.

52. Prove that $a^{12} - 1$ is divisible by 7 where $(a,7) = 1$

53. State and prove Wilson's Theorem.

54. State and prove converse Wilson's Theorem.

55. If p is a prime number then prove that $2(p-3)! + 1 \equiv 0 \pmod{p}$.

56. Show that 18! + 1 is divisible by 437.

57. Show that (28! + 233) is divisible by 29 & 31 both and hence by 899.

58. Prove that 12! + 209 is divisible by 221.

59. Prove that 12! + 1 = 13k where k is an integer.

K.L.E Society's

G. I. Bagewadi Arts, Science & Commerce College, NIPANI

Assignments For the year 2019-20

B.Sc. VI Sem.

Paper III

Assignment

1. Two Mark questions

1. Define topology and give an example.
2. Define discrete topology on nonempty set Y and give an example.
3. Write any two mutually comparable proper topologies on X = {a, b ,c}
4. Write any four non comparable topologies on X = {a, b, c}
5. Write a topology on X = {a, b, c} in which a proper nonempty subset is both open and closed.
6. Give an example to show that union of two topologies is need not be a topology.
7. Let X = {1,2,3,4} and τ = {X, ∅,{1, 2}, {3, 4}} and A= {1,2} then is it open and closed? Why?.
8. Define indiscrete topology on X with example.
9. If A = {1,2,3} be sub set of R then is it open in (R,U) ?
10. Every open interval is open set in (R, U).
11. Every closed interval is closed set in (R, U).
12. Prove that every singleton set is closed in (R, U).
13. Give an example to show that arbitrary intersection of open sets need not be open set.
14. Give an example to show that arbitrary union of closed sets need not be closed set.
15. Show that finite non-empty subset of R is not open.
16. Give an example to show that $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$.
17. Define interior, exterior and boundary of a non-empty subset A of a topological space x.
18. Let X = {1, 2, 3} and τ = {X, Φ, {1}, {2}, {1, 2}, {2, 3}} be a topology on X. If A= {1, 3} then find derived set of A.
19. Let X = {a, b, c} and τ = {X, ∅,{b}, {a,c}} then prove that set {b, c} is a nhd. of the pt. 'b'.
20. Let X = {a, b, c} and τ = {X, ∅, {a}, {a, b}} then prove that the subset {a, c} is dense in X.
21. Prove that A is closed iff $A = \overline{A}$ in a topological space (X, τ).
22. Prove that A is open iff $A = A^o$ in a topological space (X, τ).
23. For any subsets A and B of a topological space X prove that $A \subset B \Rightarrow A^o \subset B^o$
24. Let X = {a, b, c} and τ = {X, ∅,{a,b}, {a,c}, {a}} then find τ_Y where Y={a}.
25. Define separable space and show that the real space (R, U) separable.
26. Prove that subspace of indiscrete space is indiscrete.
27. Prove that the real space (R, U) is T_2.
28. Prove that subspace of T_1 is T_1.
29. Prove that subspace of T_2 is T_2.
30. Define T_1 and T_2 space.
31. Prove that every T_2 space is T_1 space.
32. Prove that every singleton set in T_2-space is closed.
33. Define base and sub-base for a topology.
34. Define T_1-space and P.T co-finite topology is T_1-space.

2. Five Mark questions

1. Show that the finite intersection of topologies on a set is again a topology on a set.
2. State and prove co-finite topology on X.
3. Give an example to show that arbitrary union of closed sets need not be closed.
4. Give an example to show that arbitrary intersection of open sets need not be open.
5. State and prove usual topology on R.
6. Let A, B are two subsets in a topological space (X, τ), then prove that $d(A \cup B) = d(A) \cup d(B)$.
7. If (X, τ) is a topological space then prove that any non-empty subset A of X is closed iff $d(A) \subset A$.
8. If A is a subset of X in a topological space (X, τ), then prove that $A \cup d(A)$ is closed and $\bar{A} = A \cup d(A)$.
9. For any subsets A and B of a topological space X prove that
   i. $A \subset B \Rightarrow \bar{A} \subset \bar{B}$
   ii. $\overline{A \cup B} = \bar{A} \cup \bar{B}$
10. For any subsets A and B of a topological space X prove that
    i. $A \subset B \Rightarrow A^o \subset B^o$
    ii. $(A \cap B)^o = A^o \cap B^o$
11. For any subsets A and B of a topological space X prove that
    i. $A \subset B \Rightarrow d(A) \subset d(B)$
    ii. $d(A \cup B) = d(A) \cup d(B)$
12. For any subsets A of a topological space X prove that
    i. $A \subset \bar{A}$
    ii. $\bar{A}$ is a closed set.
    iii. $\bar{A}$ is the smallest closed set containing A
    iv. $A = \bar{A}$ iff A is closed set..
13. For any subsets A of a topological space X prove that
    i. $A^o \subset A$
    ii. A^o is a open set.
    iii. A^o is the largest open set contained in A
    iv. $A = A^o$ iff A is open set.
14. Define Interior, Exterior and Boundary of a set in a topological space and prove that $A^o \cup B^o \neq (A \cup B)^o$.
15. Let X={a, b, c, d, e} and τ = {X, Φ, {a}, {a, b}, {a, b, c}, {a, b, c, d}}, then find interior, exterior and boundary of the set {a, b, c}.
16. Let X={a, b, c, d, e} and τ = {X, Φ, {a}, {c, d}, {a, c, d}, {b, c, d, e}}, then find derived set of A = {a, b, e}.
17. Let X={a, b, c, d} and τ = {X, Φ, {b}, {a, b}, {b, d}, {a, b, c}, {a, b, d}}be the topology on X, then for $A = \{a, c, d\} \subset X$ find $\bar{A}$, A^o, $(A')^o$ and bd(A).
18. Let X={a, b, c, d, e} and τ = {X, Φ, {a}, {a, b}, {a, b, c}, {a, b, c, d}}, then find the neighbourhood system of b.
19. Define neighbourhood of a point in (X, τ) and prove that a non-empty subset A of X is open iff it is neighbourhood of each of its points.
20. Let (X, τ) is a topological space and let A be a subset of X. Then prove that $A^o = (\overline{A'})'$

21. Let (X, τ) is a topological space and let A be a subset of X. Then prove that
   i. $bd(A) = \bar{A} \cap (\bar{A'})$ and hence the boundary of A is closed.
   ii. $\bar{A} = A^o \cup bd(A)$
22. Let (X, τ) is a topological space then prove that the subfamily β of τ is a base for τ iff or each $U \in \tau$ and for each $x \in U$ there exists $B_x \in B$ such that $x \in B_x \subset U$.
23. Let X={a, b, c} and τ = {X, Φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}} be a topology on X, then show that $\beta = \{\Phi, \{a\}, \{b\}, \{c\}\}$ is a base for τ.
24. Let X={a, b, c, d, e} and τ = {X, Φ, {a}, {b, c}, {d, e}, {a, b, c}, {a, d, e}, { b, c, d, e}} be a topology on X. Prove that β = {Φ, {a}, {b, c}, {d, e}}is a base for τ and write a sub-base for this topology.
25. Define base and sub base. If X = {a, b, c} and τ = {X, Φ, {a}, {c}, {a, b}, {a, c}} be a topology on X, then show that $\beta = \{\Phi, \{a\}, \{c\}, \{a, b\}\}$ is a base for τ.
26. Prove that $\tau_Y = \{Y \cap G : G \in \tau\}$ is a topology on $Y \subset X$ in a topological space (X, τ).
27. Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that a non-empty subset A of Y is τ_Y-closed iff A is intersection of Y with τ-closed set.
28. Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that a non-empty subset A of Y is τ_Y-open iff A is intersection of Y with τ-open set.
29. Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that the closer of a non-empty subset A in Y is intersection of Y with closer of A in X.
30. Prove that every subspace of a T_1 space is a T_1 space.
31. Define hereditary property and prove that the property of being T_2 is hereditary.
32. Define T_1-space and T_2-space. Show that every T_2-space is T_1-space.
33. Define base &subbase P.T S={ϕ,{a,b},{b,c},{a,c}} form a sub base for discrete topology on X={a,b,c}.

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DEPARTMENT OF MATHEMATICS

B.Sc V Sem

Paper I- Real Analysis Question Bank

For the year 2019-20

Paper I –Real Analysis

I Two mark Questions

1. a) Find supremum and infimum of f(x) if (i) 1+2sinx in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (ii) |x| in [-1, 1] (iii) $\frac{1}{1+2\cos^2 x}$ in $[0, \pi]$. (iv) $\frac{1}{4\cos^2 x+9\cos^2 x}$ in $[0, \pi]$.
2. Define upper and lower sums of bounded function f(x) in [a,b].
3. Calculate L(P, f) and U(P,f) for given function f(x) (i) if f(x) = sinx and $P = \{0, \frac{\pi}{3}, \frac{\pi}{2}\}$ of $[0, \frac{\pi}{2}]$ (ii) f(x) = 2x+3 if P = {1, 2, 3} of [1,3].
4. Define norm of a partition and if (i) P ={1, 1.2, 1.4, 1.7, 1.8 , 2} of [1,2] find norm of P (ii) $Q = \{0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{4}, 2\frac{\pi}{3}, \pi\}$ of $[0, \pi]$ find norm of Q
5. Define upper and lower integrals of f(x) in [a, b]
6. Prove that $L(P, f) \leq U(P, f)$
7. If M and m are bounds of the function f(x) on [a,b] then (or with usual notation) prove that $m(b-a) \leq s \leq S \leq M(b-a)$
8. Show that constant function is R-integrable.
9. 6. Give an example of a function to illustrate that every bounded
10. function in a closed interval is not R-integrable.
11. State necessary and sufficient condition for a bounded function to be R-integrable.
12. i) If f(x) = 1 when x is rational

    = 0 when x is irrational

    show that f(x) is bounded but not R integrable.

    ii) If f(x) = 1 when x is rational

    = -1 when x is irrational

    show that f(x) is bounded but not R integrable.
13. State First mean value theorem of Integral Calculus.
14. State second mean value theorem of Integral Calculus.

15. i) State Bonnet's form of second mean value theorem of Integral calculus.

ii) State Weiertass's form of second mean value theorem of Integral calculus.

16. Prove that $_a\int^b (\sin x^2) dx \le 1/a$

17. Discuss the convergence of $_0\int^1 dx/ [\sqrt{(1-x^2)}]$

18. Examine the convergence of $_0\int^1 dx/ [x^3 (1+x^2)]$

19. Show that the improper integral $_1\int^\infty [(\sin^2 x)/x^2]\, dx$ is convergent

20. Show that the improper integral $_1\int^2 dx/(2-x)$

21. Show that the improper integral $_a\int^b dx/(x-a)^n$ is convergent iff $n<1$

22. Show that the improper integral i) $_0\int^\infty dx/ [1+x^2]$ is convergent

i.ii) $_0\int^\infty dx/ [4a^2+x^2]$ is convergent

23. Show that the improper integral $_1\int^\infty dx/ [4-x^2]$ is convergent

24. Show that the improper integral $_1\int^\infty dx/ x$ is divergent

25. Prove that $_0\int^{\sqrt{2}} dx/ [\sqrt{(2-x)}]$ is convergent.

26. State Abel's Theorem for convergence of improper integral of product of two function.

27. State Dirichlet's Test for convergence of improper integral of product of two function.

28. Define Gamma – Beta functions.

29. Prove that $\Gamma(1/2) = \pi/2$

30. Prove that $n\Gamma n = {_0\int^\infty} e^{y^{-1/n}}\, dy$

31. Show that $\beta(1/2, ½) = \pi$

32. Prove that (i) $\beta(1,1) = 1$ where $0! = 1$ (ii) $\beta(2,2) = 1/6$.

33. Prove that $\beta(m,n) = \beta(n,m)$

34. Evaluate $_0\int^1 x^5(1-x^2)\, dx$

35. Evaluate $_0\int^\infty x^4(e^{-x^2})\, dx$

36. Prove that $\Gamma(n+1) = n (\Gamma n)$ or $\Gamma(n) = (n-1)\, \Gamma(n-1)$

37. Evaluate $_0\int^1 (\log x)^4 dx$

38. Prove that $_0\int^\infty [x^8(1-x^6)/(1+x^{24})]\, dx = 0$

39. Express the integral $_0\int^1 dx/\sqrt{(1-x^2)}$ in terms of Gamma function.

40. Express the integral $_0\int^1 /\sqrt{(x} /\sqrt{(1-x)}$ dx in terms of Beta function.

41. 35. Evaluate $_0\int^1 {_0\int^1}$ dx dy $/[\sqrt{(1-x^2)}\sqrt{(1-y^2)}]$

42. Evaluate $_0\int^3 {_0\int^2} x^2y^2$ dx dy

43. Evaluate $_0\int^1 {_1\int^2} x^3y^3$dx dy

44. Evaluate $_0\int^1 {_1\int^2} (x+y)$dx dy

45. Evaluate $_0\int^1 {_{\sqrt{y}}\int^1}$ dx dy

46. Evaluate $_0\int^1 {_1\int^2} (x^2 +y^2)$dx dy

47. Evaluate $_0\int^3 {_1\int^2} xy(x+y)$dx dy

48. Evaluate $_0\int^1 {_0\int^2} {_1\int^2} x^2yz$ dz dy dx

49. Evaluate $_0\int^1 {_1\int^3} {_1\int^2} xy^2z$ dx dy dz

50. Evaluate $_0\int^1 {_1\int^2} {_2\int^3}$ dx dy dz

51. Evaluate $_0\int^3 {_0\int^2} {_0\int^1} (x+y+z)$ dx dy dz

52. Evaluate $_0\int^1 {_0\int^2} {_1\int^2} x^2yz$ dx dy dz

53. Evaluate $_0\int^1 {_0\int^1} {_0\int^1} (x+y+z)$ dz dx dy

54. Evaluate $_0\int^{\pi/2} {_0\int^a} r^2 \sin\theta\, dr\, d\theta$

55. Prove that $_0\int^1 {_1\int^1} (x^2 +y^2)$dx dy $=2/3$

Q.No. II Five Mark Questions

56. If a function f(x) is R-integrable and bounded in [a,b] then prove that $m(b-a) \leq {_a\int^b} f(x)\, dx \leq M(b-a)$

57. If f is bounded on [a,b] then $\forall\varepsilon>0$ there corresponds $\delta>0$ such that for every division $D = \{a=x_0, x_1,x_2,x_3,\text{-------}x_n= b\}$ with $\|\Delta\|<\delta$ choose $\xi\in[x_{r-1}, x_r]$ such that $\left|\sum f(\xi_r)(x_r - x_{r-1}) - {_a\int^b} f(x)\, dx\right| < \varepsilon$

58. A function f defined in [0,1] as $f(x) = 1/a^{r-1}$ where $1/a^r <x<1/a^{r-1}$ for r= 1,2-
0 where x = 0 where a is an integer

59.then show that $_0\int^1 f(x)$ dx exists and equal to $a/(a+1)$

60.If f is bounded function then prove that $_a\int^b f(x)\, dx \leq {_a\overline{\int}^b} f(x)\, dx$

61.(i) If $f(x) = 1/2^n$ where $1/2^{n+1}<x<1/2^n$ for n=0,1,2,3------
& f(0) = 0, show that f(x) is integrable in [0,1] and evaluate $_0\int^1 f(x)$ dx.

62. State and prove necessary and sufficient conditions for the function f(x) to be R-integrable.

63.Prove that every continuous function defined on [a,b] is R integrable .

64.Prove that a bounded and monotonic function f : [a,b] → R is integrable

65.If f(x) = 0 when x is rational

= 1 when x is irrational defined on [a,b] , show that $f \notin R[0,1]$

66.i) If f is defined on [4,5] by f(x) = x ∀ x ∈[4,5], then prove that f ∈ R[0,1] and $_0\int^1 f(x)\,dx = 1/2$

ii) If f is defined on [0,a] by $f(x) = x^2$ ∀ x ∈[0,1], then prove that f (x) is R-integrable and $_0\int^a f(x)\,dx = a^3/3$

iii) If f is defined on [0,a] by f(x) = 2x+5 ∀ x ∈[1,2], then prove that f (x) is R-integrable and evaluate $_1\int^2 f(x)\,dx$.

67.State and prove First mean value theorem of Integral Calculus.

68.State and prove second mean value theorem of Integral Calculus.

69. State and prove Bonnet's form of second mean value theorem of Integral calculus.

70.State and prove Wiestrass form of second mean value theorem of Integral calculus.

71.State and prove Fundamental Theorem of Integral Calculus.

72.If f(x) and g(x) are bounded and integrable then i) f(x) + g(x) ii) f(x) – g(x) iii) f(x).g(x) iv) f(x) / g(x) when g(x) ≠ 0 are R-integrable in [a,b] and also prove that i) $_a\int^b [f(x) + g(x)]\,dx = {}_a\int^b f(x)dx + {}_a\int^b g(x)\,dx$

ii) $_a\int^b [f(x) - g(x)]\,dx = {}_a\int^b f(x)\,dx - {}_a\int^b f(x)\,dx$

73. If f(x) is bounded and integrable in [a,b] then prove that $|f(x)|$ is also integrable in[a,b]. Give an example to show that the converse is not true.

74.Prove that x> [log(1+x)] > [x/(1+x)] ∀ x >0

75.Show that $_0\int^\pi [x\,dx / (a \cos^2 (x/2)+ b \sin^2 (x/2))]dx$ lies between $\pi^2/2a$ & $\pi^2/2b$.

76.Show that $_0\int^\pi [x^2\,dx / (5+3\cos x)]$ lies between $\pi^3/24$ & $\pi^3/6$.

77.Prove that $\sin^{-1}x \le {}_0\int^1 [dx/ (\sqrt{1-x^2})(\sqrt{1-\lambda x^2})] \le \sin^{-1}x /(\sqrt{1-\lambda x^2})$ by first mean value theorem where λ, $x \in (0,1)$

78. Prove that $1/\pi \leq \int_0^x [\sin\pi x/(1+x^2)]\, dx \leq 2/\pi$ using mean value Theorem.

79. Show that $\frac{1}{2} < \int_0^1 dx/\sqrt{(4-x^2-x^3)} < \pi/6$

80. Using first mean value theorem of integral calculus prove that

$\pi^3/192 < \int_0^{\pi/2} [x^2 / (5+3\cos x)]dx < \pi^3/120$

81. Using first mean value theorem of integral calculus prove that

$\pi^2/9 \leq \int_{\pi/6}^{\pi/2} x/\sin x\, dx \leq (2\pi^2)/9$

82. Prove that $1/(6) \leq \int_0^1 [x^2/\sqrt{(1+3x^2)}]\, dx \leq 1/2$

83. (i) Prove that $\pi/6 \geq \int_0^{1/2} dx/\sqrt{(1+x^{2n})} \geq \frac{1}{2}$ (ii) $\pi/4 \leq \int_0^{\frac{\pi}{4}} \sec x\ dx \leq \frac{\pi}{2\sqrt{2}}$

84. Show that $\int_0^{\pi/2} \sqrt{\sin x}\ dx$ lies between 1 and $\sqrt{(\pi/2)}$

85. Prove that $\left| \int_p^q (\sin x/x)dx \right| \leq 2/p$ where $p,q > 0$.

86. The improper integral $\int_a^b dx/(x-a)^n$ is convergent iff $n<1$.

87. If f(x) and g(x) are +ve in[a,b] and $f(x) \leq g(x)\ \forall\ x \in [a,b]$ then prove that

i) if $\int_a^b g(x)\, dx$ is convergent then $\int_a^b f(x)\, dx$ is also convergent

ii) If $\int_a^b f(x)\, dx$ is divergent then $\int_a^b g(x)\, dx$ is also divergent

88. If f(x) and g(x) are two functions defined on [a,b] & a is a point of infinite discontinuity and $\text{lt}\ [f(x)/g(x)] = l$ where $l \neq 0$ & $\neq \infty$ then show that $\int_a^b f(x)\, dx$ and $\int_a^b g(x)\, dx$ behave alike.

89. State and prove Dirichlit's Test for the convergence of an of improper integral of product of two function.

90.35. State and prove Abel's Theorem for convergence of improper integral of product of two function.

91. Prove that $\int_0^\infty [(e^{-ax} \sin x)/x]\, dx$ $a>0$ is convergent.

92. Prove that $\int_0^\infty [\sin x/x]\, dx$ is convergent

93. Test the convergence of $\int_0^1 (1/x) \sin(1/x)\, dx$

94. Show that $\int_0^\infty \sin x^p\, dx$ is convergent if $p>1$

95. Discuss the convergence of i) $\int_0^\infty dx /[x^{1/3}(1+x)^{1/2}]$ ii) $\int_0^\infty e^{-x} x^{n-1}\, dx$ $n>0$

96. Test the convergence of i) $\int_0^1 dx /[x^2(1+x^2)]$ ii) $\int_0^\infty [x^{2m-1} / (1+x^{2n})]\, dx$

97. Test the convergence of i) $\int_0^1 dx /[x^{1/2}(1+x)^{1/3}]$ ii) $\int_0^\infty [\sin x/x^2]\, dx$

98. Test the convergence of $_0\int^{\infty} [(1-e^{-x})\cos x))/x^2]\, dx$ where a>0

99. Test the convergence of $_1\int^{\infty} [(e^{-x}\sin x)/x^2]\, dx$.

100. a) Examine the convergence of $_0\int^{2} [dx/(1+x)\sqrt{(2-x)}]$ ii) $_0\int^{1} dx/\sqrt{x}$

b) Examine the convergence of $_0\int^{1} [dx/(x^2)(1+x)^2]$ ii) $_0\int^{\infty} dx/\sqrt{x}(1+x)$

c) Test the convergence of (i) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x^p}$ dx (ii) $\int_0^{\frac{\pi}{2}} x^n \operatorname{cosec}^n x$ dx **(refer Real analysis N.P. Bali)**

101. Examine the convergence of $_0\int^{1} [dx/x^2(1+x)^2]$ ii) $_1\int^{\infty} dx/\sqrt{x}(1+x)$

102. Discuss the convergence of i) $_0\int^{\infty} (\sin^2 x)/x^2\, dx$ ii) $_0\int^{\infty} x^{2m}/(1+x^{2n})\, dx$

103. Establish the relation between Beta and Gamma function.

104. (i) Prove that $_0\int^{1} [(x^{m-1} + x^{n-1})/(1+x)^{m+n}]\, dx = \beta(m,n)$ where m and n are +ve reals.

105. Prove that $_0\int^{1} [x^{m-1}(1-x)^{n-1}/(a+x)^{m+n}]\, dx = \beta(m,n)/a^n(1+a)^m$ where m and n are +ve reals.

106. Test the convergence of $_0\int^{1} x^{m-1}(1-x)^{n-1}\, dx$

OR

Test the convergence of Beta function

107. Prove Duplication formula $\Gamma m\, \Gamma(m+1/2) = [\sqrt{\pi}\, \Gamma 2m]/(2m-1)$

108. Prove that $_0\int^{\infty} x\, e^{-x^8}\, dx \;\; _0\int^{\infty} x^2\, e^{-x^4}\, dx = \pi/(16\sqrt{2})$

109. Test the convergence of $_0\int^{\infty} e^{-x} x^{n-1} dx$ OR Test the convergence of Gamma function.

110. Prove that $_0\int^{\pi/2} \sin^p x \cos^q x\, dx = \frac{1}{2}\beta((p+1)/2),\ (q+1)/2))$

$= [(\Gamma(p+1)/2)\,(\Gamma(q+1)/2)]/\{2\Gamma[(p+q+2)/2$

111. Prove that (i) $_0\int^{\pi/2} \sin^p x\, dx \;\; _0\int^{\pi/2} \sin^{p+1} x\, dx = \pi/2(1+p)$

(ii) $_0\int^{\infty} e^{-tx} x^{n-1} dx = \frac{\lceil n}{t^n}$

112. Show that $_0\int^{\infty} x^4\sqrt{(a^2-x^2)}\, dx = (\pi a^6)/32$

113. Show that $_0\int^{\pi/2} d\theta/\sqrt{\sin\theta} \;\; _0\int^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$

114. Show that $_0\int^{1} x^3(1-x^2)^{5/2} = 2/63$

115. Evaluate $_0\int^{\pi/2} \sqrt{\tan\theta}\, d\theta$

116. Show that $_0\int^{\infty} {_0\int^{\infty}} x e^{x^2/y}\, dx\, dy = \frac{1}{2}$

117. Show that ${}_0\int^{\infty} \sqrt{y}\ e^{-y^2}\ dy\ {}_0\int^{\infty}(1/\sqrt{y})\ e^{-y^2}\ dy = \pi/(2\sqrt{2})$

118. With usual notations prove that $\frac{d}{dx}[{}_a\int^b f(x,y)\ dx] = {}_a\int^b f_y(x,y)\ dx$

OR

State and prove Leibnitz's Test for differentiation under integral sign.

119. Prove that ${}_0\int^{\infty} [(e^{-ax} \sin bx)/x]\ dx = \tan^{-1}(b/a)$ and deduce that ${}_0\int^{\infty} [(\sin bx)/x]\ dx = \pi/2$

120. Prove that ${}_0\int^{\pi} [\{\log(1+a\cos x)\}/\cos x]\ dx = \pi \sin^{-1}a$ by differential under integral sign.

121. Prove that ${}_0\int^{\pi/2} [\log\{(1+\cos\alpha\cos x)\}/\cos x]\ dx = (\pi^2 - 4\alpha^2)/8$

122. (i) Prove that ${}_0\int^{\pi} [\log(1+a\cos x)]\ dx = \pi \log(1+\sqrt{(1-a^2)})$ where $|a|<1$

(ii)Prove that ${}_0\int^{\infty} e^{-x^2} \cos\alpha x\ dx = (\sqrt{\pi}/2)e^{-\alpha^2/4}$

123. Prove that ${}_0\int^{\pi/2} [\log(a^2\cos^2x + b^2 \sin^2x)\ dx = \pi \log[(a+b)/2]$

124. Prove that ${}_0\int^1 [(x^a - x^b)/\log x] = \log[(b+1)/(a+1)]$

125. Evaluate ${}_0\int^1 {}_0\int^{\sqrt{1-x^2}} {}_0\int^{\sqrt{1-x^2-y^2}} xyz\ dx\ dy\ dz$

126. Evaluate $\int\int (x^2+y^2)\ dx\ dy$ over the circle $x+y\leq 1$

127. Evaluate $\int\int (x^2+y^2)^{7/2}\ dx\ dy$ over the circle $x^2+y^2=1$

128. Find the volume of tetrahedron bounded by coordinate planes & (i) the plane $x/a+ y/b + z/c = 1$ (ii) the plane $x+y+z= 1$

129. Using integration find the volume of ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 =1$

130. Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ by triple integration.

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PART- B.

2) Statement :- Let 'x' be any topology on non empty set and 'J' be the collection of empty set and all those subsets of 'x' whose compliment is countable then 'J' is topology on 'x' is called countable topology.

Proof :- Let 'x' be any non-empty set
$J = \{\phi, A \subset X / A' \text{ is countable}\}$
We have to prove 'J' is topology on x.
T_1: Let $x' = \phi$ is countable.
$x \in J$.

T_2: $\phi \in J$ (by given hypothesis).

T_3: Let $A_\lambda \in J$ be $\forall \lambda \in I$.
claim $\bigcup_{\lambda \in I} A_\lambda \in J$.

Consider $\left(\bigcup_{\lambda \in I} A_\lambda\right)' = \bigcap_{\lambda \in I} A_\lambda'$.

clearly RHS of the given expression is countable being intersection of countable set.

$\left(\bigcup_{\lambda \in I} A_\lambda\right)'$ is countable.

$\therefore \bigcup_{\lambda \in I} A_\lambda \in J$.

Arbitrary union of members of 'J' is again a members of 'J'.

T_4: Let $A_1, A_2 \in J$
$\Rightarrow A_1' \& A_2'$ is are countable.
claim $A_1 \cap A_2 \in J$.

Consider $(A_1 \cap A_2)' = A_1' \cup A_2' \in J$ (By De-morgans law)
$\Rightarrow$ Union of countable set is countable.
$(A_1 \cap A_2)'$ is countable.
$\therefore A_1 \cap A_2 \in J$.
finite intersection of members of 'J' is again a

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members of 'J' ~~only~~
Hence 'J' is topology on X.

4) Answer:- By defination.

$$F(S) = L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt .$$

Integrating on both the sides ~~from~~ 'S' to '∞' w.r.t. 'S' we get.

$$\int_S^\infty F(S)\, dS = \int_S^\infty \left[\int_0^\infty e^{-st} f(t)\, dt\right] dS$$

$$= \int_0^\infty \left[\int_S^\infty e^{-st} f(t)\, dS\right] dS. \quad [\because \text{by integration under integral sign}]$$

$$= \int_0^\infty \left[\frac{e^{-st}}{(-t)} \cdot f(t)\right]_S^\infty dt$$

$$= \int_0^\infty \left[\left(0 + \frac{e^{-st}}{t}\right)\right] f(t)\, dt .$$

$$= \int_0^\infty \frac{e^{-st}}{t} f(t)\, dt .$$

$$= L\left[\frac{f(t)}{t}\right]$$

$$\therefore \int_S^\infty F(S)\, dS = L\left[\frac{f(t)}{t}\right]$$

$$\boxed{\therefore L\left[\frac{f(t)}{t}\right] = \int_S^\infty F(S)\, dS}$$

6). Proof:- By defination we have.

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

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(3)

$$= \int_0^T e^{-st} f(t)\,dt + \int_T^{2T} e^{-st} f(t)\,dt + \int_{2T}^{3T} e^{-st} f(t)\,dt + \cdots$$

Put $t = u+T$ in second integral. and put $t = u+2T$ in third integral and so on.

if $t = T \Rightarrow u = 0$
$t = 2T \Rightarrow u = T$, } $dt = du$.

and $t = 2T \Rightarrow u = 0$
$t = 3T \Rightarrow u = T$. } $dt = du$.

$$L[f(t)] = \int_0^T e^{-st} f(t)\,dt + \int_0^T e^{-s(u+T)} f(u+T)\,du + \int_0^T e^{-s(u+2T)} f(u+2T)\,du + \cdots$$

$$L[f(t)] = \int_0^T e^{-su} f(u)\,du + e^{-sT}\int_0^T e^{-su} f(u)\,du + e^{-2sT}\int_0^T e^{-su} f(u)\,du + \cdots$$

[$\because$ f(t) is periodic fn with period f(t) is T]

$f(u+T) = f(u)$
$f(u+2T) = f(u)$

$$L[f(t)] = \left(1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right)\int_0^T e^{-su} f(u)\,du.$$

$$L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-su} f(u)\,du.$$

{ $\because (1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots)$ is an infinite Geometric series with $a=1$ & common ratio $r = e^{-sT}$. $\therefore S_\infty = \frac{a}{1-r} = \frac{1}{1-e^{-sT}}$ }

Replacing all 'u' in terms of 't'

$$L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\cdot dt.$$

i.e. $$L[f(t)] = \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}.$$

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5) Sol^n :- Given. $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$

$$\Rightarrow \frac{4s+5}{(s+1)^2(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{(s+2)} \quad \text{--- (1)}$$

⇒ Multiplying $(s+1)^2(s+2)$ we get.

$$4s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$$

⇒ put $s=-2$ we get

$$4(-2)+5 = A(-1)(0) + 0 + C(-1)^2$$

$$-8+5 = C$$

$$\boxed{C=-3}$$

⇒ Again put $s=-1$

$$-4+5 = 0 + B(1) + 0$$

$$\boxed{B=1}$$

Now. $0 = A + C$

$$A = -C$$

$$\boxed{A=3}$$

Substituting above values in eq^n (1) we get.

$$\frac{4s+5}{(s+1)^2(s+2)} = \frac{3}{(s+1)} + \frac{1}{(s+1)^2} + \frac{(-3)}{(s+2)}$$

$$L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\} = 3L^{-1}\left\{\frac{1}{(s+1)}\right\} + L^{-1}\left\{\frac{1}{(s+1)^2}\right\} - 3L^{-1}\left\{\frac{1}{(s+2)}\right\}$$

$$L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\} = 3e^{-t} + e^{-t}t - 3e^{-2t} \quad //$$

PART - C

8) a) Sol^n :- In real space $(\mathbb{R}, u)$.
Consider the closed sets
$I_n = \left[\frac{1}{n}, 2-\frac{1}{n}\right]$ for. $n \in N$.

$$\bigcup_{n\in N} I_n = \bigcup_{n\in N}\left[\frac{1}{n}, 2-\frac{1}{n}\right] = [1,1] \cup \left[\frac{1}{2}, \frac{3}{2}\right] \cup \left[\frac{1}{3}, \frac{5}{3}\right] \cup \dots$$

$$\dots \cup \lim_{n\to\infty}\left[\frac{1}{n}, 2-\frac{1}{n}\right]$$

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$= (-0, 2)$ which is not closed in $(\mathbb{R}, u)$
$\therefore$ Arbitory union of closed sets is not closed.

8) b) Sol^n :- Given $X = \{a, b, c, d\}$
$\mathcal{T} = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$

i) Let $b \in X$.
$\exists$ open set $\{a, b\}$
s.t. $b \in \{a, b\} \subset A$.

Where 'A' is any subset of 'X' containing 'b'.

$\therefore N(b) = \{\{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}$

Next Let $d \in X$.
$\exists$ an open set 'X' such that

$d \in X \subset A$.

Where 'A' is any subset of X containing 'd'.

$\therefore N(d) = \{X\}$ //.

9) a) Proof :- Let $(X, \mathcal{T})$ is T_2-space.
We have to prove $(X, \mathcal{T})$ is T_1-space.
Let $x, y \in X$ be two distinct points.
Since $(X, \mathcal{T})$ is T_2-space
$\therefore \exists$ two disjoint open sets $U \& V$ s.t.
$x \in U$ & $y \in V$.
$\Rightarrow x \in U$ & $x \notin V$.
$\Rightarrow x \in U$ & $y \notin U$
$\therefore$ for every pair of distinct point $x \neq y$, in 'X' $\exists$ an open set U containing 'x' (

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not containing y.
$\Rightarrow (X, \mathcal{J})$ is T_1-space.
Hence every T_2-space is T_1-Space.

But the converse of above Thm need not be true.
i.e every T_1-space is need not be T_2-space
for example :- The co-finite topology defined on infinite set 'X'.
Since co-finite topology is T_1-Space, but not T_2-space.
Let $x, y \in X$ & $x \neq y$
$\{x\}$ & $\{y\}$ are closed sets.
$\Rightarrow \{x\}'$ & $\{y\}'$ are open sets.

s.t $y \in \{x\}'$ & $x \in \{y\}'$.
and $\{x\}' \cap \{y\}' = X - \{x, y\} \neq \phi$.

$(X, \mathcal{J})$ is not T_2-space.

Q) b) Herediatary property :- A property 'p' of a topological space $(X, \mathcal{J})$ is said to be herediatery property if every subspace of a topological space $(X, \mathcal{J})$ have the property 'p'.

Proof :- Let $(X, \mathcal{J})$ is T_1-space and $(Y, \mathcal{J}_Y)$ is subspace of $(X, \mathcal{J})$.
claim :- $(Y, \mathcal{J}_Y)$ is T_1-space.
Let $y \in Y$ & $Y \subseteq X$.

$\Rightarrow y \in X$.
Since $(X, \mathcal{J})$ is T_1-space.
$\Rightarrow \{y\}$ is closed in X.

$\Rightarrow Y \cap \{y\}$ is closed in Y
$\Rightarrow \{y\}$ is closed in Y $\forall y \in Y$.

$\therefore (Y, \mathcal{J}_Y)$ is T_1-space.
Hence every subspace of T_1-space is T_1-space.

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10) a) Given $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$

Consider.

$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)} \quad (1)$$

Multiplying above eqn by $s(s+1)(s+2)$ we get:

$$1 = A(s+1)(s+2) + Bs(s+2) + C(s+1)s.$$

Now put $s=-1$ in above eqn.

$$1 = 0 + B(-1)\cdot(1) + 0.$$

$$\boxed{B = -1}$$

Again put $s=-2$.

$$+1 = 0 + 0 + C(-2)(-1).$$

$$+1 = 2C.$$

$$\boxed{C = +1/2}$$

Now equating the co-efficient of s^2 on both the sides we get.

$$0 = A + B + C$$

$$A = -B - C$$

$$A = 1 - 1/2$$

$$\boxed{A = 1/2}$$

Now substituting these values in eqn (1) we get.

$$\frac{1}{s(s+1)(s+2)} = \frac{1}{2s} + \frac{(-1)}{(s+1)} + \frac{1}{2(s+2)}.$$

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{1}{2}L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{s+2}\right]$$

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{1}{2}\cdot 1 - e^{-t} + \frac{1}{2}e^{-2t}.$$

i.e $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}.$

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10) b) : Given $L[f(t)] = F(S)$ and $g(t) = \begin{cases} f(t-a), & t>a. \\ 0, & t<a. \end{cases}$

proof :- Let $L[g(t)] = \int_0^\infty e^{-st} g(t)\, dt.$

$$= \int_0^a e^{-st} g(t)\, dt + \int_a^\infty e^{-st} g(t)\, dt.$$

$$= \int_0^a e^{-st}\, 0\, dt + \int_0^\infty e^{-st} f(t-a)\, dt.$$

$$L[g(t)] = \int_0^\infty e^{-st} f(t-a)\, dt.$$

put $t-a = u.$
$dt = du.$
when. $t=a, \quad u=0.$
$t=\infty, \quad u=\infty$

$$L[g(t)] = \int_0^\infty e^{-s(u+a)} f(u)\, du.$$

$$L[g(t)] = e^{-sa}\int_0^\infty e^{-su} f(u)\, du.$$

$$L[g(t)] = e^{-as} F(S).$$

12) a) statement :— If $L[f(t)] = F(S)$ & $L[g(t)] = G(S)$
then.
$$L^{-1}[F(S)G(S)] = \int_0^t f(u)\, g(t-u)\, du.$$
$$= f(t) * g(t).$$

proof :—
Let $L[f(t)] = F(S)$ & $L[g(t)] = G(S).$

Consider $L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] = \int_{t=0}^{t=\infty} e^{-st}\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] dt$

$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] = \int_{t=0}^{t=\infty}\int_{u=0}^{u=t} e^{-st}\, f(u)\, g(t-u)\, du\, dt.$$

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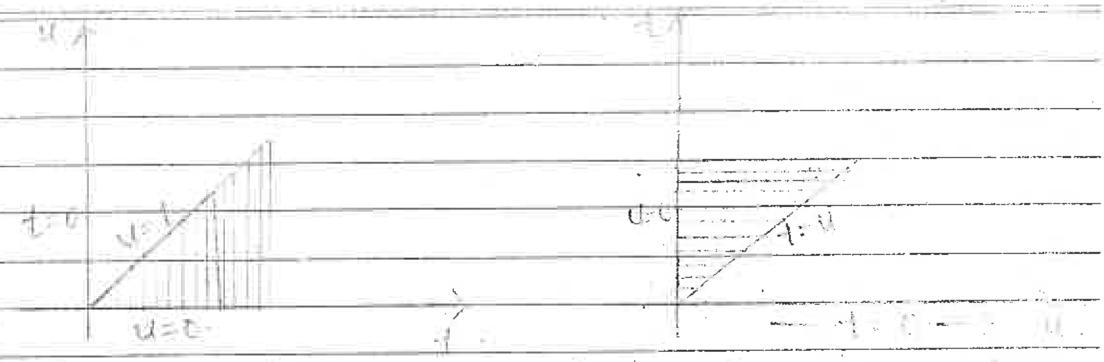

The existing reason is $t=0$ to $t=\infty$
$u=0$ to $u=t$.

On changing the co-ordinate axes (on changing the order of integration).
$u=0$ to $u=\infty$.
$t=u$ to $t=\infty$.
(In both the sides cases area remains same)
$\therefore$ Equation ① reduces to

$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] = \int_{u=0}^{u=\infty} \int_{t=u}^{t=\infty} e^{-st} f(u)\, g(t-u)\, dt\, du.$$

put $t-u=v$.
$\therefore t=u+v$.
$\therefore dt = dv$

$\therefore t=u \rightarrow v=0$ & $t=\infty \rightarrow v=\infty$.

$$\therefore L\left[\int_{u=0}^{u=t} f(u)\cdot g(t-u)\, du\right] = \int_{u=0}^{u=\infty} \int_{v=0}^{v=\infty} e^{-s(u+v)} f(u)\cdot g(v)\, dv\cdot du.$$

$$= \int_{u=0}^{u=\infty} e^{-su} f(u)\, du \cdot \int_{v=0}^{v=\infty} e^{-sv} g(v)\, dv$$

$$= L[f(t)]\, L[g(t)]$$

$$= F(s)\, G(s).$$

$$\Rightarrow L[f(t) * g(t)] = F(s)\, G(s)$$

$$\therefore L^{-1}[F(s)\, G(s)] = f(t) * g(t)$$

12) b) Soln :- Given. $L^{-1}\left[\frac{1}{(s^2+4s+1)^2}\right]$

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$= L^{-1}\left[\frac{1}{((s+2)^2-3)^2}\right]$

Let $F(s) = \frac{1}{(s+2)^2-3}$

and $G(s) = \frac{1}{(s+2)^2-3}$

$\therefore L^{-1}[F(s)] = L^{-1}\left[\frac{1}{(s+2)^2-3}\right] = e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}}$

and $L^{-1}[G(s)] = L^{-1}\left[\frac{1}{(s+2)^2-3}\right] = e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}}$

$\therefore L^{-1}[F(s)\cdot G(s)] = f(t) * g(t)$

$= L^{-1}[F(s)] * L^{-1}[G(s)]$

$= e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}} * e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}}$

$= \int_0^t e^{-2u}\frac{\sinh\sqrt{3}u}{\sqrt{3}}\cdot e^{-2(t-u)}\frac{\sinh\sqrt{3}(t-u)}{\sqrt{3}}\,du$

$= \frac{1}{3}\int_0^t e^{-2u}e^{-2(t-u)}\sinh\sqrt{3}u\cdot\sinh\sqrt{3}(t-u)\,du.$

$= \frac{1}{3}\int_0^t e^{-2t}\cdot\sinh\sqrt{3}u\,\sinh\sqrt{3}(t-u)\,du.$

$= \frac{1}{3}\int_0^t e^{-2t}\left\{\frac{e^{\sqrt{3}u}-e^{-\sqrt{3}u}}{2}\right\}\left\{\frac{e^{\sqrt{3}(t-u)}-e^{-\sqrt{3}(t-u)}}{2}\right\}du.$

$= \frac{1}{3}\cdot\frac{1}{4}\int_0^t e^{-2t}\left(e^{\sqrt{3}u}-e^{-\sqrt{3}u}\right)\left(e^{\sqrt{3}t}\cdot e^{-\sqrt{3}u} - e^{-\sqrt{3}t}\cdot e^{\sqrt{3}u}\right)du.$

$L^{-1}[F(s)\cdot G(s)] = \frac{1}{12}\int_0^t e^{-2t}\left(e^{\sqrt{3}u}-e^{-\sqrt{3}u}\right)\left(e^{\sqrt{3}t}\cdot e^{-\sqrt{3}u} - e^{-\sqrt{3}t}\cdot e^{\sqrt{3}u}\right)du.$

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PART-A

a) Proof :- Let us consider

$\{x\}' = (-\infty, x) \cup (x, \infty)$.

$\{x\}'$ = union of open sets.

$\{x\}'$ = open set

$\Rightarrow$ $\{x\}$ is closed set.

$\Rightarrow$ In real space $(\mathbb{R}, u)$, every singleton set is closed.

b) Soln :- Given. $X = \{1, 2, 3, 4\}$

i) $\mathcal{J}_1 = \{X, \phi, \{1\} \{2\} \{1, 2\}\}$

$\mathcal{J}_2 = \{X, \phi, \{1\}\}$

$\Rightarrow$ $\mathcal{J}_1$ & $\mathcal{J}_2$ are comparable.
$\because$ $\mathcal{J}_2 \subset \mathcal{J}_1$.

ii) $\mathcal{J}_3 = \{X, \phi, \{2\}\}$

$\Rightarrow$ $\mathcal{J}_1$ & $\mathcal{J}_3$ are comparable.
$\because$ $\mathcal{J}_3 \subset \mathcal{J}_1$.

iii) $\mathcal{J}_4 = \{X, \phi\}$

$\Rightarrow$ $\mathcal{J}_1$ & $\mathcal{J}_4$ are comparable.
$\because$ $\mathcal{J}_4 \subset \mathcal{J}_1$.

c) * Base :- Let $(X, \mathcal{J})$ be any topological space. Then the subfamily 'β' of '$\mathcal{J}$' is called as base for '$\mathcal{J}$' if every open set in '$\mathcal{J}$' is expressed as union of members of 'β'.

* Sub-base :— Let (X, J) be a topological space. A collection 'S' of a subset of 'X' is said to be sub-base if

i) S ⊂ J.

ii) collection of finite intersection of members of 'S' form a base.

d) T_1 - space :— A topological space (X, J) is called T_1 - space iff every singleton set is closed in (X, J).

for example :—

i) Real space (R, u) is T_1-space.

f) Laplace transform :— Let f(t) be a fn of real variable 't'. defined for t≥0. Laplace transform of f(t) is denoted by L[f(t)] and is defined as

$$\int_0^\infty e^{-st} f(t)\, dt$$ provided the integral exist

* Given $L[e^{5t}] = \left[\frac{1}{s-5}\right]$ //.

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g) Statement :- If Laplace transform of $f(t)$ is $F(s)$ then $L[e^{at} f(t)] = F(s-a)$.

proof :-

We have. $L[e^{at} f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s)$.

$$\therefore L[e^{at} f(t)] = \int_0^\infty e^{-st} \cdot e^{at} \cdot f(t)\, dt$$

$$L[e^{at} f(t)] = \int_0^\infty e^{-(s-a) \cdot t} f(t)\, dt$$

$$L[e^{at} f(t)] = F(s-a).$$

h) Soln :-

$$L[t^n] = \int_0^\infty e^{-st}\, t^n\, dt$$

put $st = z$

$t = z/s$

$dt = \frac{dz}{s}$

if $t = 0 \Rightarrow z = 0$.

$t = \infty \Rightarrow z = \infty$

$$L[t^n] = \int_0^\infty e^{-z} \cdot (z/s)^n \frac{dz}{s}$$

$$L[t^n] = \left\{ \int_0^\infty e^{-z} \cdot z^n\, dz \right\} \frac{1}{s^{n+1}}$$

$$L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$$

where 'n' is a +ve real number.

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k) Heaviside fu^n:— The unit step fu^n $u(t-a)$ or Heaviside fu^n $H(t-a)$ defined by

$$H(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$

Where 'a' is non-negative constant.

⇒ Laplace transform of heaviside fu^n is

$$L[H(t-a)] = \frac{e^{-as}}{s}$$

When $a=0$ ⇒ $L[H(t)] = \frac{1}{s}$.

l) Sol^n:— Given $\frac{d^2y}{dt^2} + y = 0$ — (1)

$$\rightarrow y'' + y = 0$$

$$\therefore L[y''+y] = L[0]$$

$$L[y''] + L[y] = L[0]$$

$$\text{i.e. } [s^2 y(s) - s\,y(0) - y'(0)] + y(s) = 0$$

$$y(s)(s^2+1) - s\,y(0) - y'(0) = 0.$$

$$\therefore y(s)(s^2+1) - s(1) - 1 = 0.$$

$$y(s)(s^2+1) = 1+s.$$

$$\therefore y(s) = \frac{1+s}{s^2+1}$$

$$\therefore L[y(t)] = \frac{1+s}{s^2+1} = F(s)$$

$$\therefore y(t) = L^{-1}\left[\frac{1+s}{s^2+1}\right]$$

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$$y(t) = L^{-1}\left[\frac{1}{(s^2+1)}\right] + L^{-1}\left[\frac{s}{s^2+1}\right]$$

$\therefore$ $y(t) = \sin t + \cos t.$

e) Soln proof :- Let $(X, \mathcal{T})$ be a topological space and $A \subset X$.

Let $\bar{A}$ is closure of A.

Then by the defn of $\bar{A}$ it is obvious that,

$A \subseteq \bar{A}$ (i.e let $A \subseteq X$, then the intersection of all the closed set containing A)

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PART-C

(9)
(a) Let (X, J) is T_2 space.
we have, to prove (X, J) is T_1 space.
let, $x, y \in X$ be two distinct points.
Since,
(X, J) is T_2 space.
$\exists$ two disjoint open set U and V
such that, $x \in U$ and $y \in V$
$\Rightarrow x \in U$ and $x \notin V$
IIIly
$x \in U$ and $y \notin U$
$\therefore$ for every pair of distinct points
x and y in X $\exists$ an open set U containing
'x' not containing y.
$\Rightarrow (X, J)$ is T_1 space.
Hence, Every T_2 space is T_1 space.

Converse, of above thm need not be true i.e Every T_1 space is need not be T_2-space.

Ex >
The cofinite topology defined on infinite set X.
→ Since, cofinite topology is T_1-space but not T_2-space.
Let $x, y \in X$ and $x \neq y$
$\{x\}$ and $\{y\}$ are closed sets.

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$\Rightarrow \{x\}'$ and $\{y\}'$ are open sets
such that,
$y \in \{x\}'$ and $x \in \{y\}'$
and $\{x\}' \cap \{y\}' = X - \{x, y\} \neq \phi$
$\therefore (X, J)$ is not T_2-space.

(9) (b) Herediatory property → A property 'P' of topological space (X J) is said to be Herediatory property if every subspace of topological space (X, J) have the property 'P'.

we have to prove,
the property of a topological space being T_1-space is herediatary.
Let, (X J) is T_1 space
and (Y, J_Y) is subspace (X, J).
claim,
(Y, J_Y) is T_1-space.
Let, $y \in Y$ and $Y \in X$
$\Rightarrow y \in X$
Since, (X, J) be T_1-space.
$\Rightarrow \{y\}$ is closed in X
$\Rightarrow Y \cap \{y\}$ is closed in Y
$\Rightarrow \{y\}$ is closed in Y $\forall y \in Y$
$\therefore (Y, J_Y)$ is T_1 space.

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(10) (a) Given $L^{-1}\left[\frac{1}{S(S+1)(S+2)}\right]$

Resolving $\frac{1}{S(S+1)(S+2)}$ into partial fraction.

$$\frac{1}{S(S+1)(S+2)} = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+2} \rightarrow (1)$$

multiplying through out by $S(S+1)(S+2)$

$$1 = A(S+1)(S+2) + BS(S+2) + C(S)(S+1)$$

Put $S=0$ then,

$$1 = A(2) + 0 + 0$$

$$\boxed{A = \frac{1}{2}}$$

Put $S=-1$ then,

$$1 = 0 + B(-1)(+1)$$

$$1 = B(-1)$$

$$\boxed{B = -1}$$

Put $S=-2$ then,

$$1 = A(0) + B(0) + C(-2)(-1)$$

$$1 = C2$$

$$\boxed{C = \frac{1}{2}}$$

eqⁿ (1) $\Rightarrow \frac{1}{S(S+1)(S+2)} = \frac{1}{2S} + \frac{-1}{S+1} + \frac{1}{2(S+2)}$

$$= \frac{1}{2S} - \frac{1}{S+1} + \frac{1}{2(S+2)}$$

~~$= \frac{a1}{2} - e$~~

$$L^{-1}\left[\frac{1}{S(S+1)(S+2)}\right] = \frac{1}{2}L^{-1}\left[\frac{1}{S}\right] - L^{-1}\left[\frac{1}{S+1}\right] + \frac{1}{2}\left[\frac{1}{S+2}\right]$$

$$= \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

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(10)(b) Let $L[f(t)] = F(s)$

and $g(t) = \begin{cases} f(t-a) & , t>a \\ 0 & , t<a \end{cases}$ → (1)

we have,

$$L[g(t)] = \int_0^\infty e^{-st} g(t)\, dt$$

$$L[g(t)] = \int_0^a e^{-st} g(t)\, dt + \int_a^\infty e^{-st} g(t)\, dt$$

$$= \int_0^a e^{-st}\, 0\, dt + \int_a^\infty e^{-st} f(t-a)\, dt$$

{∵ from (1)}

$$L[g(t)] = \int_a^\infty e^{-st} f(t-a)\, dt$$

put $t-a = u$

$t = u+a$

$dt = du$

if $t = a$ then $u=0$

if $t = \infty$ then $u=\infty$

$$L[g(t)] = \int_0^\infty e^{-s(a+u)} f(u)\, du$$

$$= e^{-sa} \int_0^\infty e^{-su} f(u)\, du$$

$$= e^{-sa} F(s)$$

$$\therefore L[g(t)] = e^{-sa} F(s)$$

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(a) given, $f(t) = \begin{cases} \sin t & 0 < t \leq \pi/2 \\ \cos t & t > \pi/2 \end{cases}$

By standard result,

$f(t) = \sin t + [\cos t - \sin t]\, u(t - \pi/2)$

$\therefore L[f(t)] = L[\sin t] + L[(\cos t - \sin t)\, u(t - \pi/2)] \rightarrow (1)$

$\therefore L[\sin t] = \frac{1}{s^2+1} \rightarrow (2)$

and $L[(\cos t - \sin t)\, u(t - \pi/2)]$

$= L[f_1(t - \pi/2)\, u(t - \pi/2)]$

Here,

$f_1(t - \pi/2) = \cos t - \sin t$

$f_1(t) = \cos(t + \pi/2) - \sin(t + \pi/2)$

$= (-\sin t) - \cos t$

$= -(\sin t + \cos t)$

$L[f_1(t)] = -L[\sin t + \cos t]$

$= -\{L[\sin t] + L[\cos t]\}$

$= -\left[\frac{1}{s^2+1} + \frac{s}{s^2+1}\right]$

$= -\left[\frac{s+1}{s^2+1}\right] \Rightarrow F(s)$

$L[(\cos t - \sin t)\, u(t - \pi/2)] = e^{-as} F(s)$

$= -e^{-\frac{\pi}{2}s} \cdot \frac{s+1}{s^2+1}$

$= -e^{-\pi/2 s} \cdot \frac{s+1}{s^2+1} \rightarrow (3)$

from (1), (2) and (3),

$L[f(t)] = \frac{1}{s^2+1} - e^{-\pi/2 s} \cdot \frac{(s+1)}{(s^2+1)}$

$= \frac{1 - e^{-\pi/2 s}(s+1)}{s^2+1} = \frac{1 - e^{-\frac{\pi}{2}s} \cdot s - e^{-\frac{\pi}{2}s}}{s^2+1}$

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(11)(b) given $\int_0^\infty t^2 e^t \cos t \, dt$.

we have,

$$\int_0^\infty e^{-st} t^2 \cos t \, dt = L[t^2 \cos t] \longrightarrow (1)$$

we have $L[t^2 f(t)] = (-1)^2 F''(s)$

in eqⁿ (1) ⇒ $f(t) = \cos t$.

$$F(s) = \frac{s}{s^2+1}$$

$$L[t^2 \cos t] = (-1)^2 \frac{d^2}{ds^2} F(s)$$

$$= (-1)^2 \frac{d^2}{ds^2}\left(\frac{s}{s^2+1}\right)$$

$$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{s}{s^2+1}\right)\right]$$

$$= \frac{d}{ds}\left[\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$$

$$= \frac{[(s^2+1)^2](-2s) - (1-s^2)2(s^2+1)(2}{[(s^2+1)^2]^2}$$

$$= \frac{(s^2+1)[(s^2+1)(-2s) - 4s(1-s^2)]}{(s^2+1)^4}$$

$$= \frac{(s^2+1)[-2s^3-2s-4s+4s^3]}{(s^2+1)^4}$$

$$L[t^2 \cos t] = \frac{[2s^3-6s]}{(s^2+1)^3} = \frac{2s[s^2-3]}{(s^2+1)^3} \rightarrow ($$

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~~eq^n ① ⇒~~ from ① and ②,

$$\int_0^\infty e^{-st} t^2 \cos t \, dt = L[t^2 \cos t]$$

① $$\int_0^\infty e^{-st} t^2 \cos t \, dt = \frac{2s[s^2-3]}{(s^2+1)^3}$$

Put, $s = (-1)$

$$\Rightarrow \int_0^\infty e^{-(-1)t} t^2 \cos t \, dt = \frac{2(-1)\,[(-1)^2-3)}{[(-1)^2+1]^3}$$

$$\int_0^\infty e^{t} t^2 \cos t \, dt = \frac{2(-1)[1-3]}{[1+1]^3}$$

$$= \frac{2(-1)(-2)}{2^3}$$

$$= \frac{2(2)}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\therefore \int_0^\infty e^{t} t^2 \cos t \, dt = \frac{1}{2}$$

(12) (a) Statement of convolution theorem ⇒

If $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$

then, $L^{-1}[F(s) \cdot G(s)] = \int_0^t f(u)\, g(t-u)\, du$

$= f(t) * g(t)$

Proof ⇒

Let $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$

Consider,

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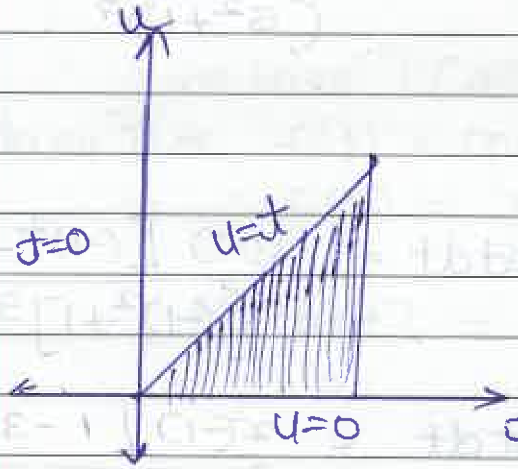
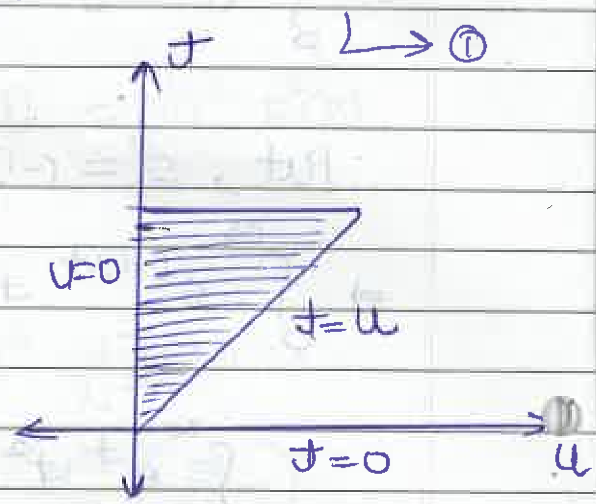
$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\,du\right] = \int_{t=0}^{t=\infty} e^{-st}\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\,du\right] dt$$

$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\,du\right] = \int_{t=0}^{t=\infty}\int_{u=0}^{u=t} e^{-st} f(u)\, g(t-u)\,du\,dt \quad \to (1)$$





The existing region is $t=0$ to $t=\infty$
$u=0$ to $u=t$

on changing the co-ordinate axes
(on changing the order of integration)
$u=0$ to $u=\infty$
$t=u$ to $t=\infty$
(In both the cases area remains same)
$\therefore$ eqn (1) reduces to,

$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\,du\right] = \int_{u=0}^{u=\infty}\int_{t=u}^{t=\infty} e^{-st} f(u)\, g(t-u)\,du$$

Put $t-u=v$

$\therefore t = u+v$

$\therefore dt = dv$

$\therefore t=u \to v=0$

and $t=\infty \to v=\infty$

$$\therefore L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\,du\right] = \int_{u=0}^{u=\infty}\int_{v=0}^{v=\infty} e^{-s(u+v)} f(u)\, g(v)\,dv\,du$$

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$$= \int_{U=0}^{U=\infty} e^{-SU} f(U)dU . \int_{V=0}^{V=\infty} e^{-SV} g(V)dV$$

$$= L[f(t)] . L[g(t)]$$

$$= F(S) . G(S).$$

$$L[f(t) * g(t)] = F(S) . G(S).$$

$$\therefore L^{-1}[F(S) . G(S)] = \int_0^t f(U) g(t-U)dU$$

$$= f(t) * g(t)$$

(12)

(b) Given that,

$$L^{-1}\left[\frac{1}{(S^2+4S+1)^2}\right]$$

Here,

$$F(S) = \frac{1}{S^2+4S+1} \quad \text{and} \quad G(S) = \frac{1}{S^2+4S+1}$$

we have to find

$$L^{-1}[F(S)] = f(t) \quad \text{and} \quad L^{-1}[G(S)] = g(t)$$

$$L^{-1}\left[\frac{1}{S^2+4S+1}\right] = L^{-1}\left[\frac{1}{S^2+4S+4-4+1}\right]$$

$$= L^{-1}\left[\frac{1}{(S+2)^2-4+1}\right]$$

$$= L^{-1}\left[\frac{1}{(S+2)^2-(\sqrt{3})^2}\right]$$

$$= e^{-2t} \frac{\sinh\sqrt{3}t}{\sqrt{3}}$$

$$L^{-1}[F(S)] = f(t) = e^{-2t} \frac{\sinh\sqrt{3}t}{\sqrt{3}}$$

$$\text{and } g(t) = e^{-2t} \frac{\sinh\sqrt{3}t}{\sqrt{3}}$$

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$$L^{-1}[F(s).G(s)] = f(t) * g(t).$$

$$= e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}} * e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}.\sqrt{3}}\int_0^t e^{-2u}\sinh\sqrt{3}u . e^{-2(t-u)}\sinh\sqrt{3}(t-$$

$$\Rightarrow \frac{1}{\sqrt{3}.\sqrt{3}}\int_0^t e^{-2u}.e^{-2t}e^{2u}.\sinh\sqrt{3}u.\sinh\sqrt{3}(t-$$

$$= \frac{1}{\sqrt{3}.\sqrt{3}}\int_0^t e^{(-2u-2t+2u)}.\sinh\sqrt{3}u.\sinh\sqrt{3}(t-u)$$

$$= \frac{1}{\sqrt{3}\sqrt{3}}\int_0^t e^{-2t}[\cos$$

$$= \frac{e^{-2t}}{3}\int_0^t \sinh\sqrt{3}u.\sinh\sqrt{3}(t-u)du \; du.$$

$$= \frac{e^{-2t}}{3}\int_0^t \sin(\sqrt{3}t+\sqrt{u}) \sin(\sqrt{3}t-\sqrt{3}u)d$$

$$\Rightarrow \frac{e^{-2t}}{3}\frac{-\cos(\sqrt{3}t-\sqrt{3}u)}{-\sqrt{3}}. + c$$

$$= \frac{e^{-2t}.\cos(\sqrt{3}t-\sqrt{3}u)}{3\sqrt{3}}$$ //

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PART - B.

(2) statement of co-countable topology on X.

Let X be any nonempty sets and 'J' be collection of empty set and all those subsets of X, whose compliment is countable then 'J' is topology on X is called co-countable topology.

Proof ⇒

let, X be any non empty sets.

$J = \{ \phi, A \subset X / A \text{ is countable} \}$

we have, to prove J is topology on X.

T_1 : Let $X' = \phi$ is countable.

$X \in J$

T_2 : $\phi \in J$ [By given hypothesis]

T_3 : Let $A_\lambda \in J$ for all $\lambda \in I$

claim,

$$\bigcup_{\lambda \in I} A_\lambda \in J$$

Consider,

$$\left(\bigcup_{\lambda \in I} A_\lambda \right)' = \bigcap_{\lambda \in I} A_\lambda'$$

clearly, RHS of above expression is countable. being intersection of countable sets.

$[\because A_\lambda \in J \Rightarrow A_\lambda' \text{ is countable } \forall \lambda \in I]$

$$\left(\bigcup_{\lambda \in I} A_\lambda \right)' \text{ is countable.}$$

$$\therefore \bigcup_{\lambda \in I} A_\lambda \in J$$

i.e arbitrary union of member of J is again a members of J.

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T_4 : Let $A_1, A_2 \in J$

$\Rightarrow A_1'$ and A_2' are countable

claim, $A_1 \cap A_2 \in J$

Consider,

$$(A_1 \cap A_2)' = A_1' \cup A_2'$$

= Union of countable set is countable

$\therefore (A_1 \cap A_2)'$ is countable

$\therefore A_1 \cap A_2 \in J$

ie finite intersection of members of J is again a member of J.

$\therefore$ Hence,

'J' is Topology on X.

④ Given, $L[f(t)] = F(s)$

we ~~and~~ have,

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

Integrating on both side from 's' to '∞' w.r.t 's' we get.

$$\int_s^\infty F(s)\,ds = \int_s^\infty \left[\int_0^\infty e^{-st} f(t)\,dt\right] ds$$

$$= \int_0^\infty \left[\int_s^\infty e^{-st} f(t)\,ds\right] dt$$

$\because$ by integrat' under integral sign.

$$\therefore \int_s^\infty F(s)\,ds = \int_0^\infty \left. \frac{e^{-st}}{-t} f(t) \right]_s^\infty dt$$

$$= \int_0^\infty (-1)\left[0 - \frac{e^{-st}}{t} f(t)\right] dt$$

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$$\int_s^\infty F(s)\,ds = \int_0^\infty e^{-st}\frac{f(t)}{t}\,dt$$

$$\Rightarrow \int_s^\infty F(s)\,ds = L\left[\frac{f(t)}{t}\right]$$

$$\therefore L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds.$$

(5) given, $L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}\right]$

Resolving, $\frac{4s+5}{(s+1)^2(s+2)}$ into partial fractions.

$$\frac{4s+5}{(s+1)^2(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{s+2} \rightarrow (1)$$

multiplying both side by $(s+1)^2(s+2)$

$\rightarrow (2)$

$$4s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$$

$$4s+5 = A[s^2+3s+2] + Bs + B2 + Cs^2 + C + 2Cs$$

$$= As^2 + 3As + 2A + Bs + 2B + Cs^2 + C + 2Cs$$

$$4s+5 = (A+C)s^2 + (3A+B+2C)s + (2A+2B+C)$$

equating coe. of s^2

$$(A+C) = 0$$

~~A = -C~~

~~A = -(-3)~~

~~A = 3~~

Put $s = -1$, in eq^n (2),

$$1 = 0 + B(1) + 0$$

$\boxed{B=1}$

Put $s = -2$ in eq^n (2),

$$-3 = 0 + 0 + C(1)$$

$\boxed{C=-3}$

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equating coe. of s^2

$(A+C) = 0$

$A = -C$

$A = (-1)(-3)$

$\boxed{A = 3}$

eq(1) ⟹ $\frac{4s+5}{(s+1)^2(s+2)} = \frac{3}{s+1} + \frac{1}{(s+1)^2} - \frac{3}{(s+2)}$

$L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}\right] = L^{-1}\left[\frac{3}{s+1}\right] + L^{-1}\left[\frac{1}{(s+1)^2}\right] - L^{-1}\left[\frac{3}{s+2}\right]$

$= 3e^{-t} + te^{t} - 3e^{-2t}$ //

(6) Given,

If $f(t)$ be a periodic function of period $T>0$ then we have to prove,

$L[f(t)] = \frac{\int_0^T e^{-st} f(t)\, dt}{1-e^{-sT}}$

By defn, we have,

$L[f(t)] = \int_0^\infty e^{-st} f(t) dt.$

$L[f(t)] = \int_0^T e^{-st} f(t)dt + \int_T^{2T} e^{-st} f(t) dt + \int_{2T}^{3T} e^{-st} f(t)dt$

$+ \_\_\_\_\_\_$

Put $t = u+T$ in second integral and put $t = u+2T$ in 3^{rd} integral & so on

If $t=T \Rightarrow u=0$ | $t=2T \Rightarrow u=0$

and $t=2T \Rightarrow u=T$ | $t=3t \Rightarrow u=T$

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$dt = du$ | $dt = du$

$$L[f(t)] = \int_0^T e^{-st} f(t)\,dt + \int_0^T e^{-s(u+T)} f(u+T)\,du$$

$$+ \int_0^T e^{-s(u+2T)} f(u+2T)\,du + \cdots$$

$$L[f(t)] = \int_0^T e^{-su} f(u)\,du + e^{-sT}\int_0^T e^{-su} f(u)\,du$$

$$+ e^{-2sT}\int_0^T e^{-su} f(u)\,du + \cdots$$

$\because$ $f(t)$ is a periodic function with period T

$f(u+T) = f(u)$

$f(u+2T) = f(u)$.

$$L[f(t)] = \left(1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right) \cdot \int_0^T e^{-su} f(u)\,du$$

{ Geometric series with ~~finite~~ infinite term

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-e^{-sT}}$$

$a = 1$, $r = e^{-sT}$ }

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-su} f(u)\,du$$

replace 'u' by 't'

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$ //

+2)

3
s+2

2t //

od

t f(t)

so on

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PART - A

①

ⓒ Base → Let (X, J) be any topological space then the subfamily β of J is called as base for J iff every open set in J is expressed as union of members of β.

Subbase → Let $(X J)$ be topological space A collection S of subset of X is said to be subbase,

(i) ~~Se~~ $S \subset J$

(ii) The collection of finite intersection of members of S forms a base.

ⓓ T_1 - space →

A topological space $(X J)$ is called T_1 space iff every singleton set is closed in (X, J)

~~for ex~~ →

① Discreate topological space (X, J) is T_1 space

② Real space (IR, U) is T_1-space.

ⓔ give (X, J) be a topological space and $A \subseteq X$. then, we have to prove $A \subset \bar{A}$

$A \subseteq \bar{A}$ (i.e by defn of closure of set A)

~~let (X, J) be a topological space and $\bar{A}$ is closure of A.~~

$A \subseteq \bar{A}$ By the defn of closure of set A.

$A \subseteq X$, the intersection of all the closed set containing A.

(f) Laplace transform →

Let f(t) be a function of real variable 't' defined for $t \geq 0$, laplace transform of f(t) is denoted by, L[f(t)] and is defined as,

$$\int_0^\infty e^{-st} f(t)\, dt$$

Provided the integral exist. where s is a parameter of real or complex number.

The operator L is called the laplace transform operator & clearly the L[f(t)] is a function of the parameter 's'. It is denoted by, F(s)

i.e

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt \quad , t \geq 0$$

$$L[f(t)] = F(s)$$

and $L[e^{5t}] = \frac{1}{s-5}$

(g) statement of first shifting property →

If laplace transform of f(t) is F(s) then $L[e^{at}.f(t)] = F(s-a)$

Proof →

we have,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

$$\therefore L[e^{at}. f(t)] = \int_0^\infty e^{-st} e^{at} f(t)\, dt$$

$$L[e^{at}. f(t)] = \int_0^\infty e^{-(s-a)t} f(t)\, dt.$$

$$L[e^{at}. f(t)] = F(s-a)$$

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b) $L[t^n] \Rightarrow$

we have $L[f(t)] = \int_0^\infty e^{-st} f(t)dt$

~~L[f(t)]~~ $L[t^n] = \int_0^\infty e^{-st} t^n dt$

Put $st = z$

$t = \frac{z}{s}$

$dt = \frac{dz}{s}$ | if $t=0 \Rightarrow z=0$, if $t=\infty \Rightarrow z=\infty$

$L[t^n] = \int_0^\infty e^{-z} \left(\frac{z}{s}\right)^n \frac{dz}{s}$

$L[t^n] = \frac{1}{s^{n+1}} \int_0^\infty e^{-z} z^n dz$

$L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$ where 'n' is a positive real number.

If 'n' is a positive integer then,

$L[t^n] = \frac{n!}{s^{n+1}}$

(i) given, f(t) is of exponential order.

we have,

$L[f(t)] = \int_0^\infty e^{-st} f(t)dt$

$\therefore L[f'(t)] = \int_0^\infty e^{-st} f'(t)dt$ { integrating by parts

$L[f'(t)] = e^{-st} . f(t) \Big]_0^\infty - \int_0^\infty e^{-st}(-s) f(t)dt$

$= 0 - f(0)] + s\int_0^\infty e^{-st} f(t)dt$

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$$L[f'(t)] = -f(0) + sL[f(t)]$$

$$\therefore \quad L[f'(t)] = sF(s) - f(0).$$

i) given, $L[t \sin^2 mt] \rightarrow (1)$

we have

$$L[t \cdot f(t)] = (-1)F'(s)$$

in eqⁿ (1) $\Rightarrow f(t) = \sin^2 mt =$

$$f(t) = \left[\frac{1 - \cos m2t}{2}\right]$$

$$L[f(t)] = \frac{1}{2}\{L[1] - L[\cos 2mt]\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+2^2m^2}\right\}$$

$$L[t.\sin^2 mt] = (-1)F'(s)$$

$$= -\frac{1}{2}\frac{d}{ds}\left(\frac{1}{s} - \frac{s}{s^2+4m^2}\right)$$

$$= -\frac{1}{2}\left[\right.$$

$$= -\frac{1}{2}\left[-\frac{1}{s^2} - \left(\frac{(s^2+4m^2)(1) - s(2s)}{(s^2+4m^2)^2}\right)\right]$$

$$= -\frac{1}{2}\left[-\frac{1}{s^2} - \left[\frac{s^2+4m^2-2s^2}{(s^2+4m^2)^2}\right]\right]$$

$$= -\frac{1}{2}\left[-\frac{1}{s^2} - \left[\frac{4m^2-s^2}{(s^2+4m^2)^2}\right]\right].$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{s^2} + \left(\frac{4m^2-s^2}{(s^2+4m^2)^2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{s^2} - \left(\frac{(s^2+4m^2)}{(s^2+4m^2)^2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{s^2} - \frac{1}{s^2+4m^2}\right]$$

$$= \frac{1}{2}\left[\frac{s^2+4m^2-s^2}{\quad}\right] = \frac{1}{2}\left[\frac{4m^2}{\quad}\right]$$

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k) ~~Defi~~

Heaviside function →

The unit step function $u(t-a)$ or Heaviside function $H(t-a)$ defined by,

$$H(t-a) = \begin{cases} 0, & t \le a \\ 1, & t > a \end{cases}$$; where a is non nagetive constant.

clearly this is dicontinuous function, discontinuous at the point $t = a$.

If $a = 0$ the the function $H(t-a) = H(t)$

Also $H(t) = \begin{cases} 0, & t \le 0 \\ 1, & t > 0 \end{cases}$

and

$L[H(t-a)]$ ⇒

By definatⁿ of Heaviside function, we have,

$$H(t-a) = \begin{cases} 0 & t \le a \\ 1 & t > a. \end{cases} \longrightarrow (1)$$

we have,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$\therefore L[H(t-a)] = \int_0^\infty e^{-st} \cdot H(t-a) \cdot dt$$

$$L[H(t-a)] = \int_0^a e^{-st} \, 0\, dt + \int_a^\infty e^{-st} \, 1 \, dt$$

↳ from (1)

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty = \left[\frac{e^{\infty}}{-s} - \frac{e^{-sa}}{-s}\right]$$

$$= -\frac{1}{s}\left[0 - e^{-as}\right]$$

$$L[H(t-a)] = \frac{e^{-as}}{s}$$ //

In particular when $a = 0$, $L[H(t)] = \frac{1}{s}$

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(d) Given,

$$y'' + y = 0 \quad \text{and} \quad y(0) = 1$$
$$y'(0) = 1$$

$$L[y'' + y] = L[0].$$

$$L[y''] + L[y] = 0$$

$$[s^2 y(s) - s y(0) - y'(0)] + y(s) = 0$$

$$y(s)[s^2+1] - s y(0) - y'(0) = 0$$

$$y(s)[s^2+1] - s(1) - 1 = 0$$

$$y(s)(s^2+1) = 1+s$$

$$y(s) = \frac{1+s}{s^2+1}$$

$$L[y(t)] = \frac{1+s}{s^2+1} = F(s)$$

$$y(t) = L^{-1}\left[\frac{1+s}{s^2+1}\right]$$

$$y(t) = L^{-1}\left[\frac{1}{s^2+1}\right] + L^{-1}\left[\frac{s}{s^2+1}\right]$$

$$y(t) = \sin t + \cos t$$

(a) Let $(\mathbb{R}, U)$ is a real space and $\{x\}$ be any singleton set, then consider.

$$\{x\}' = (-\infty, x) \cup (x, \infty)$$

$\{x\}'$ = Union of open set in $(\mathbb{R}, U)$ (8)

$\{x\}'$ = open set.

$\Rightarrow \{x\}$ is closed in $(\mathbb{R}, U)$.

$\Rightarrow$ every singleton set is closed in $(\mathbb{R}, U)$

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PART-C

(8) (a) In real space $(\mathbb{R}, U)$

Consider, the closed set

$I_n = \left[\frac{1}{n}, 2-\frac{1}{n}\right]$ for $n \in N$

$$\bigcup_{n\in N} I_n = \bigcup_{n\in N}\left[\frac{1}{n}, 2-\frac{1}{n}\right]$$

$$= [1,1] \cup \left[\frac{1}{2}, \frac{3}{2}\right] \cup \left[\frac{1}{3}, \frac{5}{3}\right] \cup \cdots \cup \lim_{n\to\infty} [0,2]$$

$$\bigcup_{n\in N} I_n = [1,1] \cup \left[\frac{1}{2}, \frac{3}{2}\right] \cup \left[\frac{1}{3}, \frac{5}{3}\right] \cup \cdots \cup \lim_{n\to\infty}\left[\frac{1}{n}, 2-\frac{1}{n}\right]$$

$$\bigcup_{n\in N} I_n = (0,2)$$

which is not closed in $(\mathbb{R}, U)$

Hence,

an arbitrary union of closed set need not be closed.

(8)(b) Let,

Given $X = \{a, b, c, d\}$

$\mathcal{T} = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}\}$

Let $b \in X$

$\exists$ an open set $\{a,b\}$ such that,

$b \in \{a,b\} \subset A$

where A is any subset of X containing 'b'

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$\therefore N(b) = \{ \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\} \}$

Next, let $d \in X$

$\exists$ an open set X such that
$d \in X \subset A$
where, A is any subset of X containing d.

$\therefore N(d) = \{X\}$

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Topology and laplace transforms

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PART - B

27 Co-countable topology

let X be any non empty set & J be collection of empty sets & all those subsets of X whose complement is countable then J is topology on X is called co-countable topology

proof - let X be any non empty set

$J = \{ \phi . A \subset X / A' \text{ is countable} \}$

so we have to prove J is topology on X

⟹ T_1 : let $X' = \phi$ is countable

→ $X \in J$

T_2 : $\phi \in J$

T_3 : let $A \in J \ \forall \ \lambda \in I$

claim $\bigcup_{\lambda \in I} A_\lambda \in J$

Consider $\left(\bigcup_{\lambda \in I} A_\lambda \in J \right)' = \cap A'_\lambda \ \lambda \in I$

clearly RHS is countable being intersection of countable sets

(ie if $A_\lambda \in J \Rightarrow A'_\lambda$ is countable $\forall \lambda \in I$)

∴ $(\bigcup_{\lambda \in I} A_\lambda)'$ is countable

∴ $\bigcup_{\lambda \in I} A_\lambda \in J$

∴ Arbitary union of members of J is members of J

T_4 : let $A_1 A_2 \in J$

⟹ $A_1' \& A_2'$ are countable

claim $A_1 \cap A_2 \in J$

consider $(A_1 \cap A_2)' = A_1' \cup A_2'$

= union of countable set is countable set is countable

ie $(A_1 \cap A_2)'$ is countable

claim $A_1 \cap A_2 \in J$

∴ finite intersection of members of J

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~~Is a members of T~~
~~since T is Topology of X~~

5) $L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}\right]$

$\Rightarrow \frac{4s+5}{(s+1)^2(s+2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+2)}$ —①

$4s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$

put $s=-1$

$B=1$

put $s=-2$

$C=-3$

$0 = A+B$

$A=3$

from ① $= \frac{3}{s+1} + \frac{1}{(s+1)^2} - \frac{3}{(s+2)}$

$= 3L^{-1}\left[\frac{1}{s+1}\right] + L^{-1}\left[\frac{1}{(s+1)^2}\right] - 3L^{-1}\left[\frac{1}{s+2}\right]$

$= 3e^{-t} + e^{-t}t - 3e^{-2t}$

6) $L[f(t)] = \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$

$\Rightarrow$ by defn $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$= \int_0^T e^{-st} f(t)\,dt + \int_0^{2T} e^{-st} f(t)\,dt + \int_{2T}^{3T} e^{-st} f(t)\,dt + \dots$

put $t = u+T$

if $t=T \Rightarrow u=0$ & $t=2T \Rightarrow u=T$

put $t=u+T$ in 2nd integral

if $t=T \Rightarrow u=0$ & $t=2T \Rightarrow u=T$

$= \int_0 e^{-st} f(t)\,dt + \int_0^T e^{-s(u+T)} f(u+T)\,du + \int e^{-s(u+2T)} f(u+2T)\,du + \dots =$

$= \int_0^T e^{-su} f(u)\,du + e^{-sT}\int_0^T e^{-su} f(u)\,du + e^{-2sT}\int^T e^{-st} f(u)\,du + \dots \quad \because f(t)$

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Since f(t) is periodic fn with period T

$= \left(1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right) \int_0^T e^{-su} f(u) du$

$= \frac{1}{1-e^{-sT}} \int_0^T e^{-su} f(u) du$

$= 1 + e^{-sT} + e^{-2sT} + \cdots$ is an infinite geometric Series with 1st term a = 1 & common ratio $r = e^{-sT}$

$= \frac{1}{1-e^{sT}} \int_0^T e^{-sT} f(t) dt$

$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt$

47) If $L[f(t)] = F(s)$ then $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s) ds$

Proof - By defn of LT we have

$L[f(t)] = \int_0^\infty e^{-st} f(t) dt = F(s)$

$\therefore F(s) = \int_0^\infty e^{-st} f(t) dt$

Integrating on both side w.r.t s from s to ∞

$\int_s^\infty F(s) ds = \int_s^\infty \left[\int_0^\infty e^{-st} f(t) dt\right] ds$

changing the order of integration as s and t are independent

$\int_s^\infty F(s) ds = \int_0^\infty \left[\int_s^\infty e^{-st} ds\right] f(t) dt$

$= \int_0^\infty \left[\frac{e^{-st}}{-t}\right]_s^\infty f(t) dt$

$= \int_0^\infty \left[0 - \frac{e^{-st}}{-t}\right]_s^\infty f(t) dt$

$= \int_0^\infty e^{-st} \left[\frac{f(t)}{t}\right] dt.$

$= L\left[\frac{f(t)}{t}\right]$

PART - C

97 a) (a) Every T_2 space is T_1 space

Proof. let $(X, \mathfrak{I})$ is T_2 space

we have to prove $(X, \mathfrak{I})$ is T_1 space

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let $x, y \in X$ be two distinct point
by by Def^n since $(X.\mathcal{T})$ is T_2 space
at ∃ two disjoint open sets U & V
st $x \in U$ & $y \in V$
$\Rightarrow x \in U$ & $x \notin V$
$x \in U$ & $y \notin U$
∴ for every pair of distinct pt x and y
in X ∃ open set V containing x and y in X
∃ open set V containing x not containg y
∴ $(X.\mathcal{T})$ is T_1 space
Hence every T_2 space is T_1 space

Converse of above theorem need not be true
ie Every T_1 space is need not be T_2 space

b) A property 'P' of a topological space $(X.\mathcal{T})$ is said to be herediatory property if every subspace of a topological space $(X.\mathcal{T})$ have the property P.

c) Theorem - the property of topological space being T_1 space is herediatory property
ie Every subspace of T_1 space is T_1 space
Proof - $(X.\mathcal{T})$ is T_1 space
and $(Y, \mathcal{T}_Y)$ is subbase of $(X.\mathcal{T})$
claim $(Y, \mathcal{T}_Y)$ is T_1 space
let $y \in Y$ & $Y \subset X$
$y \in X$
Since $(X.\mathcal{T})$ is T_1 space
$\Rightarrow \{y\}$ is closed in X
$\Rightarrow Y \cap \{y\}$ is closed in Y
$\Rightarrow \{y\}$ is closed in Y $\forall y \in Y$
$(Y.\mathcal{T}_Y)$ is T_1 space

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10) a) $L^{-1}\left[\frac{1}{S(S+1)(S+2)}\right]$

$\frac{1}{S(S+1)(S+2)} = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+2}$ — (1)

$1 = A(S+1)(S+2) + B(S)(S+2) + S(S+1)$

Put $S=-1$, $[B=-1]$

Put $S=-2$ $[C=\frac{1}{2}]$

$0 = A+B+C$

$0 = A-1+\frac{1}{2}$, $[A=\frac{1}{2}]$

from (1)

$= \frac{1}{2}L^{-1}\left[\frac{1}{S}\right] + (-1)L^{-1}\left[\frac{1}{S+1}\right] + L^{-1}\frac{1}{2}\left[\frac{1}{S+2}\right]$

$= \frac{1}{2}(1) - e^{-t} + \frac{1}{2}e^{-2t}$

b) If $L[f(t)] = F(S)$ · $g(t) = \begin{cases} f(t-a) & \text{if } t>1 \\ 0 & \text{if } t<a \end{cases}$

then $L[g(t)] = e^{-as}F(S)$

Proof - By defn of laplace transform

$L[g(t)] = \int_0^\infty e^{-st} f(t) dt$

$= \int_0^a e^{-st} g(t) dt + \int_a^\infty e^{-st} f(t) dt$

$= 0 + \int_a^\infty e^{-st} f(t-a) dt$

$L[g(t)] = \int_a^\infty e^{-st} f(t-a) dt.$

Put $t-a=y \Rightarrow dt=dy$ when $t=a$, $y=0$, $t=\infty$ $y=\infty$

$L[g(t)] = \int_0^\infty e^{-s(y+a)} f(y) dy$

$= \int_0^\infty e^{-sy-sa} f(y) dy = e^{-sa}\int_0^\infty e^{-sy} f(y) dy$

$L[g(t)] = e^{-sa} F(S)$

8) a) Soln - In real space (IR, u)
Consider the closed sets

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$I_n = \left[\frac{1}{n}, 2-\frac{1}{n}\right]$ for $n \in N$

$\bigcup_{n\in N} I_n = \bigcup_{n\in N} \left[\frac{1}{n}, 2-\frac{1}{n}\right]$

$= [1,1] \cup \left[\frac{1}{2}, \frac{3}{2}\right] \cup \left[\frac{1}{3}, \frac{5}{3}\right] \cup \ldots$

$\ldots \lim_{n\to\infty} \left[\frac{1}{n}, 2-\frac{1}{n}\right]$

(0 1/2 1/3 . . . . 3/2 5/2 ... 2

(0, 2) which is not closed in (IR, u)

b) $X = \{a, b, c, d\}$ and $t = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ b

$b \in X$. ∃ open set $\{a,b\}$ such that $b \in \{a\,b\} \subset A$

where A is any subset of X containing 'b'

$N(b) = \{\{a\,b\}, \{a\,b\,c\}, \{a\,b\,c\,d\}\}$

$d \in X$ ∃ open set $\{d\}$ such that $d \in \{X\} \subset A$

$N(b) = \{X\}$

12 a) Statement - If $L[f(t)] = F(s)$ & $L[g(t)] = G(s)$
Then $L^{-1}[F(s)G(s)] = \int_0^t f(u)\, g(t-u)\,du$

$= f(t) \cdot g(t)$

Proof - let $L[f(t)] = F(s)$ & $L[g(t)] = G(s)$
Consider

$\left[\int_{u=0}^{u=t} f(u) g(t-u)\,du\right] = \int_{t=0}^{t=\infty} e^{-st} \left[\int_{u=0}^{u=t} f(u)\, g(t-u)\,du\right] dt$

$\Rightarrow L\left[\int_{u=0}^{t=t} f(u)\, g(t-u)\,du\right] = \int_{t=0}^{t=\infty} \int_{u=0}^{u=t} e^{-st} f(u)\, g(t-u)\,du\,dt$ — (1)

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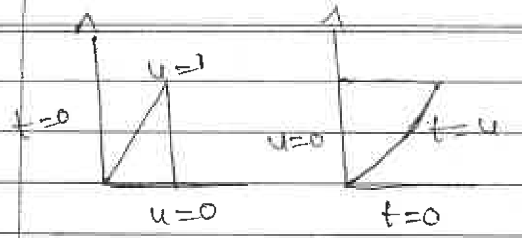

The existing region is
$t=0$, to $t=\infty$
$u=0$ to $u=t$

on changing the coordinates
$u=0$, to $u=\infty$
$t=u$ to $t=\infty$

∴ equation (1) reduces to

$$L\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right] = \int_{u=0}^{u=\infty}\int_{t=u}^{t=\infty} e^{-st} f(u)\,g(t-u)\,dt\,du$$

put $t-u=v$ ∴ $t=u+v$
∴ $dt=dv$

$t=u \rightarrow v=0$ & $t=\infty \rightarrow v=\infty$

$$\therefore L\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right] = \int_{u=0}^{u=\infty}\int_{v=0}^{v=\infty} e^{-s(u+v)} f(u)\,g(v)\,du\,dv$$

$$= \int_{u=0}^{u=\infty} e^{-su} f(u)\,du \cdot \int_{v=0}^{v=\infty} e^{-sv} g(v)\,dv$$

$= L[f(t)]\,L[g(t)]$

$= F(s)\,G(s)$

$L[f(t).g(t)] = F(t)\,g(t)$

PART - A

a) consider $\{x\}' = (-\infty, x) \cup (x, \infty)$
union of open set is open set
$\{x\}' =$ open set

c) Base :- let (X, J) be any topological space then the subfamily B of J is called as base for J if every open set in J is expressed as union of members of B

Subbase - let (X, J) be topological space a collection S of subset of X is to be subbase

i) $S \subset J$

ii) The collection of finite intersection of members of S form a base

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d) T_1 Space - A topological space $(X, \mathcal{T})$ is called T_1 space if every singleton set closed in $(X, \mathcal{T})$

eg - Discrete topological space $(X, \mathcal{T})$ is T_1 space

e) Let $(X, \mathcal{T})$ be topological space
$\bar{A}$ is closure of of A

f) $L[e^{st}] = \frac{1}{s-s}$

g) Proof - we have $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = f(s)$

$\therefore$ $L[e^{at} f(t)] = \int_0^\infty e^{-st} e^{at} f(t)\,dt$

$= \int_0^\infty e^{-(s-a)t} f(t)\,dt$

$\therefore L[e^{at} f(t)] = F(s-a)$

h) $L[t^n] = \int_0^\infty e^{-st} t^n\,dt$

put $s \cdot t = z$ $t = \frac{z}{s} \Rightarrow dt = \frac{dz}{s}$

if $t = 0 \Rightarrow z = 0$.
& $t = \infty \Rightarrow z = \infty$

$= \int_0^\infty e^{-z} \left(\frac{z}{s}\right)^n dt \frac{dz}{s}$

$= \frac{1}{s^{n+1}} \int_0^\infty e^{-z} z^n dz$

$L(t^n) = \frac{\Gamma n+1}{s^{n+1}}$

~~j) L[t sin at]~~

k) Defn - The unit step function $u(t-a)$ or Heaviside fun $H(t-a)$ is defined by

$u(t-a) = \begin{cases} 0, & 0 < t \le a \\ 1, & t > a, \end{cases}$

$L[H(t-a)] = \frac{e^{-at}}{s}$.

l) $\frac{d^2y}{dt^2} + y = 0$ $y(0) = 1$ & $y'(0) = 1$

$\Rightarrow y'' + y = 0$ — (1)

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$\therefore L[y''+y] = L[0]$

$\therefore L[y''] + L[y] = 0$

i.e $[s^2 y(s) - s\,y(0) - y'(0)] + y(s) = 0$

$y(s)(s^2+1) - s\,y(0) - y'(0) = 0$

$y(s)(s^2+1) = 1+s$

$\therefore y(s) = \frac{1+s}{s^2+1}$

$\therefore L[y(t)] = \frac{1+s}{s^2+1} = f(s)$

$y(t) = L^{-1}\left[\frac{1+s}{s^2+1}\right]$

$= L^{-1}\left[\frac{1}{s^2+1}\right] + L^{-1}\left[\frac{s}{s^2+1}\right]$

$y(t) = \sin t + \cos t$

b) $X = \{1, 2, 3, 4\}$

$J_1 = \{X, \phi, \{1\}\}$, $J_2 = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$.

$t_3 = \{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,3\}\}$

$t_4 = \{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}\}$

$J_1 \subset J_2$, $J_2 \subset J_3$, $J_3 \subset J_4$.

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Part C

8) a) Let, $f(z) = u + iv$ be analytic and ψ be a function of 'x' and 'y'.
By partial differentiation,

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial \psi}{\partial v}\frac{\partial v}{\partial x} \quad \text{--- (1)}$$

and,

$$\frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial \psi}{\partial v}\frac{\partial v}{\partial y}$$

$$\therefore \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial u}\left(\frac{-\partial v}{\partial x}\right) + \frac{\partial \psi}{\partial v}\left(\frac{\partial u}{\partial x}\right) \quad \text{--- (2)}$$

Squaring and adding eqn (1) & (2) we get.

$$\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 = \left(\frac{\partial \psi}{\partial u}\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial v}\frac{\partial v}{\partial x}\right)^2 +$$

$$2\frac{\partial \psi}{\partial u}\frac{\partial u}{\partial x}\frac{\partial \psi}{\partial v}\frac{\partial v}{\partial x} + \left(\frac{\partial \psi}{\partial u}\left[-\frac{\partial v}{\partial x}\right]\right)^2 +$$

$$\left(\frac{\partial \psi}{\partial v}\frac{\partial u}{\partial x}\right)^2 - 2\frac{\partial \psi}{\partial u}\frac{\partial v}{\partial x}\frac{\partial \psi}{\partial v}\frac{\partial u}{\partial x}$$

$$= \left(\frac{\partial \psi}{\partial u}\right)^2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] + \left(\frac{\partial \psi}{\partial v}\right)^2$$
$$\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2\right]$$

$$= \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right]\left[\left(\frac{\partial \psi}{\partial u}\right)^2 + \left(\frac{\partial \psi}{\partial v}\right)^2\right]$$

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$$= \left[\left(\frac{\partial \phi}{\partial u} \right)^2 + \left(\frac{\partial \phi}{\partial v} \right)^2 \right] |f'(z)|^2$$

i.e. $\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 = \left[\left(\frac{\partial \phi}{\partial u} \right)^2 + \left(\frac{\partial \phi}{\partial v} \right)^2 \right] |f'(z)|^2$ //

8. b) Given function,

$$f(z) = \frac{(x^3-y^3) + i(x^3+y^3)}{(x^2+y^2)}, \quad z \neq 0$$

i.e. $f(z) = \frac{x^3-y^3}{x^2+y^2} + i \frac{x^3+y^3}{x^2+y^2}$

$f(z) = u + iv$.

$\therefore u = \frac{x^3-y^3}{x^2+y^2}$ and $v = \frac{x^3+y^3}{x^2+y^2}$, $z \neq 0$.

$f(z) = f(0) = 0$.

i.e $u = 0$ & $v = 0$ at $(x,y) = (0,0)$

Clearly u and v are rational functions with non zero denomenator and hence they exists and continuous.

$\Longrightarrow$ $f(z)$ is continuous.

Now. C-R eqns at origin :-

$$u_x(0,0) = \lim_{x\to 0} \frac{u(x,0) - u(0,0)}{x-0}$$

$$= \lim_{x\to 0} \frac{x-0}{x} = 1.$$

llly,

$$u_y(0,0) = \lim_{y\to 0} \frac{u(0,y) - u(0,0)}{y-0}$$

$$= \lim_{y\to 0} \frac{-y-0}{y} = -1.$$

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And

$$V_x(0,0) = \lim_{x \to 0} \frac{V(x,0) - V(0,0)}{x-0}$$

$$= \lim_{x \to 0} \frac{x-0}{x} = 1$$

and,

$$V_y(0,0) = \lim_{y \to 0} \frac{V(0,y) - V(0,0)}{y-0}$$

$$= \lim_{y \to 0} \frac{y-0}{y} = 1$$

Clearly $U_x(0,0) = V_y(0,0)$ & $u_y(0,0) = -V_x(0,0)$

$\Rightarrow$ u and V satisfy the C-R eqns at origin.

Next,

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z-0}$$

$$= \lim_{(x,y) \to 0} \frac{\left(\frac{(x^3 - y^3) + i(x^3 + y^3)}{x^2+y^2} - 0 \right)}{x+iy}$$

Choose $z \to 0$ along $y = mx$ we have

$$f'(0) = \lim_{x \to 0} \frac{(x^3 - m^3x^3) + i(x^3 + m^3y^3)}{(x^2+m^2x^2)(x+imx)}$$

$$= \lim_{x \to 0} \frac{(1-m^3) + i(1+m^3)}{(1+m^2)(1+im)}$$ depends on m

$\Rightarrow$ $f'(0)$ is not unique

$\Rightarrow$ $f'(0)$ doesn't exist

$\Rightarrow$ f(z) is not differentiable at $z=0$

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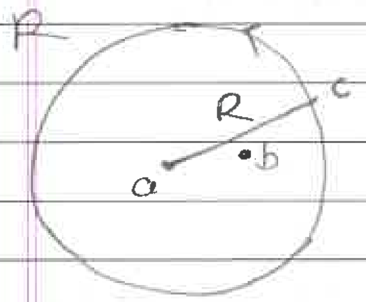
9) a) Statement :- " If $f(z)$ is analytic, $\forall z$ in the complex plane and is den bounded then $f(z)$ is constant.

proof :-



Given that $f(z)$ is analytic in Complex plane, $\forall z$, and is bounded.

$\therefore \exists$ a +ve real number M such that.

$$|f(z)| \leq M, \quad \forall z.$$

Let 'a' and 'b' be two arbitrary points in the complex plane. Let 'c' be the circle of radious R having centre at 'a' enclosing the point b.

$$\therefore \quad c: |z-a| = R$$

and $|a-b| < R$

Since $f(z)$ is analytic and a, b are inside c.

$\therefore$ By Cauchy's integral formula, for the analytic function, we have.

$$f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{(z-b)} dz$$

Consider,

$$|f(a) - f(b)| = \left| \frac{1}{2\pi i} \left\{ \int_c \frac{f(z)}{(z-a)} dz - \int_c \frac{f(z)\,dz}{(z-b)} \right\} \right|$$

$$\leq \frac{1}{2\pi} \int_c |f(z)| \left| \frac{1}{z-a} - \frac{1}{z-b} \right| |dz|$$

$$\leq \frac{1}{2\pi} \int_c M \left| \frac{1}{z-a} - \frac{1}{z-b} \right| |dz|$$

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$$\leq \frac{1}{2\pi} \int_C M \left| \frac{a-b}{(z-a)(z-b)} \right| |dz|$$

$$= \frac{M}{2\pi} |a-b| \int_C \frac{|dz|}{|z-a||z-b|} \quad \text{--- (1)}$$

Clearly,

$$|z-a| = R$$

$$\Rightarrow |z-b| = |z-a+a-b|$$

$$\geq |z-a| - |a-b|$$

$$= R - |a-b|$$

$$\Rightarrow \frac{1}{|z-b|} \leq \frac{1}{R - |a-b|}$$

Then eqn (1) becomes,

$$|f(a) - f(b)| \leq \frac{M|a-b|}{2\pi} \int_C \frac{|dz|}{R[R-|a-b|]}$$

$$= \frac{M}{2\pi R} \frac{|a-b|}{(R-|a-b|)} \int_C |dz|$$

$$= \frac{M}{2\pi R} \frac{|a-b|}{(R-|a-b|)} \quad (\text{length of curve } c)$$

$$= \frac{M}{2\pi R} \frac{|a-b|}{(R-|a-b|)} \cdot 2\pi R$$

$$= \frac{M|a-b|}{R-|a-b|} \longrightarrow 0 \text{ as } R \to \infty.$$

As $R \to \infty$, circle C tends to whole complex plane

$$|f(a) - f(b)| = 0$$

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$\Rightarrow f(a) - f(b) = 0$

$\Rightarrow f(a) = f(b)$

$\forall$ a, b $\in$ complex plane.

It is true only when f(z) is constant.

Thus every bounded analytic function in complex plane is constant.

9 b) Given function is.

$$\int_C \frac{z^2}{(z-1)(z-3)} dz. \quad , C: |z| = 4.$$

Let,

$$\int_C f(z) dz = \int_C \frac{z^2}{(z-1)(z-3)} dz$$

Here,

$$f(z) = \frac{z^2}{(z-1)(z-3)}$$

clearly z = 1, and z = 3 are the poles of order 1. and both the poles z = 1 and z = 3 lies inside the circle C, C: |z| = 4.

Let R_1 be residue at z = 1.

$$\Rightarrow R_1 = \lim_{z \to 1} (z-1) f(z)$$

$$\therefore R_1 = \lim_{z \to 1} (z-1) \frac{z^2}{(z-1)(z-3)}$$

$$R_1 = \lim_{z \to 1} \frac{z^2}{z-3} = \frac{-1}{2} \text{ is residue at}$$

z = 1. and

Let $R_2 = \lim_{z \to 3} (z-3) f(z)$

$$R_2 = \lim_{z \to 3} (z-3) \frac{z^2}{(z-1)(z-3)}$$

4

$R_2 = \lim_{z \to 3} \frac{z^2}{(z-1)} = \frac{9}{2}$ is residue at $z=3$.

$\therefore$ By Cauchy's integral formula,

$$\int_C f(z)\, dz = 2\pi i\,(R_1 + R_2)$$

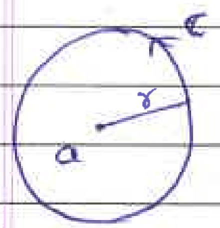
$$= 2\pi i \left(\frac{-1}{2} + \frac{9}{2} \right)$$

$$= 2\pi i \left(\frac{8}{2} \right)$$

$$= 8\pi i$$

---

10) a) Statement :- "If $f(z)$ is analytic inside a circle $C : |z-a| = r$ then for any point z inside C.



$$f(z) = f(a) + (z-a) f'(a) + \frac{(z-a)^2}{2!} f''(a) + \ldots$$

i.e, $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$

Where $a_n = \frac{f^n(a)}{n!}$ "

Proof :- Given that,
$f(z)$ is analytic in the region D bounded by the closed curve $C : |z-a| = r$.
$\Rightarrow f(\xi)$ is analytic in D, $\forall\, \xi \in D$.
We have the identity,

$$\frac{1}{\xi - z} = \frac{1}{(\xi - a) + (a - z)} = \frac{1}{(\xi - a)\left[1 - \frac{z-a}{\xi - a}\right]}$$

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$$= \frac{1}{(\xi - a)} \left[1 - \frac{z-a}{\xi - a} \right]^{-1}$$

$$= \frac{1}{\xi - a} \left[1 + \left(\frac{z-a}{\xi - a}\right) + \frac{(z-a)^2}{(\xi - a)^2} + \cdots \cdots + \frac{(z-a)^n}{(\xi - a)^n} + \cdots \right]$$

$$= \frac{1}{(\xi - a)} + \frac{(z-a)}{(\xi - a)^2} + \frac{(z-a)^2}{(\xi - a)^3} + \frac{(z-a)^3}{(\xi - a)^4} + \cdots \cdots + \frac{(z-a)^n}{(\xi - a)^{n+1}} + \frac{(z-a)^{n+1}}{(\xi - a)^{n+2}} + \cdots \quad (1)$$

$$= \frac{1}{(\xi - a)} + \frac{(z-a)}{(\xi - a)^2} + \frac{(z-a)^2}{(\xi - a)^3} + \cdots + \cdots$$

$$\frac{(z-a)^n}{(\xi - a)^n} \left[\frac{1}{(\xi - a)} + \frac{(z-a)}{(\xi - a)^2} + \frac{(z-a)^2}{(\xi - a)^3} + \cdots \cdots + \frac{(z-a)^n}{(\xi - a)^{n+1}} + \cdots \right]$$

ie.

$$\frac{1}{\xi - z} = \frac{1}{(\xi - a)} + \frac{z-a}{(\xi - a)^2} + \frac{(z-a)^2}{(\xi - a)^3} + \cdots \cdots + \frac{(z-a)^n}{(\xi - a)^n} \left(\frac{1}{\xi - z}\right) \quad \text{from } (1).$$

$$\Rightarrow \frac{1}{\xi - z} = \sum_{n=0}^{n} \frac{(z-a)^n}{(\xi - a)^{n+1}} + \frac{(z-a)^n}{(\xi - a)^n} \frac{1}{(\xi - z)} \quad (2)$$

∴ We have the identity,

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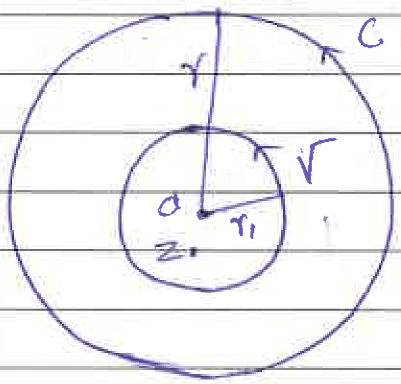
$$\frac{1}{\xi - z} = \sum_{n=0}^{n} \frac{(z-a)^n}{(\xi-a)^{n+1}} + \frac{(z-a)^n}{(\xi-a)^n} \frac{1}{(\xi - z)}$$

Let z be any point inside C construct circle γ with centre at a and enclosing the point z.



Let $f(\xi)$ be analytic in the region bounded by C and point z' is inside C.

$\therefore$ By cauchy's integral formula for analytic function we have,

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(\xi)}{(\xi - z)} d\xi$$

$$= \frac{1}{2\pi i} \int_C f(\xi) \frac{1}{(\xi - z)} d\xi$$

$$= \frac{1}{2\pi i} \int_C f(\xi) \left[\sum_{n=0}^{n} \frac{(z-a)^n}{(\xi-a)^{n+1}} + \frac{(z-a)^n}{(\xi-a)^n} \frac{1}{(\xi-z)} \right] d\xi$$

$$= \frac{1}{2\pi i} \int_C \sum_{n=0}^{n} \frac{(z-a)^n}{(\xi-a)^{n+1}} f(\xi)\, d\xi + \frac{1}{2\pi i} \int_C \frac{(z-a)^n}{(\xi-a)^n} \frac{1}{\xi - z} d\xi \; f(\xi)$$

$$= \sum_{n=0}^{n} \frac{(z-a)^n}{2\pi i} \int_C \frac{f(\xi)}{(\xi-a)^{n+1}} d\xi + R_n$$

Where

$$R_n = \frac{1}{2\pi i} \int_C f(\xi) \frac{(z-a)^n}{(\xi-a)^n} \frac{1}{(\xi - z)} d\xi$$

$$= \sum_{n=0}^{n} (z-a)^n \frac{f^n(a)}{n!} + R_n$$

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Where by cauchy's integral formula for the n^{th} derivative,

$$f^n(a) = \frac{n!}{2\pi i} \int_c \frac{f(\xi)}{(\xi-a)^{n+1}} d\xi$$

taking limit as $n \to \infty$ we have,

$$|R_n| \leq \frac{1}{2\pi} \int_c |f(\xi)| \frac{|z-a|^n}{|\xi-a|^n} \frac{|d\xi|}{|\xi-z|}$$

$$= \frac{1}{2\pi} \int_c |f(\xi)| \left(\frac{r}{r_1}\right)^n \frac{|d\xi|}{|\xi-z|}$$

tends to 0 as $n \to \infty$.

as $\left(\frac{r}{r_1}\right)^n \longrightarrow 0$ as $n \longrightarrow \infty$

from (3),

$$f(z) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (z-a)^n + 0$$

$$\Rightarrow f(z) = f(a) + f'(a)(z-a) + (z-a)^2 \frac{f''(a)}{2!} + \text{------} + \text{---} //$$

10) b) Given function,

$$f(z) = \frac{z}{(z-1)(z+2)}$$

Let,

$$f(z) = \frac{z}{(z+1)(z+2)} = \frac{A}{(z-1)} + \frac{B}{(z+2)}$$

$$\Rightarrow z = A(z+2) + B(z-1)$$

put $z = -2$.

$$-2 = B(-3)$$

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6.

$\Rightarrow \quad 3B = 2$

$\Rightarrow \quad B = \frac{2}{3}$

put $z = 1$

$\Rightarrow \quad 1 = A3$

$A = \frac{1}{3}$

$\therefore \quad f(z) = \frac{1/3}{(z-1)} + \frac{2/3}{(z+2)}$

$\Rightarrow \quad f(z) = \frac{1}{3(z+1)} + \frac{2}{3(z+2)} \quad \text{--- } (1)$

(a) i) :-

$|z| < 1, \Rightarrow |z| < 2$

$\therefore \quad f(z) = \frac{1}{-3(1-(z))} + \frac{2}{2.3(1+\frac{z}{2})}$

$f(z) = \frac{1}{-3(1-z)} + \frac{1}{3(1+z/2)}$

$f(z) = \frac{-1}{3}(1-z)^{-1} + \frac{1}{3}(1+\frac{z}{2})^{-1}$

$= \frac{-1}{3}[1 + (1-z) + (1-z)^2 +$

$f(z) = \frac{-1}{3}[1+z+z^2+z^3+ \cdots\cdots] + \frac{1}{3}[1+\frac{z}{2}+\frac{z^2}{4} + \frac{z^3}{8} + \cdots\cdots]$

$f(z) = \frac{-1}{3}\sum_{n=0}^{\infty} z^n + \frac{1}{3}\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{z}{2}\right)^{n-1}$

which is taylor's series.

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case ii) :-

If $1 < |z| < 2$

i.e. $|z| > 1$ and $|z| < 2$

$\Rightarrow \frac{1}{|z|} < 1$ and $\frac{|z|}{2} < 1$

Given function,

$$f(z) = \frac{1}{3(z-1)} + \frac{2}{3(z+2)}$$

i.e. $$f(z) = \frac{1}{3z(1 - 1/z)} + \frac{2}{3(z+2)}$$

i.e. $$f(z) = \frac{1}{3z}\left(1 - \frac{1}{z}\right)^{-1} + \frac{2}{2 \cdot 3(1 + z/2)}$$

$$f(z) = \frac{1}{3z}\left(1 - \frac{1}{z}\right)^{-1} + \frac{1}{3}\left(1 + \frac{z}{2}\right)^{-1}$$

i.e.
$$f(z) = \frac{1}{3z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right) +$$

i.e.
$$\frac{1}{3}\left(1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \cdots\right)$$

i.e. $$f(z) = \frac{1}{3}\left(\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{z^4} + \cdots\right) + \frac{1}{3}\left(1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \cdots\right)$$

i.e. $$f(z) = \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{z^n} + \frac{1}{3}\sum_{n=0}^{\infty} (-1)^n \left(\frac{z}{2}\right)^n$$

Which is Lorentz's series

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case iii) :-

If $|z| > 2 \Rightarrow |z| > 1$

$$\Rightarrow \frac{2}{|z|} < 1 \quad \text{and} \quad \frac{1}{|z|} < 1$$

$$f(z) = \frac{1}{3(z-1)} + \frac{2}{3(z+2)}$$

$$f(z) = \frac{1}{3z(1-1/z)} + \frac{2}{3z(1+2/z)}$$

$$f(z) = \frac{1}{3z}\left(1-\frac{1}{z}\right)^{-1} + \frac{2}{3z}\left(1+\frac{2}{z}\right)^{-1}$$

$$f(z) = \frac{1}{3z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+ \cdots\right) + \frac{2}{3z}\left(1-\frac{2}{z}+\frac{4}{z^2}-\frac{8}{z^3}+ \cdots\right)$$

$$=$$

$$f(z) = \frac{1}{3}\left[1+\frac{1}{z^2}+\frac{1}{z^3}+\frac{1}{z^4}+ \cdots\right] + \frac{2}{3}\left[\frac{1}{z}-\frac{2}{z^2}+\frac{2^2}{z^3}-\frac{2^3}{z^4}+ \cdots\right]$$

$$\therefore f(z) = \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{z^n} + \frac{2}{3}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{z^{n-1}}{z^n}$$

Which is laurent's series.

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4) a) Statement :- Let $f(z)$ be analytic within on closed countour C except at finite number of poles z_1, z_2, z_3 ---- z_n inside C. then,

$$\int_C f(z)dz = 2\pi i \, (R_1 + R_2 + R_3 + \cdots + R_n)$$

$$= 2\pi i \text{ (Sum of residues at these poles inside C)}$$

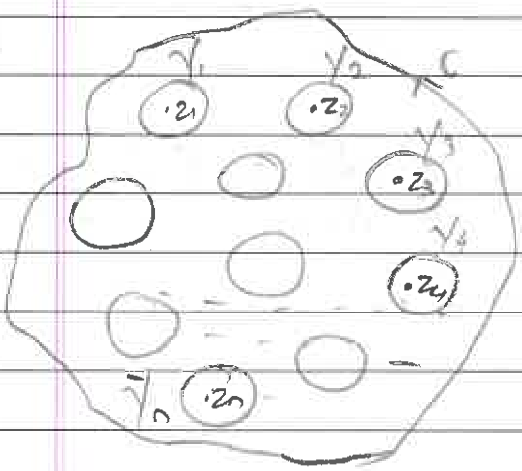
Where,

R_1, R_2, R_3 ------ R_n are residues at poles z_1, z_2, z_3 ------ z_n resp."

Proof:-

By hypothesis z_1, z_2, z_3 ---- z_n poles of $f(z)$ inside C. Therefore function $f(z)$ is not analytic at these points inside C. Hence construct small circles. ~~inside C~~.

$\gamma_1, \gamma_2, \gamma_3$ ------ γ_n around these point bounded by closed curves C, $\gamma_1, \gamma_2, \gamma_3$ ------ γ_n.



By cauchy's theorem for multiconnected region we have

$$\int_C f(z)dz = \int_{\gamma_1} f(z)dz + \int_{\gamma_2} f(z)dz + \int_{\gamma_3} f(z)dz + \int_{\gamma_4} f(z)dz$$

$$+ \cdots \int_{\gamma_n} f(z)dz \quad \text{—— (1)}$$

By the def^n of residue of f(z) we have,

$$R_1 = \frac{1}{2\pi i}\int_{\gamma_1} f(z)\,dz$$ where γ_1 is circle.

around the pole z_1 and R_1 is residue.

$$\therefore \int_{\gamma_1} f(z)\,dz = 2\pi i R_1$$

Simillary,

$$\int_{\gamma_2} f(z)\,dz = 2\pi i R_2$$

$$\int_{\gamma_3} f(z)\,dz = 2\pi i R_3$$

⋮

$$\int_{\gamma_n} f(z)\,dz = 2\pi R_n$$

Then eq^n ① becomes,

$$\int_C f(z)\,dz = 2\pi i R_1 + 2\pi i R_2 + 2\pi i R_3 + \cdots\cdots + 2\pi i R_n$$

i.e. $$\int_C f(z)\,dz = 2\pi i\,[R_1 + R_2 + R_3 + \cdots\cdots + R_n]$$

$$= 2\pi i\,[\text{sum of residues at these poles inside } C]$$

Thus if f(z) be analytic within and on closed contour C except at finite number of poles $z_1, z_2, z_3, \cdots\cdots, z_n$ inside C. Then,

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$$\int_C f(z)\,dz = 2\pi i\,(R_1 + R_2 + \cdots + R_n)$$

$$= 2\pi i\,[\text{Sum of residue at these poles inside } C]$$

Where, $R_1, R_2, R_3, \ldots, R_n$ are residues at poles $z_1, z_2, z_3, \ldots, z_n$ respectively.

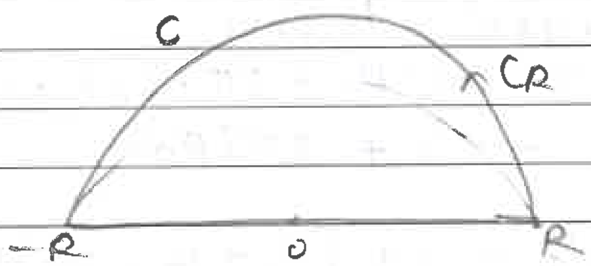
11) b) Given integral,

$$\int_0^\infty \frac{x}{(x^2+4)(x^2+25)}\,dx$$

Consider integral,

$$\int_C f(z)\,dz = \int_C \frac{z}{(z^2+4)(z^2+25)}\,dz$$

taken around the closed contour C consisting of upper half large circle $C_R : |z| = R$ and real line from $-R$ to $+R$



Here,

$$f(z) = \frac{z}{(z^2+4)(z^2+25)}$$

$$\Rightarrow f(z) = \frac{z}{(z+2i)(z-2i)(z+5i)(z-5i)}$$

Clearly,

$z = -2i$, $z = 2i$, & $z = 5i$, $z = -5i$ are the simple poles.

Clearly the pole, $z = 2i$, $z = 5i$. lies inside C and poles $z = -2i$, $z = -5i$. are lies outside C.

If R_1 is residue at $z = 2i$.

$\therefore R_1 = \lim_{z \to 2i} (z-2i) f(z)$

$= \lim_{z \to 2i} (z-2i) \frac{z}{(z-2i)(z+2i)(z^2+25)}$

$= \lim_{z \to 2i} \frac{z}{(z+2i)(z^2+25)} = \frac{1}{42}$

$R_1 = \frac{1}{4}$ is required at $z = 2i$.

If R_2 is residue at $z = 5i$. then,

$R_2 = \lim_{z \to 5i} (z-5i) f(z)$

$= \lim_{z \to 5i} (z-5i) \frac{z}{(z^2+4)(z-5i)(z+5i)}$

$= \lim_{z \to 5i} \frac{z}{(z^2+4)(z+5i)}$

$= \frac{5i.}{(-21)(10i)} = \frac{-1}{42}.$

Then by couchy's residue thm,

$\int_C f(z)\, dz = 2\pi i (R_1 + R_2)$

$= 2\pi i \left(\frac{1}{42} + \frac{1}{42} \right) = ~~2\pi i~~ ($

$= 0$ //

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i.e. $\int_{C_R} f(z)dz + \int_{-R}^{R} f(x)dx = 0$

$\Rightarrow \lim_{R\to\infty} \int_{C_R} f(z)dz + \lim_{R\to\infty} \int_{-R}^{R} f(x)dx = 0$

$\Rightarrow \int_{-\infty}^{\infty} f(x)dx = -\lim_{R\to\infty} \int_{C_R} f(z)dz.$

Consider,

$$\left| \int_{C_R} f(z)dz \right| \leq \int_{C_R} |f(z)| \, |dz|$$

$$= \int_{C_R} \left| \frac{z}{(z^2+4)(z^2+25)} \right| |dz|$$

$$= \int_{C_R} \frac{|z| \, |dz|}{|z^2+4| \, |z^2+25|}$$

$$\leq \int_{C_R} \frac{R \, |dz|}{(|z|^2-4)(|z|^2-25)}$$

$$\leq \int_{C_R} \frac{R}{(R^2-4)(R^2-25)} |dz|$$

$$\frac{R}{(R^2-4)(R^2-25)} \int_{C_R} |dz|.$$

$= \frac{R}{(R^2-4)(R^2-25)} \times$ length of semi circle C_R

$$= \frac{R}{(R^2-4)(R^2-25)} \times \pi R$$

$\to 0$ as $R \to \infty$

$$\Rightarrow \lim_{R\to\infty} \left| \int_{C_R} f(z)\,dz \right| = 0$$

$$= \int_{-\infty}^{\infty} f(x)\,dx = 0$$

$$= 2\int_{0}^{\infty} f(x)\,dx = 0$$

$$\Rightarrow \int_{0}^{\infty} \frac{x}{(x^2+4)(x^2+25)}\,dx = 0 \;//$$

Part B.

2) Necessary condition :- for f(z) to be analytic :-
"If the function f(z) is analytic in the domain 'D' then the conditions $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are satisfy.

proof :-
Let $w = f(z) = u(x,y) + i\,v(x,y)$ be analytic in the domain D.
ie.

$$f'(z) = \lim_{\Delta z\to 0} \frac{f(z)+\Delta z) - f(z)}{\Delta z}$$

exist along any path we choose for $\Delta z \to 0$

$$f'(z) = \lim_{(\Delta x,\Delta y)\to(0,0)} \frac{\left[u(x+\Delta x, y+\Delta y) + i v(x+\Delta x, y+\Delta y) - [u(x,y) + i v(x,y)]\right]}{(\Delta x + i\Delta y)}$$

— ①

As the derivative exists, this limit must be unique irrespective of path in which $\Delta z \to 0$.

i) let $\Delta z \to 0$ along the x-axis.
Along. x-axis, $y = 0$.
$\therefore \Delta z = \Delta x$, $\Delta y = 0$. from (1).

$$f'(z) = \lim_{\Delta x \to 0} \frac{[u(x+\Delta x, 0) + iv(x+\Delta x, 0)] - [u(x,0) + iv(x,0)]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[u(x+\Delta x, 0) - u(x,0)]}{\Delta x} + i \lim_{\Delta x \to 0} \frac{[v(x+\Delta x, 0) - v(x,0)]}{\Delta x}.$$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \quad \text{--- (2)}$$

ii) Let $\Delta z \to 0$. along y-axis.
Let along y-axis, $x = 0$. $\Delta z = i\Delta y$.
Then eq^{2} (1) becomes,

$$f'(z) = \lim_{\Delta y \to 0} \frac{[u(0 - y+\Delta y) + iv(0, y+\Delta y)] - [u(0,y) + iv(0,y)]}{i\Delta y}$$

$$\Rightarrow f'(z) = \lim_{\Delta y \to 0} \frac{[u(0, y+\Delta y) - u(0,y)]}{i\Delta y} + \frac{i}{i}$$

$$\lim_{\Delta y \to 0} \frac{[v(0, y+\Delta y) - v(0,y)]}{\Delta y}.$$

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$\Rightarrow f'(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$

$\Rightarrow f'(z) = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ —— (3)

from (2) & (3),

$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$

$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

or $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Thus if $w = f(z) = u(x,y) + iv(x,y)$ is analytic then C-R eqns.

$u_x = v_y$ and $u_y = -v_x$ are satisfied.

6) Given integral,

$$\int_0^{2\pi} \frac{d\theta}{5/4 + \sin\theta} \quad \text{—— (1)}$$

put $e^{i\theta} = z \Rightarrow d\theta = \frac{dz}{iz}$

$\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$ and $c: |z| = 1$ in (1)

$$\int_0^{2\pi} \frac{d\theta}{5/4 + \sin\theta} = \int_c \frac{dz}{iz\left[\frac{5}{4} + \frac{1}{2i}\left(z - \frac{1}{z}\right)\right]}$$

$$= \int_c \frac{dz}{iz\left[\frac{5}{4} + \frac{z}{2i} - \frac{1}{2iz}\right]}$$

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$$= \int_C \frac{dz}{iz\left[\frac{5iz+2z^2-2}{4iz}\right]}$$

$$= 4\int_C \frac{dz}{2z^2+5iz-2}$$

$$= \frac{4}{2}\int_C \frac{dz}{z^2+\frac{5i}{2}z-1}$$

$$= 2\int_C \frac{dz}{z^2+\frac{5i}{2}z-1}$$

$$= 2\int_C \frac{dz}{(z-\alpha)(z-\beta)}$$

$$\therefore \int_0^{2\pi} \frac{d\theta}{5/4+\sin\theta} = 2\int_C f(z)\,dz \quad \text{——} (2)$$

Where $f(z) = \frac{1}{(z-\alpha)(z-\beta)}$

clearly, $z=\alpha$ & $z=\beta$ are the simple poles

$$\alpha = -\frac{5i}{2} + \sqrt{-\frac{25}{4}+4} \Big/ 2$$

$$\alpha = \frac{-5}{2}i + \sqrt{\frac{-25+16}{4}} \Big/ 2$$

$$\alpha = \frac{-5i+3i}{4}$$

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$$\alpha = \frac{-2i}{4}$$

$$\alpha = \frac{-i}{2}$$

& $\beta = \frac{-5i-3i}{4} = \frac{-8i}{4}$

$\beta = -2i$

Clearly $z=\alpha$ lies inside circle C and $z=\beta$ lies outside circle $C: |z|=1$

$\because |\alpha| = \left|\frac{-i}{2}\right| < 1$

$|\beta| = |-2i| > 1$

$\therefore$ If R_1 is residue at $z=\alpha$ then,

$$R_1 = \lim_{z\to\alpha} (z-\alpha) f(z)$$

$$R_1 = \lim_{z\to\alpha} (z-\alpha) \frac{1}{(z-\alpha)(z-\beta)}$$

$$R_1 = \frac{1}{(\alpha-\beta)} = \frac{1}{-i/2+2i}$$

$$R_1 = \frac{1}{i[2-1/2]} = \frac{1}{i[3/2]}$$

$R_1 = \frac{2}{3i}$ is residue at $z=\alpha$.

$\therefore$ By C.R thm, we have,

$$\int_C f(z)dz = 2\pi i (R_1) = 2\pi i \frac{2}{3}$$

$$= \frac{4\pi}{3}$$

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∴ from ②,

$$\int_0^{2\pi} \frac{d\theta}{5/4 + \sin\theta} = 2 \int_C f(z)\, dz = 2 \times \frac{4\pi}{3}$$

$$= \frac{8\pi}{3} \;//$$

5) Let f(z) be analytic and z = a be zero of the function f(z) of order m. then by the defn of

$f(z) = (z-a)^m [\phi(z)]$ where $\phi(a) \neq 0$

ie $\phi(z)$ is analytic and non zero in the neighbourhood of z = a. Also $(z-a)^m \neq 0$ for all values of $z \neq a$.

Thus there exists no other points in the neighbourhood of z = a. at which f(z) = 0
Hence the zero z = a is isolated. It is true for all zeros of f(z).
∴ Zeros of f(z) are isolated.

Next

Let z = a, be a pole of order m of f(z) then, by the defn of pole principal part of f(z) in the laurentz's expansion, have m no. of terms.
ie.

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \frac{b_1}{(z-a)} + \frac{b_2}{(z-a)^2} + \frac{b_3}{(z-a)^3}$$
$$+ \cdots\cdots + \frac{b_m}{(z-a)^m}$$

ie. $$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \frac{b_m}{(z-a)^m} + \frac{b_{m-1}}{(z-a)^{m-1}} +$$
$$\cdots + \frac{b_2}{(z-a)^2} + \frac{b_1}{(z-a)}.$$

ie $f(z) = \frac{1}{(z-a)^m} \left\{ \sum_{n=0}^{\infty} a_n (z-a)^{n+m} + b_m + b_{m-1}(z-a) \right.$

$+ b_{m-2}(z-a)^2 + \cdots + b_1 (z-a)^{m-1}$

ie,

$$f(z) = \frac{1}{(z-a)^m} \phi(z) \quad \text{---(1)}$$

where $\phi(z) = \left[\sum_{n=0}^{\infty} a_n (z-a)^{n+m} + b_m + b_{m-1}(z-a) \right.$

$\left. + b_{m-2}(z-a)^2 + \cdots + b_1 (z-a)^{m-1} \right]$

Clearly $\phi(z)$ does not tend to infinity for any finite value of z as powers of $(z-a)$ are positive.

$\Rightarrow$ There is no other pole in the neighbourhood of $z=a$.

Thus poles of $f(z)$ are isolated.

3) Given function,

$$u = (x-1)^3 - 3xy^2 + 3y^2 \quad \text{---(1)}$$

$u_x = 3(x-1)^2 - 3y^2$

$u_{xx} = 6(x-1)$

$u_y = -6xy + 6y$

$u_{yy} = -6x + 6 = -6(x-1)$

Clearly $u_{xx} + u_{yy} = 0$ which is satisfying laplace equation & is continuous also as it is polynomial function.

Hence given function u is harmonic.

Let 'v' be the harmonic conjugate of u so that $f(z) = u + iv$.

Then we have,

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$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$$

By C-R eqns.

$$dv = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$

$$\rightarrow dv = -[-6x + 6y] dx + [3(x-1)^2 - 3y^2] dy$$

which is exact.

Hence it's soln. is given by.

$$dv = (6xy - 6y) dx + (x^2 - 2x + 3 - 3y^2) dy$$

∴ soln is,

$$V = \int (6xy - 6y)\, dx + \int (3 - 3y^2)\, dy + c$$

$$\Rightarrow V = 6y \frac{x^2}{2} - 6yx + 3y - y^3 + c.$$

$$\Rightarrow V = 3x^2y - 6xy + 3y - y^3 + c.$$

$$\Rightarrow V = 3(x^2y - 2xy + y) - y^3 + c$$

$$\Rightarrow V = 3(x^2 - 2x + 1)y - y^3 + c$$

$$V = 3(x-1)^2 y - y^3 + c.$$

Now,

let $u = (x-1)^3 - 3xy^2 + 3y^2$

$u_x = 3(x-1)^2 - 3y^2 = \phi_1(x,y)$

$u_y = -6xy + 6y = \phi_2(x,y)$

Then by milne's method we have,

$f'(z) = \phi_1(z,0) - i\,\phi_2(z,0)$

$= 3(z-1)^2 + i\,(0)$

$= 3(z-1)^2 = 3(z^2 - 2z + 1)$

$f'(z)\ 3(z^2 - 2z + 1)$

$f(z) = 3\int (z^2 - 2z + 1)$

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$$f(z) = 3\left(\frac{z^3}{3} - \frac{2z^2}{2} + c\right)$$

$$f(z) = 3\frac{(z^3 - z^2)}{3} + c$$

$$f(z) = \frac{(z-1)^3}{3} + c$$

$$f(z) = (z-1)^3 + c$$

Part A

a) A function $w = f(z)$ is said to be ~~one~~ regular at $z = z_0$ if $f(z)$ is single valued and differentiable in the neighbourhood of z_0.
~~And on~~
For ex :-

$$f(z) = z.\bar{z}.$$

c) Let $f(z) = u + iv$ be analytic. So that,
$v(x,y) = \text{constant}$
Let $v = K$
$\Rightarrow v_x = 0, \quad v_y = 0$
Since $f(z)$ is analytic and hence C-R eqns are satisfied.

$\therefore$ ~~v_y~~ $u_x = 0 \rightarrow u_x = 0$

and,

$v_x = -u_y = 0 \Rightarrow u_y = 0$

$\therefore u_x = u_y = 0$

$\Rightarrow u$ is constant

$\Rightarrow f(z) = u + iv$ is constant.

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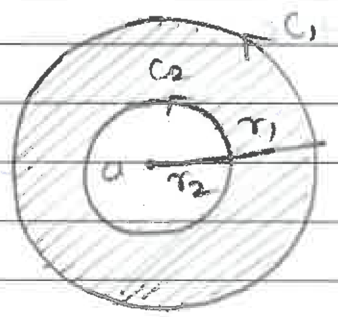
d) **Laurent's theorem** :- If f(z) is analytic inside and on the boundary of the ring shaped origine R bounded by two concentric circles C_1 and C_2 with centre at 'a' and radii r_1 & r_2, then,



$\forall z \in R$,

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} b_n (z-a)^n$$

where,

$$a_n = \frac{1}{2\pi i} \int_{C_1} \frac{f(\xi)}{(\xi-a)^{n+1}} d\xi, \quad \forall n = 0, 1, 2 - - - -$$

and

$$b_n = \frac{1}{2\pi i} \int_{C_2} \frac{f(\xi)}{(\xi-a)^{n+1}} d\xi \quad \forall n_1 = 0, 1, 2, 3, - - -$$

f) **Statement of Jordon's lemma** :- "If $f(z) \to 0$ uniformly as $|z| \to \infty$ (i.e. region tends to whole plane), then,

$$\lim_{R \to \infty} \int_{C_R} e^{imz} f(z) dz = 0$$

Where,

C_R denotes semi circle $|z| = R$,

$|z| > 0$.

g) **Statement of Morera's theorem** :- "If f(z) is continuous function in a region D and if $\int_C f(z) dz = 0$, taken around closed contour C in D. then f(z) is an analytic in D.

j) Let R be a ring. A non empty set 'S' of a ring R is said to be subring of R if 'S' itself is a ring under two induced operation.
Ex:- Set of integers 'z'.

b) Given,

$$f(z) = \frac{z-3}{(z^2+1)}$$

Zero of f(z) = 3
pole of f(z) = ± i. of order 1

i) Given,

$$f(z) = \frac{\sin z}{\left(z - \frac{\pi}{4}\right)^2}$$

Here $z = \frac{\pi}{4}$ is a pole of order 2.
∴
∴ If R_1 is residue at $z = \pi/4$.
then,

$$R_1 = \frac{1}{(2-1)!} \lim_{z \to \pi/4} \frac{d^{2-1}}{dz^{2-1}} \left(z - \frac{\pi}{4}\right)^2 f(z)$$

$$= \frac{1}{(2-1)!} \lim_{z \to \pi/4} \frac{d}{dz} \left(z - \frac{\pi}{4}\right)^2 \frac{\sin z}{(z - \pi/4)^2}$$

$$= \frac{1}{(2-1)!} \lim_{z \to \pi/4} \frac{d}{dz} \sin z$$

$$R_1 = \cos\frac{\pi}{4} \quad //$$

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e) let $f(z) = \frac{e^z}{(z+2)^2}$

$z = -2$ is pole of order 2. inside the circle $|z| = 3$

∴ Residue at $z = -2$.

~~$R_1 = \lim_{z \to -2} (z+2) \cdot f(z)$~~

~~$= \lim_{z \to 2}$~~

∴ $R_1 = \frac{1}{1!} \lim_{z \to -2} \frac{d}{dz} (z+2)^2 f(z)$

$= \frac{1}{1} \lim_{z \to -2} \frac{d}{dz} (z+2)^2 \frac{e^z}{(z+2)^2}$

$= \lim_{z \to -2} \frac{d}{dz} e^z$

$= \lim_{z \to -2} e^z$

$= e^{-2}$

∴ $R_1 = \frac{1}{e^2}$ is residue at $z = -2$.

∴ By cauchy's residue thm,

$\int_c \frac{e^z}{(z+2)^2} dz = 2\pi i \; R_1$

$= 2\pi i \frac{1}{e^2}$

$= \frac{2\pi i}{e^2}$ //

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16.

b) Given function,

$$f(z) = ze^{z}$$

ie $f(z) = (x+iy)\, e^{(x+iy)}$

$= (x+iy)\,(e^{x}\, e^{iy})$

$= e^{x}\,(x+iy)\,(\cos y + i\sin y)$

$= e^{x}\,[x\cos y + xi\sin y + iy\cos y - y\sin y]$

$= e^{x}\,[x\cos y - y\sin y] + i\,e^{x}[x\sin y + y\cos y]$

Here, $u = e^{x}\,x\cos y - e^{x}\,y\sin y$

$V = e^{x}.\, x\sin y + e^{x}\, y.\cos y$

$u_x = \cos y\, e^{x}\, x + \cos y\, e^{x} - y\sin y\, e^{x}$

$u_y = -e^{x}\, x\sin y - e^{x}\, y\cos y - e^{x}\sin y$

$V_x = e^{x}\sin y + e^{x}\, x\sin y + e^{x}\, y\cos y$

$V_y = e^{x}.\, x\cos y + e^{x}\cos y - e^{x}\sin y.$

$\therefore$ $u_x = V_y$ & $u_y = -V_x$

$\therefore$ C-R eqns are satisfied

hence $f(z) = ze^{z}$ is analytic.

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PART - C

8) a) → In real space $(\mathbb{R}, u)$
Consider the closed set,

$$I_n = \left[\frac{1}{n}, 2 - \frac{1}{n}\right] \text{ for } n \in N$$

$$\bigcup_{n \in N} I_n = \bigcup_{n \in N} \left[\frac{1}{n}, 2 - \frac{1}{n}\right] = [1,1] \cup \left[\frac{1}{2}, \frac{3}{2}\right]$$

$$\cup \left[\frac{1}{3}, \frac{5}{3}\right] \cup \text{---} \cup \lim_{n \to \infty} \left[\frac{1}{n}, 2 - \frac{1}{n}\right]$$

$= (0, 2)$ which is not closed in $(\mathbb{R}, u)$

8) b) Let Given,

$X = \{a, b, c, d\}$

$\mathcal{T} = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}\}$

Let, $b \in X$

$\exists$ an open set $\{a,b\}$ such that

$b \in \{a,b\} \subset A$

where A is any subset of X containing b.

$\therefore N(b) = \left\{ \{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\} \right\}$

Next, Let $d \in X$

$\exists$ an open set X such that

$d \in X \subset A$

where A is any subset of X containing d.

$\therefore N(d) = \{X\}$

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9) a) ⇒ let $(X, \mathcal{T})$ is T_2 - space.
We have to prove $(X, \mathcal{T})$ is T_1 - space.
Let, $x, y \in X$ be two distinct points.
Since, $(X, \mathcal{T})$ is T_2 - space.
$\therefore \exists$ two disjoint open sets U and V such that $x \in U$ and $y \in V$
⇒ $x \in U$ and $x \notin V$
⇒ $x \in U$ and $y \notin U$
⇒ for every pair of distinct points x and y in X, $\exists$ an open set U containing x, not containing y.
⇒ $(X, \mathcal{T})$ is T_1 - space.
Hence every T_2 - space is T_1 - space.
And
Converse of above theorem need not to be true.
ie. Every T_1 - space is need not to be T_2 - space.
for ex. ⇒ The co-finite topology defined on infinite set X.
Since, co-finite topology is T_1 - space.
but it is not T_2 - space.
Let, $x, y \in X$ and $x \neq y$
$\{x\}$, $\{y\}$ are closed sets [By T_1 - space]
⇒ $\{x\}'$ & $\{y\}'$ are open sets
such that,
$x \in \{y\}'$ and $y \in \{x\}'$
and $\{x\}' \cap \{y\}' = X - \{x, y\} \neq \phi$
$\therefore$ $(X, \mathcal{T})$ is not T_2 - space.

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9) b) ⇒ <u>Hereditary property</u> :- A property 'p' of topological space $(X, \mathcal{J})$ is said to be Hereditary property if every subspace of a topological space $(X, \mathcal{J})$ have the property p.

⇒

Let $(X, \mathcal{J})$ is T_1 - space and $(Y, \mathcal{J}_Y)$ is subspace of $(X, \mathcal{J})$

<u>Claim</u> :- $(Y, \mathcal{J}_Y)$ is T_1 - space.

Let, $y \in Y$ and $Y \subset X$

$\Rightarrow y \in X$

Since, $(X, \mathcal{J})$ is T_1 - space.

$\Rightarrow \{y\}$ is closed in X.

$\Rightarrow Y \cap \{y\}$ is closed in Y

$\Rightarrow \{y\}$ is closed in Y., $\forall y \in Y$

$\rightarrow (Y, \mathcal{J}_Y)$ is T_1 - space

Hence, every subspace of T_1 - space is T_1 - space.

10) a) ⇒ Given, $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$

Resolving $\frac{1}{s(s+1)(s+2)}$ into partial fractions

$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)} \quad \text{--- (1)}$$

multiplying $[s(s+1)(s+2)]$ on both sides of eqn (1) we get,

$$1 = A(s+1)(s+2) + B\,s(s+2) + C\,s(s+1)$$

Put $s=0$

$\Rightarrow 1 = A(1)(2)$

$\Rightarrow A = \frac{1}{}$

Put $s = -1$

$\Rightarrow 1 = B(-1)(1)$

$\Rightarrow B = -1$

Put $s = -2$

$\Rightarrow 1 = C(-2)(-1)$

$\Rightarrow C = \frac{1}{2}$

from (1),

$$\frac{1}{s(s+1)(s+2)} = \frac{1}{2s} - \frac{1}{(s+1)} + \frac{1}{2(s+2)}$$

$$\Rightarrow L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{1}{2} L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{(s+1)}\right] + \frac{1}{2} L^{-1}\left[\frac{1}{s+2}\right]$$

$$= \frac{1}{2}(1) - e^{-t} + \frac{1}{2} e^{-2t}$$

$$\Rightarrow L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t}$$

10) b) $\Rightarrow$ Second shifting property statement :- "If $L[f(t)] = F(s)$ and

$$g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t < a \end{cases}$$ then

$$L[g(t)] = e^{-as} F(s)."$$

Proof $\Rightarrow$ By the defn,

$$L[g(t)] = \int_0^\infty e^{-st} g(t)\,dt$$

$$\therefore L[g(t)] = \int_0^a e^{-st} g(t)\,dt + \int_a^\infty e^{-st} g(t)\,dt$$

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$$\therefore L[g(t)] = \int_0^a e^{-st}(0)\,dt + \int_a^\infty e^{-st} g(t)\,dt$$

$$\Rightarrow L[g(t)] = 0 + \int_a^\infty e^{-st} f(t-a)\,dt$$

$$\Rightarrow L[g(t)] = \int_a^\infty e^{-st} f(t-a)\,dt$$

Put $t-a = u \Rightarrow t = a+u$

$\Rightarrow dt = du$

when $t = a$, $u = 0$

$t = \infty$, $u = \infty$

$$\Rightarrow L[g(t)] = \int_0^\infty e^{-s(a+u)} f(u)\,du$$

$$\Rightarrow L[g(t)] = e^{-as}\int_0^\infty e^{-su} f(u)\,du$$

$$\Rightarrow L[g(t)] = e^{-as} F(s) ///$$

12) a) <u>Statement of convolution theorem</u> :- " If $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$ then,

$$L^{-1}[F(s)\cdot G(s)] = \int_0^t f(u)\,g(t-u)\,du$$

$$= f(t) * g(t)$$

<u>Proof</u> $\Rightarrow$ Let $L[f(t)] = F(s)$ and

$L[g(t)] = G(s)$

Consider,

$$L\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right] =$$

$$\int_{t=0}^{t=\infty} e^{-st}\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right]dt$$

$$\therefore L\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right.$$

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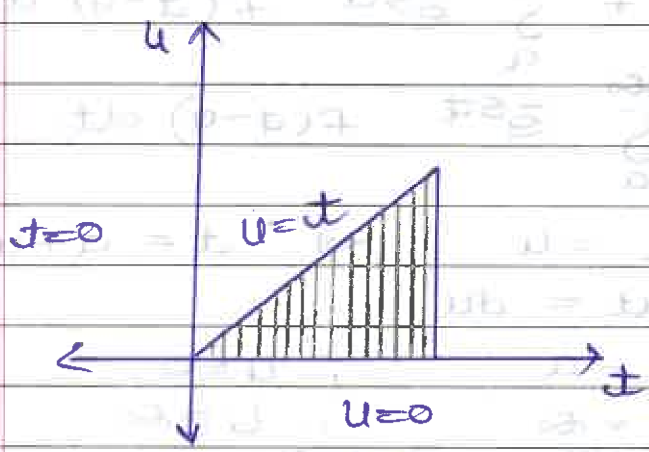
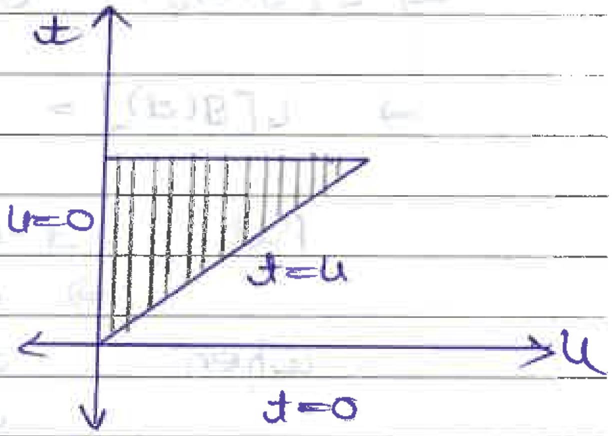
$$= \int_{t=0}^{t=\infty} \int_{u=0}^{u=t} e^{-st} f(u)\, g(t-u)\, du\, dt$$





The existing reason is, $t=0$ to $t=\infty$
$u=0$ to $u=t$

On changing the coordinate axes,
(On changing the order of integration)
$u=0$ to $u=\infty$
$t=u$ to $t=\infty$

In both cases area remains the same.
$\therefore$ eq^n ① reduces to,

$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] =$$

$$\int_{u=0}^{u=\infty} \int_{t=u}^{t=\infty} e^{-st} f(u)\, g(t-u)\, dt\, du$$

Put $t-u = v$ $\therefore$ $t = u+v$
$\therefore$ $dt = dv$

if $t=u \Rightarrow v=0$
and $t=\infty \Rightarrow v=\infty$

$$\therefore L\left[\int_{u=0}^{u=t} f(u)\, g(t-u) du\right] =$$

$$= \int_{u=0}^{u=\infty} \int_{v=0}^{v=\infty} e^{-s(u+v)} f(u)\, g(v)\, dv\, du$$

$$= \int_{u=0}^{u=\infty} e^{-su} f(u)\, du \int_{v=0}^{v=\infty} e^{-sv} g(v)\, dv$$

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$$= L[f(t)] \cdot L[g(t)]$$
$$= F(s) \cdot G(s)$$
$$\therefore L[f(t) * g(t)] = F(s) \cdot G(s)$$
$$\therefore L^{-1}[F(s) \cdot G(s)] = f(t) * g(t) ///$$

12) b) Given, $L^{-1}\left[\frac{1}{(s^2+4s+1)^2}\right]$

Here $F(s) = \frac{1}{(s^2+4s+1)}$

$G(s) = \frac{1}{(s^2+4s+1)}$

then, $L^{-1}[F(s)] = e^{-2t} \frac{\sinh\sqrt{3}t}{\sqrt{3}} = f(t)$

$L^{-1}[G(s)] = e^{-2t} \frac{\sinh\sqrt{3}t}{\sqrt{3}} = g(t)$

By convolution thm,

$L^{-1}[F(s) \cdot G(s)] = f(t) * g(t)$

$$= \int_0^t \frac{e^{-2u} \sinh\sqrt{3}u}{\sqrt{3}} \frac{e^{-2(t-u)}}{\sqrt{3}} \sinh\sqrt{3}(t-u)\,du$$

$$= \frac{1}{3} \int_0^t e^{-2u} \sinh\sqrt{3}u \; e^{-2t} e^{2u} \sinh\sqrt{3}(t-u)\,du$$

$$= \frac{1}{3} e^{-2t} \int_0^t \sinh\sqrt{3}u \; \sinh\sqrt{3}(t-u)\,du$$

$$= -\frac{e^{-2t}}{6} \int_0^t \left[\cosh\sqrt{3}t - (\cosh(2\sqrt{3}u - \sqrt{3}t))\right]du$$

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$$= -\frac{e^{-2t}}{6}\left(u\cosh\sqrt{3}t - \frac{\sinh(2\sqrt{3}u-\sqrt{3}t)}{2\sqrt{3}}\right)_0^t$$

$$= -\frac{e^{-2t}}{6}\left(t\cosh\sqrt{3}t - \frac{\sinh\sqrt{3}t}{2\sqrt{3}} - \frac{\sinh\sqrt{3}t}{2\sqrt{3}}\right)$$

$$= -\frac{e^{-2t}}{6}\left(t\cosh\sqrt{3}t - \frac{2\sinh\sqrt{3}t}{2\sqrt{3}}\right)$$

$$= -\frac{e^{-2t}}{6}\left(t\cosh\sqrt{3}t - \frac{\sinh\sqrt{3}t}{\sqrt{3}}\right)$$

$$= \frac{e^{-2t}}{6}\left(\frac{\sinh\sqrt{3}t}{\sqrt{3}} - t\cosh\sqrt{3}t\right)$$

$$\therefore L^{-1}\left[\frac{1}{(s^2+4s+1)^2}\right] =$$

$$\frac{e^{-2t}}{6}\left(\frac{\sinh\sqrt{3}t}{\sqrt{3}} - t\cosh\sqrt{3}t\right)$$

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* PART - B *

2) → Co- countable topology statement :-

" Let X be any non empty set & $\mathcal{J}$ be collection of empty set and all those subsets of X whose compliment is countable then $\mathcal{J}$ is topology on X is called as Co- countable topology."

Proof → Let X be any non empty set.

$\mathcal{J} = \{ \phi, A \subset X \mid A \text{ is countable} \}$

We have to prove $\mathcal{J}$ is topology on X

T_1 : $X' = \phi$ is countable

$\Rightarrow X \in \mathcal{J}$

T_2 : $\phi \in \mathcal{J}$ {By the hypothesis}

T_3 : Let $A_\lambda \in \mathcal{J}$ for all $\lambda \in I$

Claim $\bigcup_{\lambda \in I} A_\lambda \in \mathcal{J}$

Consider, $\left(\bigcup_{\lambda \in I} A_\lambda\right)' = \bigcap_{\lambda \in I} A_\lambda'$

Clearly RHS of above expression is countable being intersection of the countable sets.

[$\because \exists$ $A_\lambda \in \mathcal{J} \Rightarrow A_\lambda'$ is countable $\forall \lambda \in I$]

$\Rightarrow \left(\bigcup_{\lambda \in I} A_\lambda\right)'$ is countable

$\Rightarrow \bigcup_{\lambda \in I} A_\lambda$ is member of $\mathcal{J}$

ie. $\bigcup_{\lambda \in I} A_\lambda \in \mathcal{J}$

ie. Arbitrary union of members of $\mathcal{J}$ is again a member of $\mathcal{J}$

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T4 : Let $A_1, A_2 \in \mathcal{J}$

$\Rightarrow A_1', A_2'$ are countable.

Claim : $(A_1 \cap A_2)$ is member of $\mathcal{J}$

ie. $(A_1 \cap A_2) \in \mathcal{J}$

Consider, $(A_1 \cap A_2)' = A_1' \cup A_2'$

= union of countable sets is again countable sets

$\therefore (A_1 \cap A_2)'$ is countable.

$\rightarrow (A_1 \cap A_2)$ is member of $\mathcal{J}$

$\Rightarrow (A_1 \cap A_2) \in \mathcal{J}$

ie. Finite intersection of members of $\mathcal{J}$ is again member of $\mathcal{J}$

Hence $\mathcal{J}$ is topology on X

4) $\Rightarrow$ By the defn we have,

$$F(s) = L[f(t)] = \int_0^{\infty} e^{-st} f(t)\, dt$$

integrating on both sides from s to ∞ w.r.t. s we get,

$$\int_s^{\infty} F(s)\, ds = \int_s^{\infty} \left[\int_0^{\infty} e^{-st} f(t)\, dt \right] ds$$

$$= \int_0^{\infty} \left[\int_s^{\infty} e^{-st} f(t)\, ds \right] dt$$

By integration under integral sign

$$= \int_0^{\infty} \left[\frac{e^{-st}}{-t} f(t) \right]_s^{\infty} dt$$

$$= \int_0^{\infty} (-1) \left[0 - \frac{e^{-st}}{t} f(t) \right] dt$$

$$= \int_0^{\infty} e^{-st} \frac{f(t)}{t}\, dt$$

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$$= \int_0^\infty e^{-st} \frac{f(t)}{t} dt$$

$$= L\left[\frac{f(t)}{t}\right]$$

$$\Rightarrow \int_s^\infty F(s)\, ds = L\left[\frac{f(t)}{t}\right]$$

$$\therefore L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\, ds \quad ///$$

5) ⇒ Resolving $\frac{4s+5}{(s+1)^2(s+2)}$ into partial fractions.

$$\frac{4s+5}{(s+1)^2(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{(s+2)} \leftarrow (1)$$

multiplying $(s+1)^2 (s+2)$,

$$4s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$$

Put $s = -1$

$\Rightarrow 1 = 0 + B(1) + 0$

$\Rightarrow \underline{B = 1}$

Put $s = -2$

$\Rightarrow -3 = 0 + 0 + C(1)$

$\Rightarrow \underline{C = -3}$

$$4s+5 = A[s^2+3s+2] + Bs + B2 + Cs^2 + C + 2Cs$$

$$\Rightarrow 4s+5 = As^2 + 3As + 2A + Bs + 2B + Cs^2 + C + 2Cs$$

$$\therefore 4s+5 = (A+C)s^2 + (3A+B+2C)s + (2A+2B+C)$$

equating coeffs of s^2

$\Rightarrow A + C = 0$

$\Rightarrow A = -C$

$\Rightarrow \underline{A = 3}$

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$\therefore$ ① $\rightarrow$

$$\frac{4s+5}{(s+1)^2(s+2)} = \frac{3}{(s+1)} + \frac{1}{(s+1)^2} - \frac{3}{(s+2)}$$

$$\therefore L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}\right] = L^{-1}\left[\frac{3}{(s+1)}\right] + L^{-1}\left[\frac{1}{(s+1)^2}\right] + L^{-1}\left[\frac{-3}{(s+2)}\right]$$

$$\therefore L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}\right] = 3e^{-t} + te^{-t} - 3e^{-2t}$$ ///

6) $\rightarrow$

<u>Proof</u> $\Rightarrow$ By the defn we have,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^T e^{-st} f(t)\,dt + \int_T^{2T} e^{-st} f(t)\,dt + \int_{2T}^{3T} e^{-st} f(t)\,dt + - - - -$$

Put $T = u+T$ in 2^{nd} integral and put $t = u+2T$ in 3^{rd} integral and so on

If $t = u+T \Rightarrow dt = du$

if $t = T, u = 0$

$t = 2T, u = T$

Put $t = u+2T \Rightarrow dt = du$

if $t = 2T, u = 0$

$t = 3T, u = T$

$$= \int_0^T e^{-st} f(t)\,dt + \int_0^T e^{-s(u+T)} f(u+T)\,du + \int_0^T e^{-s(u+2T)} f(u+2T)\,du + ---$$

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Since $f(t)$ is periodic function with period is T

$$\Rightarrow f(u+T) = f(u)$$

$$f(u+2T) = f(u), \ldots$$

$$= (1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \ldots) \int_0^T e^{-su} f(u)\,du$$

$$= \frac{1}{1-e^{-sT}} \int_0^T e^{-su} f(u)\,du$$

$$= \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

$$\therefore L[f(t)] = \int_0^T \frac{e^{-sT} f(t)\,dt}{1-e^{-sT}}$$

* PART - A *

1)

a) $\Rightarrow$ Let, x be any arbitrary element $\in \mathbb{R}$

Consider,

$$\{x\}' = (-\infty, x) \cup (x, \infty)$$

$$= \text{union of open sets}$$

$$\{x\}' = \text{open set}$$

$$\Rightarrow \{x\} \text{ is closed set.}$$

hence, every singleton set is closed.

c) Base = Let $(X, \mathcal{J})$ be any topological space then subfamily B of $\mathcal{J}$ is called base if every open set in $\mathcal{J}$ is expressible as union of members of B.

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subbase :- Let $(X, \mathcal{T})$ be topological space. A collection 'S' of subsets of X is said to be subbase if,
i) $S \subset \mathcal{T}$
ii) The collection of finite intersection of members of S forms a base.

d) ⇒ A topological space $(X, \mathcal{T})$ is called as T_1 - space iff every singleton set is closed in $(X, \mathcal{T})$

for ex. ⇒
1) Discrete topological space $(X, \mathcal{T})$ is T_1 - space.
2) $(\mathbb{R}, \mathcal{U})$ is T_1 - space.

e) Let $(X, \mathcal{T})$ be topological space and $A \subseteq X$

By the defn of closure of set A, $\bar{A}$ is intersection of all the closed sets of $(X, \mathcal{T})$ containing A.
∴ By the defn,
$$A \subset \bar{A}$$
Thus if $A \subseteq X$ ⇒ $A \subset \bar{A}$

f) ⇒ Let $f(t)$ be a function of real variable 't' defined for $t \geq 0$, Laplace transform of $f(t)$ is denoted by $L[f(t)]$ & is defined as $\int_0^\infty e^{-st} f(t)\, dt$, provided the integral exist, where 'S' is the parameter real or complex number.

and $L[e^{5t}] = \frac{1}{s-5}$

h) ⇒ We have, $L[t^n] = \int_0^\infty e^{-st}\, t^n\, dt$

Put $st = z \Rightarrow t = \frac{z}{s}$

$\therefore dt = \frac{dz}{s}$

when $t=0$, $z=0$
$t=\infty$, $z=\infty$

$\therefore L[t^n] = \int_0^\infty e^{-z} \left(\frac{z}{s}\right)^n \frac{dz}{s}$

$\therefore L[t^n] = \int_0^\infty \frac{e^{-z} z^n\, dz}{s^{n+1}}$

$\therefore L[t^n] = \frac{1}{s^{n+1}} \int_0^\infty e^{-z} z^n\, dz$

$L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$ /// where 'n' is +ve real number

g) ⇒ First shifting property :- " If the Laplace transform of $f(t)$ is $F(s)$ then,
$L[e^{at} f(t)] = F(s-a)$."

We have, $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s)$

$\therefore L[e^{at} f(t)] = \int_0^\infty e^{-st} e^{at} f(t)\, dt$

$\therefore L[e^{at} f(t)] = \int_0^\infty e^{-(s-a)t} f(t)\, dt$

$\therefore L[e^{at} f(t)] = F(s-a)$ ///

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k) The heaviside function $H(t-a)$ or unit step function $u(t-a)$ is defined by,

$$H(t-a) = \begin{cases} 0 & , t \leq a \\ 1 & , t > a \end{cases}$$, where

a is non negative constant.

And Laplace transform of heaviside function is,

$$L[H(t-a)] = \frac{e^{-as}}{s}$$

l) Given DE, $\frac{d^2y}{dt^2} + y = 0$

ie. $y'' + y = 0$

$L[y'' + y] = L[0]$

ie. $L[y''] + L[y] = 0$

ie. $(s^2 y(s) - s\, y(0) - y'(0)) + y(s) = 0$

ie. $y(s)\ (s^2+1) - s\ y(0) - y'(0) = 0$

ie. $y(s)\ (s^2+1) - s(1) - 1 = 0$ $\quad \begin{cases} y(0) = 1 \\ y'(0) = 1 \end{cases}$

ie. $y(s)\ (s^2+1) - s = 1$

$\therefore\ y(s) = \frac{1+s}{s^2+1}$

$\therefore\ L[y(t)] = \frac{1+s}{s^2+1}$

$\therefore\ y(t) = L^{-1}\left[\frac{1+s}{s^2+1}\right]$

$y(t) = L^{-1}\left[\frac{1}{s^2+1}\right] + L^{-1}\left[\frac{s}{s^2+1}\right]$

$y(t) = \sin t + \cos t$ //

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(i) We have,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

and hence,

$$L[f'(t)] = \int_0^\infty e^{-st} f'(t)\, dt$$

By using parts per integration,

$$L[f'(t)] = e^{-st} f(t)\Big]_0^\infty - \int_0^\infty f(t)\, e^{-st}(-s)\, dt$$

$$= [0 - f(0)] + s \int_0^\infty e^{-st} f(t)\, dt$$

ie. $L[f'(t)] = s F(s) - f(0)$

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Name : Soundarya Patil

Part - B

2] Co-Countable Topology :

Statement : Let X be any non-empty set and J be the collection of empty sets and all those subsets of X, whose compliment is countable & J is topology on X is called co-countable topology.

Proof : Let X be any non-empty set
$J = \{\phi, A \subset X / A' \text{ is countable}\}$
we have to prove J is topology on X

T_1 = Let $X' = \phi$ is countable
$\therefore X \in J$

T_2 $\phi \in J$ (By the given hypothesis)

T_3 : Let $A_\lambda \in J$ for all $\lambda \in I$

Claim : $\bigcup_{\lambda \in I} A_\lambda \in J$

Consider $\left(\bigcup_{\lambda \in I} A_\lambda\right)' = \bigcap_{\lambda \in I} A'_\lambda$

Clearly R.H.S of above expression is countable being intersection of contable sets.
[If $A_\lambda \in J \Rightarrow A'_\lambda$ is countable $\forall \lambda \in I$]

$\left(\bigcup_{\lambda \in I} A_\lambda\right)'$ is countable.

$\therefore \bigcup_{\lambda \in I} A_\lambda \in J$

Orbitary union of members of J is again a members of J.

T_4 : Let $A_1, A_2 \in \mathcal{J}$.
$\Rightarrow A_1' \& A_2'$ are countable.
claim : $A_1 \cap A_2 \in \mathcal{J}$.
consider $(A_1 \cap A_2)' = A_1' \cup A_2'$.

Union of countable set is countable
$(A_1 \cap A_2)'$ is countable
$\therefore A_1 \cap A_2 \in \mathcal{J}$

finite intersection of members of $\mathcal{J}$ is again a members of $\mathcal{J}$

Hence $\mathcal{J}$ is topology on X.

4]. By the definition we have
$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt.$$

Integrating on both the side from s to ∞ with respect to s we get

$$\int_s^\infty F(s)\,ds = \int_s^\infty \left[\int_0^\infty e^{-st} f(t)\,dt\right] ds$$

$$= \int_0^\infty \left[\int_s^\infty e^{-st} f(t)\,ds\right] dt$$ $\because$ since by integration under integral sign.

$$= \int_0^\infty \left[\frac{e^{-st}}{-t} f(t)\right]_s^\infty dt.$$

$$= \int_0^\infty \left[\frac{1}{-t}\left[e^{-\infty} - e^{-st} f(t)\right]\right] dt.$$

$$= \int_0^\infty \left[-\frac{1}{t} \left[0 - \left(e^{-st} f(t)\right) \right] \right] dt$$

$$= \int_0^\infty \left[\frac{e^{-st} f(t)\, dt}{t} \right]$$

$$= L \left[\frac{f(t)}{t} \right].$$

$$\therefore \int_s^\infty F(s)\, ds = L \left[\frac{f(t)}{t} \right]$$

$$L \left[\frac{f(t)}{t} \right] = \int_s^\infty F(s)\, ds. \;//$$

5] Given $L^{-1} \left[\frac{4s+5}{(s+1)^2 (s+2)} \right].$

Reducing $\frac{4s+5}{(s+1)^2(s+2)}$ into partial functions

$$\frac{4s+5}{(s+1)^2 (s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{(s+2)} \quad \text{--- (1)}$$

multiply $(s+1)^2 \, (s+2)$ we get

$$4s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$$

Now put $s = -1$

$$4(-1) + 5 = A(-1+1)(-1+2) + B(1) + C(0).$$

$$= 0 + B + 0$$

$$\boxed{B = 1}$$

put $s = -2$

$4(-2)+5 = A(-2+1)(0) + B(0) + C(1)$

$-3 = 0 + 0 + C$

$\boxed{C = -3}$

Equating Coefficient of s^2 on both side

$0 = A + C$

$0 = A - 3$

$\boxed{A = 3}$

Eqn (1) $\Rightarrow$

$$\left[\frac{4s+5}{(s+1)^2(s+2)}\right] = L^{-1}\left\{\frac{3}{s+1}\right\} + L^{-1}\left[\frac{1}{(s+1)^2}\right] + L^{-1}\left[\frac{-3}{(s+2)}\right]$$

$$= 3L^{-1}\left[\frac{1}{s+1}\right] + L^{-1}\left[\frac{1}{(s+1)^2}\right] - 3L^{-1}\left[\frac{1}{s+2}\right]$$

$$= 3e^{-t} + e^{-t}.t - 3e^{-2t}$$

$$\left[\frac{4s+5}{(s+1)^2(s+2)}\right] = 3e^{-t} + e^{-t}.t - 3e^{-2t} \,//$$

6) By definition, $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$= \int_0^T e^{-st} f(t)\, dt + \int_T^{2T} e^{-st} f(t)\, dt + \int_{2T}^{3T} e^{-st} f(t)\, dt + \text{------}$$

put $t = u + T$ in 2nd integral and $t = u + 2T$ in 3rd integral and so on

if $t = T \Rightarrow u = 0$ | $t = 2T \Rightarrow u = 0$

& $t = 2T \Rightarrow u = T$ | $t = 3T \Rightarrow u = T$

$dt = du$.

$$= \int_0^T e^{-st} f(t)\,dt + \int_0^T e^{-s(u+T)} f(u+T)\,du + \int_0^T e^{-s(u+2T)} f(u+2T)\,du + \cdots \quad (2)$$

$$= \int_0^T e^{-su} f(u)\,du + e^{-sT}\int_0^T e^{-su} f(u)\,du + \int e^{-s2t} \int_0^T e^{-su} f(u)\,du + \cdots$$

[∵ $f(t)$ is a periodic function with period T
$f(u+T) = f(u)$,
$f(u+2T) = f(u)$]

$$= \left(1 + e^{-sT} + e^{-s2T} + e^{-3sT} + \cdots\right) \int_0^T e^{-su} f(u)\,du$$

$$= \frac{1}{1-e^{-sT}} \int_0^T e^{-su} f(u)\,du \quad \left[\because S_\infty = \frac{a}{1-r} \right].$$

[$1 + e^{-sT} + e^{-2sT} + \cdots$ is infinite geometric series with $a = 1$ and common ratio $r = e^{-sT}$]

S_∞, $S_\infty = \dfrac{a}{1-r} = \dfrac{1}{1-e^{-sT}}$

Replace all the a terms by 't'

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

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PART - C

9) (a). Proof :- Let (X.J) is T_2 - Space.
we have to prove (X.J) is T_1 - Space.

Let x.y ∈ X be two distinct points.
Since (X.J) is T_2 - Space.
∴ ∃ two disjoint open sets U & V.
Such that x ∈ U & y ∈ V.
⇒ x ∈ U & x ∉ V.
⇒ y ∈ V & y ∉ U.
(X.J) is T_1 - Space.

∴ ∀ pair of distinct points x and y in X. ∃ an open set 'U' containing x and not containing y.
Hence, every T_2 Space is T_1-Space.

Converse of above theorem need not be true
i.e., Every T_1-Space is need not be T_2-Space.
Ex: The cofinite topology is T_1-Space
The cofinite topology defined on infinite set X
Since cofinite topology is T_1- space.
but not T_2-space.

Let x.y ∈ X & x ≠ y
{x} & {y} are closed sets (By T_1 - Space)

⇒ {x}' & {y}' are open sets.
Such that y ∈ {x}' & x ∈ {y}'
& {x}' ∩ {y}' = X - {x.y} ≠ φ.

therefore $(X.\mathfrak{J})$ is not T_2 - space.
Hence T_1 - space is need not be T_2-space.

9) b) Definition : A property P of topological space $(X.\mathfrak{J})$ is said to be Heridiatory property if every subspace of topological $(X.\mathfrak{J})$ have the property P.

The property of topological space being T_1-space is herediatory property.

Proof : Let $(X.\mathfrak{J})$ is T_1 - space and $(Y.\mathfrak{J}_Y)$ is subspace of $(X.\mathfrak{J})$

claim : $(Y.\mathfrak{J}_Y)$ is T_1 - space
Let $y \in Y$ and $Y \subset X$.
$\Rightarrow y \in X$
Since $(X.\mathfrak{J})$ is T_1 - space
$\Rightarrow \{y\}$ is closed in X
$\Rightarrow Y \cap \{y\}$ is closed in Y.
$\Rightarrow \{y\}$ is closed in Y. $\forall\, y \in Y$
$(Y.\mathfrak{J}_Y)$ is T_1 - space

Hence, every subspace of T_1 - space is T_1 - space.

10) (a). Given $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$

Reducing $\frac{1}{s(s+1)(s+2)}$ in partial fraction

$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)} \quad \text{---(1)}$$

multiply $s(s+1)(s+2)$

$$1 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

put $s = -1$

$$1 = A(0) + B(-1)(1) + 0(C)$$

$$1 = -B$$

$$\boxed{B = -1}$$

put $s = -2$

$$1 = A(-1)(0) + B(0) + C(-1)(-2)$$

$$1 = 2C$$

$$\boxed{C = 1/2}$$

put $s = 0$

$$1 = A(1)(2) + B(0) + C(0)$$

$$1 = 2A$$

$$\boxed{A = 1/2}$$

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = L^{-1}\left[\frac{1/2}{s}\right] + L^{-1}\left[\frac{-1}{(s+1)}\right] + L^{-1}\left[\frac{1/2}{s+2}\right]$$

$$= \frac{1}{2}L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{s+2}\right]$$

$$= \frac{1}{2}(1) - e^{-t} + \frac{1}{2}e^{-2t}$$

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{1}{2} - e^{-t} + \frac{e^{-2t}}{2} \;//$$

10) b). $L[g(t)] = \int_0^\infty e^{-st} g(t)\,dt$

$$= \int_0^a e^{-st} g(t)\,dt + \int_a^\infty e^{-st} g(t)\,dt$$

$$= \int_0^a e^{-st}\, 0\, dt + \int_a^\infty e^{-st} f(t-a)\,dt .$$

$$= 0 + \int_a^\infty e^{-st} f(t-a)\,dt$$

put $t-a = u$

$du = dt$

when $t = a \Rightarrow u = 0$

$t = \infty \Rightarrow u = \infty$

$$= \int_0^\infty e^{-s(u+a)} f(u)\,du .$$

$$= \int_0^\infty e^{-as} . e^{-su} f(u)\,du$$

$$= e^{-as} \int_0^\infty e^{-su} f(u)\,du$$

$$L[g(t)] = e^{-as} F(s).$$

12) [1] a]. State : If $L[f(t)] = F(s)$ & $L[g(t)] = G(s)$ then

$$L^{-1}[F(s)\,G(s)] = \int_0^t f(u)\, g(t-u)\,du$$

$$= f(t) * g(t)$$

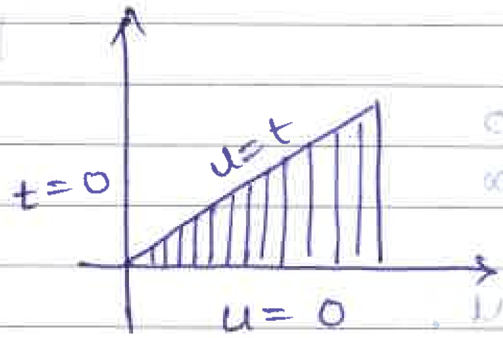
Proof :- Let $L[f(t)] = F(s)$ &

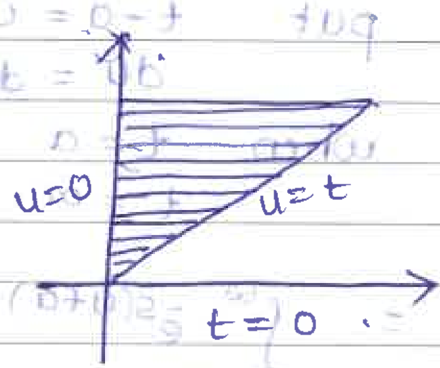
$L[g(t)] = G(s)$

Consider $L\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right]$

$$\int_{t=0}^{t=\infty} e^{-st}\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right]dt.$$

$$= \int_{t=0}^{t=\infty}\int_{u=0}^{u=t} e^{-st} f(u)\,g(t-u)\,du.dt \quad \text{—(1)}$$





The existing region is

$t=0$ to $t=\infty$

$u=0$ to $u=t$

On changing the co-ordinate axis

$u=0$ to $u=\infty$

$t=u$ to $t=\infty$

eqn (1) reduces to

$$L\left[\int_{u=0}^{u=t} f(u)\,g(t-u)\,du\right] = \int_{u=0}^{u=\infty}\int_{t=u}^{t=\infty} e^{-st} f(u)\,g(t-u)\,dt.du$$

put $t-u = v$

$t = u+v.$

$dt = dv$

$t = u \Rightarrow v=0$

$t = \infty \Rightarrow v = \infty$

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$\therefore L\left[\int_{u=0}^{u=\infty} f(u)\,g(t-u)\,du\right] = \int_{u=0}^{u=\infty}\int_{v=0}^{v=\infty} e^{-s(u+v)} f(u) \cdot g(v)\,dv\,du$

$$= \int_{u=0}^{u=\infty} e^{-su} f(u)\,du \cdot \int_{v=0}^{v=\infty} e^{-sv} g(v)\,dv$$

$$= L[f(t)] \cdot L[g(t)]$$

$$= F(s) \cdot G(s)$$

$\therefore L[f(t) \cdot g(t)] = F(s) \cdot G(s)$

$L^{-1}[F(s) \cdot G(s)] = f(t) \cdot g(t)$

$\therefore L^{-1}[F(s) \cdot G(s)] = \int_0^t f(u)\,g(t-u)\,du$

$L^{-1}[F(s) \cdot G(s)] = f(t) * g(t)$

12) b). $L^{-1}\left[\dfrac{1}{(s^2+4s+1)^2}\right]$

Let $F(s) = \dfrac{1}{s^2+4s+1}$ & $G(s) = \dfrac{1}{s^2+4s+1}$

$f(t) = L^{-1}[F(s)] = L^{-1}\left[\dfrac{1}{s^2+4s+1}\right]$

$$= L^{-1}\left[\frac{1}{(s+2)^2-3}\right]$$

$$= e^{-2t}\,\frac{\sinh 3t}{3}$$

$$= \frac{e^{-2t}}{3}\,\sinh 3t$$

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$g(t) = L^{-1}[G(s)] = L^{-1}\left[\frac{1}{(s^2+4s+1)}\right] = L^{-1}\left[\frac{1}{(s+2)^2-3}\right]$

$= \frac{e^{-2t}\sinh 3t}{3}$

$L^{-1}[F(s).G(s)] = \int_0^t f(u)\, g(t-u)\, du$

$= \int_0^t e^{-2u} \frac{\sinh 3u}{3} . e^{-2(t-u)} \frac{\sinh(3t-u)}{3} du$

$= \frac{1}{3}\int_0^t e^{-2u} . \sinh 3u . e^{-2t} . e^{2u} \sinh(3t-u) du$

$= \frac{e^{-2t}}{3}\int_0^t e^{-2u} \sinh 3u . e^{2u} . \sinh(3t-u) du$

$= \frac{e^{-2t}}{3}\int_0^t \sinh 3u . \sinh(3t-u) du$

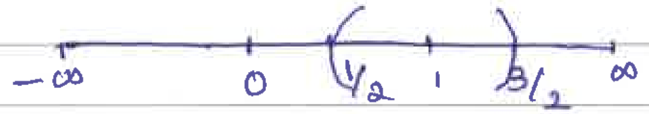
$= \frac{e^{-2t}}{3}\left\{\int_0^t \frac{1}{2} \cos\right.$

8] a] In Real space (IR, U)
consider the closed sets
$I_n = \left[\frac{1}{n}, 2-\frac{1}{n}\right]$ for $n \in N$.

$$\Rightarrow \bigcup_{n\in N} I_n = \bigcup_{n\in N}\left[\frac{1}{n}, 2-\frac{1}{n}\right]$$

$$= [1.1] \cup \left[\frac{1}{2}.\frac{3}{2}\right] \cup \left[\frac{1}{3}.\frac{5}{3}\right] \cup \dots$$

$$\dots \cup \lim_{n\to\infty}\left[\frac{1}{n}, 2-\frac{1}{n}\right]$$



which is not closed in (IR.U).

Hence The arbitary union of closed set is need not be closed.

8] b] $X = \{a.b.c.d\}$
$\mathcal{J} = \{X, \phi, \{a.b\}, \{a.b.c\}, \{a.b.c.d\}\}$
be a topology on X.

neighbourhood of system of
(i) The point 'b'
For $b \in X$, $\exists$ an open set $\{a.b\}$
such that $b \in \{a.b\} \subset A$.
where A is subset of X containing b.

$\therefore N(b) = \{\{a.b\}, \{a.b.c\}, \{a.b.d\}, X\}$.

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neighbourhood system

ii) The point 'd'

For $d \in X$, $\exists$ an open set $\{a.b.c.d\}$ such that $d \in \{a.b.c.d\} \subset A$

where 'A' is the subset of X containing d

$\therefore N(d) = \{\{a.b.d\}, \{b.c.d\}, \{a.c.d\}, X\}$.

PART - A :

1) $\frac{d^2y}{dt^2} + y = 0$ & $y(0) = 1, y'(0) = 1$

Given $\frac{d^2y}{dt^2} + y = 0$ — ①

$$y'' + y = 0$$

$$L[y'' + y] = L[0]$$

$$L[y''] + L[y] = 0$$

$$[s^2 y(s) - s y(0) - y'(0)] + [y(s)] = 0$$

$$y(s)[s^2+1] - s y(0) - y'(0) = 0$$

$$y(s)(s^2+1) - s(1) - (1) = 0$$

$$y(s)(s^2+1) - s - 1 = 0$$

$$y(s)(s^2+1) = s+1$$

$$y(s) = \frac{s+1}{s^2+1}$$

$$L[y(t)] = \frac{s+1}{s^2+1} \; //$$

$$y(t) = L^{-1}\left[\frac{s+1}{s^2+1}\right]$$

$$y(t) = L^{-1}\left[\frac{s}{s^2+1}\right] + L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$y(t) = \sin t + \cos t$$

$$L[y(t)] = \frac{s+1}{s^2+1} \; //$$

k) Definition : The unit step function $u(t-a)$ or Heaviside function $H(t-a)$ defined by

$$H(t-a) = \begin{cases} 0, & t \le a \\ 1, & t > a \end{cases}$$ where a is non negative constant.

This is discontinuous function, discontinuous at the point $t = a$.

Laplace Transform of Heaviside function:
By the defn of Heaviside function

$$H(t-a) = \begin{cases} 0, & t \le a \\ 1, & t > a \end{cases}$$

$$L[(H-a)] = \int_0^\infty e^{-st} H(t-a)\,dt$$

$$= \int_0^a e^{-st} H(t-a)\,dt + \int_a^\infty e^{-st} H(t-a)\,dt$$

$$= \int_0^a e^{-st}\, 0\,dt + \int_a^\infty e^{-st}\, 1\,dt$$

$$= 0 + \int_a^\infty e^{-st}\,dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty = -\frac{1}{s}\left[e^{-st}\right]_a^\infty$$

$$= \frac{1}{s}\left[-e^{-as}\right]$$

$$L[H(t-a)] = \frac{e^{-as}}{s}$$

b) $L[t^n] = \int_0^\infty e^{-st} t^n \, dt$.

put $st = z \Rightarrow t = z/s$.

$dt = \frac{dz}{s}$

$$= \int_0^\infty e^{-z} \left[\frac{z}{s}\right]^n \frac{dz}{s}.$$

If $t \to 0 \quad z \to 0$

$t \to \infty \quad z \to \infty$

$$= \int_0^\infty e^{-z} \left[\frac{z}{s}\right]^n \frac{dz}{s}$$

$$= \int_0^\infty e^{-z} \frac{z^n}{s^{n+1}} dz.$$

$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-z} z^n dz$$

$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-z} z^{(n+1)-1} dz$$

$L[t^n] = \frac{\Gamma n+1}{s^{n+1}}$ where n is +ve real number.

g) State : If the laplace transform of $f(t)$ is $F(s)$ then $L[e^{at} f(t)] = F[s-a]$

Proof : we have $L[f(t)] = \int_0^\infty e^{-st} f(t) dt$

$= F(s)$

$\therefore L[e^{at} f(t)] = \int_0^\infty e^{-st} e^{at} f(t) \, dt$

$= \int_0^\infty e^{-(s-a)t} f(t)\, dt$

$= F(s-a)$

$= L[e^{at} f(t)]$

$L[e^{at} f(t)] = F(s-a)$

a) Consider $\{x\}' = (-\infty, x) \cup (x, \infty)$

$=$ union of a open set in $(\mathbb{R}, u)$

$\{x\}' =$ open set

$\{x\}$ is closed set

Clearly in real space $(\mathbb{R}, u)$ singleton Set $\{x\}$ is closed.

b) $X = \{1, 2, 3, 4\}$

$J_1 = \{X, \phi, \{1\}, \{1, 3\}\}$

$J_2 = \{X, \phi, \{1\}\}$

$J_3 = \{X, \phi\}$

$\therefore J_3 \subset J_2 \subset J_1$

Hence J_1, J_2 & J_3 are mutually Comparable topologies.

i) We have

$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$L[f'(t)] = \int_0^\infty e^{-st} f'(t)\, dt$

By using parts we get

$$= \left[e^{-st} f(t)\right]_0^\infty - \int_0^\infty f(t)\,(-s)\,e^{-st}\,dt$$

$$= [0 - f(0)] + s\int_0^\infty f(t)\,e^{-st}\,dt$$

$$= -f(0) + s\,L[f(t)]$$

$$L[f'(t)] = sF(s) - f(0).$$

c) **Base** : let $(X \cdot J)$ be any topological space, then the sub family β of J is called as base for J if every open set in J is expressed as union of members of β

Sub-base : let $(X \cdot J)$ be topological space. A collection of subset of X is said to be subbase.

i) $S \subset J$

ii) The collection of finite intersection of members of S forms a base.

d) T_1 - Space :

A topological space $(X \cdot J)$ is called T_1 - Space iff every singleton set is closed in $(X \cdot J)$

Ex:- Discrete topological space $(X \cdot J)$ is T_1 - space.

e) let $(X \cdot J)$ be topological space and $\bar{A}$ is closure of A then $A \subset \bar{A}$ by the definition of closure of set A.

f) Laplace transform.

Let f(t) be a function of real variable 't' defined for $t \geq 0$ Laplace transform of f(t) is denoted by,

L[f(t)] and is defined as,

$$L[f(t)] = \int_0^\infty e^{-st} f(t) dt, \quad t \geq 0$$

Provided the Integral exists.

$$L[e^{st}] = \frac{1}{s-5} = \frac{1}{s-5}$$ //

PART - C

11] a] given function.

$$f(t) = \begin{cases} \sin t & 0 < t \leq \pi/2 \\ \cos t & t > \pi/2 \end{cases}$$

By standard result

$$f(t) = \sin t + [\cos t - \sin t] \; u[t - \pi/2]$$

$$\therefore L[f(t)] = L[\sin t] + L[\cos t - \sin t] \; u(t - \pi/2) \quad -(1)$$

$$L[\sin t] = \frac{1}{s^2+1} \quad -(2) \quad f$$

$L[\cos t - \sin t]$.

$$u(t - \pi/2) = L[f_1]$$

$$(t - \pi/2) \; u(t - \pi/2)$$

where $f_1(t-\pi/2) = \cos t - \sin t$

$$f_1(t) = \cos(t+\pi/2) - \sin(t+\pi/2)$$
$$= -\sin t - \cos t.$$

$$f_1(t) = -[\sin t + \cos t]$$

$$L[f(t)] = L[-\sin t - \cos t].$$
$$= -\frac{s}{s^2+1} - \frac{1}{s^2+1}$$
$$= \frac{-(s+1)}{s^2+1} = F(s).$$

$$L[(\cos t - \sin t)\, u(t-\pi/2)] = e^{-as} F(s)$$
$$a = \pi/2$$

$$\therefore = -e^{-\pi/2 s} \frac{s+1}{s^2+1}$$

$$= - e^{-\pi/2 s} \frac{s+1}{s^2+1} \quad \text{--- (3)}.$$

from (1), (2) & (3).

$$L[f(t)] = \frac{1}{s^2+1} - e^{-\frac{\pi s}{2}} \frac{s+1}{s^2+1}.$$

$$L[f(t)] = \frac{1 - e^{-\frac{\pi s}{2}}(s+1)}{s^2+1} \quad //$$

1] b] $\int_0^\infty e^{t} t^2 \cos t \, dt$

We have $\int_0^\infty e^{-st} t^2 \cos t \, dt = L[t^2 \cos t]$ — ①

$$L[\cos t] = \frac{s}{s^2+1}$$

$$L[t^2 \cos t] = (-1)^2 F''(s)$$

$$= (-1)^2 \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$= \frac{d}{ds}\left[\frac{(s^2+1) - s(2s)}{(s^2+1)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$$

$$= \frac{(s^2+1)^2(-2s) - (1-s^2)\,2(s^2+1)(2s)}{(s^2+1)^4}$$

$$= (s^2+1)\left[\frac{(s^2+1)(-2s) - (1-s^2)4s}{(s^2+1)^4}\right]$$

$$= \frac{(s^2+1)(-2s) - 4s(1-s^2)}{(s^2+1)^3}$$

$$= \frac{-2s^3 - 2s - 4s + 4s^3}{(s^2+1)^3}$$

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$$= \frac{2s^3 - 6s}{(s^2+1)^3} \quad \text{--- (2)}$$

from (1) & (2)

$$\int_0^\infty e^{-st} t^2 \cos t \, dt = \frac{2s^3 - 6s}{(s^2+1)^3}.$$

put $s = -1$

$$\int_0^\infty t^2 e^{t} \cos t \, dt = \frac{2(-1)^3 - 6(-1)}{[(-1)^2+1]^3}$$

$$= \frac{-2+6}{[1+1]^3}$$

$$= \frac{4}{2^3} = \frac{4}{8}$$

$$= \frac{1}{2}$$

$$\int_0^\infty e^{t} t^2 \cos t \, dt = \frac{1}{2} //$$

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PART-A

1] b] $X = \{1, 2, 3, 4\}$

$J_1 = \{X, \emptyset, \{1\} \{2\} \{2,3\} \{3\}\}$

$J_2 = \{X \ \emptyset, \{2\}\}$

$J_3 = \{X, \emptyset, \{3\}\}$

J_1, J_2 & J_3 are mutually comparable.

$J_3 \subset J_2 < J_1$

c] let (X, J) be any Topological Space. then Sub family β of J is called as base for J if every open set in J is expressed as union of member of β is called Base for (X, J).

let (X, J) be Topological Space A collection S of subsets of X is said to Be subbase
i] $S \subset J$
ii] The collection of finite intersection of member of S forms a base.

d] A topological Space (X, J) is called as a T_1-Space iff every singleton set is closed in (X, J)

Ex:- Real space $(\mathbb{R}, u)$ is T_1-Space.

f] let $f(t)$ be a function of real variable t defined for $t \geq 0$ laplace transform of $f(t)$ is denoted by $L[f(t)]$ and is defined as $\int_0^\infty e^{-st} f(t)\, dt$ provided the integral exist where S is parameter a real or complex number the operator L is called laplace transform

$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

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$$L[e^{5t}] = \int_0^\infty e^{-st} e^{5t}\, dt$$

$$= \int_0^\infty e^{-t(s-5)}\, dt$$

$$= \left[\frac{e^{-t(s-5)}}{-(s-5)}\right]_0^\infty$$

$$= 0 + \frac{1}{(s-5)}$$

$$L[e^{5t}] = \frac{1}{(s-5)}$$

g) Statement: If the laplace transform of $f(t)$ is $F(s)$ then $L[e^{at} f(t)] = F(s-a)$

→ we have $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$L[e^{at} f(t)] = \int_0^\infty e^{-st} e^{at} f(t)\, dt$$

$$= \int_0^\infty e^{-t(s-a)} f(t)\, dt$$

$$L[e^{at} f(t)] = F(s-a)$$

h) $L[t^n]$

$$L[t^n] = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^\infty e^{-st} t^n\, dt$$

put $st = z$

$$t = \frac{z}{s}$$

$$dt = \frac{dz}{s}$$

if $t = 0$, $z = 0$ & $t = \infty$, $z = \infty$

$$L[t^n] = \int_0^\infty e^{-z} \left(\frac{z}{s}\right)^n \frac{dz}{s}$$

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$$L[t^n] = \frac{1}{s^{n+1}} \int_0^\infty e^{-z} z^n dz$$

$$L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$$

j) $L[t \sin^2 mt]$

$$f(t) = \sin^2 mt = \frac{1 - \cos 2mt}{2}$$

$$= \frac{1}{2} - \frac{\cos 2mt}{2}$$

$$= \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+4m^2}$$

$$F(s) = \frac{1}{2s} - \frac{s}{2(s^2+4m^2)}$$

$$F(s) = \frac{s^2+4m^2 - s^2}{2s(s^2+4m^2)}$$

$$F(s) = \frac{4m^2}{2s(s^2+4m^2)}$$

$$L[t \sin^2 mt] = (-1)^1 F'(s)$$

$$= (-1) \frac{4m^2}{2s(s^2+4m^2)}$$

$$= \frac{-4}{2}\left[\frac{s(s^2-4m^2) - [4m^2 s(2s) - (s^2+4m^2)]}{s^2(s^2+4m^2)^2}\right]$$

$$= -2\left[\frac{-4m^2[2s^2 - s^2 + 4m^2]}{s^2(s^2+4m^2)^2}\right]$$

$$= -2\left[\frac{-4m^2[s^2+4m^2]}{s^2(s^2+4m^2)^2}\right]$$

$$= -2\left[\frac{-4m^2}{s^2(s^2+4m^2)}\right]$$

k) The unit step function $u(t-a)$ or Heaviside function $H(t-a)$ defined by

$$H(t-a) = \begin{cases} 0 & t \leq a \\ 1 & t > a \end{cases}$$ when a is non negative

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$L[H(t-a)] = \frac{e^{-as}}{s}$

1) $\frac{d^2y}{dt^2} + y = 0 \quad y(0) = 1 \quad y'(0) = 1$

$y'' + y = 0$

$s^2 y(s) - s(y'(0)) - y(0) + [y(s)] = 0$

$y(s)[s^2+1] - s - 1 = 0$

$y(s) = \frac{s+1}{s^2+1}$

$L[y(t)] = \frac{s}{s^2+1} + \frac{1}{s^2+1}$

$y(t) = L^{-1}\left[\frac{s}{s^2+1}\right] + L^{-1}\left[\frac{1}{s^2+1}\right]$

$y(t) = \cos t + \sin t.$

PART - B

2) Statement :- let X be any non-empty set & $\mathfrak{J}$ be collection of empty sets and all those subset of X whose compliment is countable Then $\mathfrak{J}$ is topology on X is called co-countable Topology

Proof :- let X be any non empty set

$\mathfrak{J} = \{\phi, A \subset X \mid A' \text{ is countable}\}$

we have to prove X is topology on X

T_1: let $X' = \phi$ is countable

$\therefore X \in \mathfrak{J}$

T_2: $\phi \in \mathfrak{J}$

T_3: let $A_\lambda \in \mathfrak{J}$ for all $\lambda \in I$

claim $\bigcup_{\lambda \in I} A_\lambda \in \mathfrak{J}$

Consider $\left(\bigcup_{\lambda \in I} A_\lambda\right)' = \bigcap_{\lambda \in I} A'_\lambda$

Clearly RHS of above expression is countable being intersection of countable sets

Modern

$\left(\bigcup_{\lambda\in I} A_\lambda\right)'$ is countable.

$\therefore \bigcup_{\lambda\in I} A_\lambda \in \mathcal{J}$

arbitary union of member of J again member of J

T₃: let $A_1, A_2 \in \mathcal{J}$
$\Rightarrow A_1' \& A_2'$ are countable
claim $A_1 \cap A_2 \in \mathcal{J}$
consider $(A_1\cap A_2)' = A_1' \cup A_2'$
= union of countable set is countable

$(A_1 \cap A_2)'$ is countable.
$\therefore A_1\cap A_2 \in \mathcal{J}$
i.e. finite intersection of member of J again member of J
Hence J is Topology on X

4) $L[f(t)] = F(s)$
$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$

By defination we have
$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

integrating on both side from s to ∞ w.r.t s we get
$\int_s^\infty F(s)\,ds = \int_s^\infty \left[\int_0^\infty e^{-st} f(t)\,dt\right] ds$

$= \int_0^\infty \left[\int_s^\infty e^{-st} f(t)\,ds\right] dt$

$= \int_0^\infty \left[\frac{e^{-st}}{-t} f(t)\right]_s^\infty dt$

$= \int_0^\infty (-1)\left[0 - \frac{e^{-st}}{t} f(t)\right] dt$

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$$\int_0^\infty e^{-st} \frac{f(t)}{t} dt$$

$$= L\left[\frac{f(t)}{t}\right]$$

$$\therefore \int_s^\infty F(s) ds = L\left[\frac{f(t)}{t}\right]$$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s) ds$$

6) f(t) is periodic function of period $T>0$ then show that $L[f(t)] = \int_0^T \frac{e^{-st} f(t)}{1-e^{-sT}} dt$

Proof :- By the defination

$$L[f(t)] = \int_0^\infty e^{-st} f(t) dt.$$

$$L[f(t)] = \int_0^T e^{-st} f(t) dt + \int_T^{2T} e^{-st} f(t) dt$$

$$+ \dots + \int$$

put $t = u+T$ in second integral.

and put $t = u+2T$ in third integral.

$dt = du$ $\qquad$ $dt = du$

$t = T \Rightarrow u = 0$ $\qquad$ if $t = 3T$ $\quad u = 2T$

$t = 2T \Rightarrow u = T$

$$L[f(t)] = \int_0^T e^{-st} f(t) dt + \int_0^T e^{-s(u+T)} du\, f(u+T)$$

$$+ \int_0^T e^{-s(u+2T)} f(u+2T) du + \dots$$

$$L[f(t)] = \int_0^T e^{-su} f(u) du + \int_0^T e^{-s(u+T)} f(u+T) du$$

$$+ \int_0^T e^{-s(u+2T)} f(u+2T) du + \dots$$

$$L[f(t)] = \int_0^T e^{-su} f(u) du + e^{-sT} \int_0^T e^{-su} f(u+T) du$$

$$+ e^{-s2T} \int_0^T e^{-su} f(u+2T) du + \dots$$

$\because$ f(t) is periodic function with period T

$L[f(t)] = (1 + e^{-sT} + e^{-2sT} + \dots) \int_0^T e^{-su} f(u)\,du$

$= \frac{1}{1 - e^{-sT}} \int_0^T e^{-su} f(u)\,du.$

Since $(1 + e^{-sT} + e^{-2sT} + \dots)$ is an infinite generation series with first term $a = 1$ & common ratio $r = e^{-st}$

$S_\infty = \frac{1}{1 - e^{-sT}}$

replace all this term interms of t

$L[f(t)] = \frac{1}{1 - e^{-st}} \int_0^1 e^{-st} f(t)\,dt.$

7) $L[y'' + 2y' - 3y] = L[\sin t]$ $y(0) = 0,\ y'(0) = 0.$

$s^2[y(s)] - sy'(0) - y(0) + 2[s(y(s)] - y(0)] - 3y(s) = \frac{1}{s^2+1}$

$y(s)[s^2 + 2s - 3] = \frac{1}{s^2+1}$

$y(s) = \frac{1}{(s^2+1)(s^2+2s-3)}$

$= \frac{1}{(s^2+1)(s+3)(s-1)}$

$L[y(t)] = \frac{As+B}{s^2+1} + \frac{C}{s+3} + \frac{D}{s-1}$

$y(t) = L^{-1}\left[\frac{As+B}{s^2+1} + \frac{C}{s+3} + \frac{D}{s-1}\right]$

$\frac{1}{(s^2+1)(s+3)(s-1)} = (As+B)(s+3)(s-1) + C(s^2+1)(s-1) + D(s^2+1)(s+3)$

$1 = (As+B)(s+3)(s-1) + C(s^2+1)(s-1) + D(s^2+1)(s+3)$

put $s = 1$

$1 = D(8)$

$D = 1/8$

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put $s = -3$

$1 = C(9+1)(-4)$

$1 = C(10)(-4)$

$C = -1/40$

equating power of s^3

$0 = A + C + D$

$0 = A - \frac{1}{40} + \frac{1}{8}$

$= A + \frac{1}{10}$

$A = -\frac{1}{10}$

equating power of s^2

$0 = B - C + 3D$

$0 = B + \frac{1}{40} + \frac{3}{8}$

$= B + \frac{2}{5}$

$B = -\frac{2}{5}$

$y(t) = L^{-1}\left[\frac{-\frac{1}{10}s - \frac{2}{5}}{s^2+1} - \frac{1}{40(s+3)} + \frac{1}{8(s-1)}\right]$

$= L^{-1}\left[-\frac{s}{10(s^2+1)} - \frac{2}{5(s^2+1)}\right]$

$- L^{-1}\left[\frac{1}{40(s+3)}\right] + L^{-1}\left[\frac{1}{8(s-1)}\right]$

$y(t) = -\frac{1}{10}\cos t - \frac{2}{5}\sin t - \frac{1}{40}e^{-3t} + \frac{1}{8}e^{t}$

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PART-C

9] a] Every T_2-Space is T_1-Space.
let $(X, \mathcal{J})$ is T_2-Space
we have to prove $(X, \mathcal{J})$ is T_1-Space
let $x, y \in X$ be two distinct points
Since $(X, \mathcal{J})$ is T_2-Space
$\therefore \exists$ two disjoint open set U & V
Such that $x \in U$ & $y \in V$
$\Rightarrow x \in U$ & $x \notin V$
$\Rightarrow x \in U$ & $y \notin U$
$\therefore \forall$ every pair of distinct point x & y in X $\exists$ an open set U containing x not containing y
$(X, \mathcal{J})$ is T_1-Space
Hence every T_2-Space is T_1-Space.

Converse of the theorem need not be true.
i.e every T_1-Space is need not be T_2-Space
For example:- The Co-finite topology defined on infinite set X.
Since Co-finite topology is T_1-Space but not T_2-Space
let $x, y \in X$ & $x \neq y$
$\{x\}$ & $\{y\}$ are closed sets
$\Rightarrow \{x\}'$ & $\{y\}'$ are open sets
Such that $y \in \{x\}'$ & $x \in \{y\}'$
& $\{x\}' \cap \{y\}' = X - \{x, y\} \neq \emptyset$
$\therefore (X, \mathcal{J})$ is not T_2-Space
$\therefore$ Every T_2-Space is T_1-Space.

b] A property P of a topological space $(X, \mathcal{J})$ is said to be hereditary property if every subspace of a topological space $(X, \mathcal{J})$ have the property P.

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property of topological space being T_1-space is hereditary property.
let $(X, \mathfrak{I})$ is T_1-space.
and $(Y, \mathfrak{I}_Y)$ subspace of $(X, \mathfrak{I})$
let $y \in Y$ & $Y \subset X$
$\Rightarrow y \in X$
since $(X, \mathfrak{I})$ is T_1-space
$\Rightarrow \{y\}$ is closed in X
$\Rightarrow Y \cap \{y\}$ is closed in Y
$\Rightarrow \{y\}$ is closed in Y $\forall y \in Y$
$\therefore (Y, \mathfrak{I}_Y)$ is T_1-space
Hence every subspace of T_1-space.

10 a) $L^{-1}\left[\frac{1}{S(S+1)(S+2)}\right]$

let $\frac{1}{S(S+1)(S+2)} = \frac{A}{S} + \frac{B}{(S+1)} + \frac{C}{(S+2)}$

$1 = A(S+1)(S+2) + B(S)(S+2) + C(S)(S+1)$

put $S = -1$
$1 = B(-1)(1)$
$B = -1$

put $S = -2$
$1 = C(-2)(-1)$
$C = \frac{1}{2}$

put $S = 0$
$1 = 2A$
$A = \frac{1}{2}$

$$\frac{1}{S(S+1)(S+2)} = \frac{1/2}{S} + \frac{(-1)}{S+1} + \frac{1/2}{(S+2)}$$

applying L^{-1} on both side

$$L^{-1}\left[\frac{1}{S(S+1)(S+2)}\right] = L^{-1}\left(\frac{1}{2S}\right) - L^{-1}\left[\frac{1}{(S+1)}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{S+2}\right]$$

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$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \frac{t}{2} - e^{-t} + \frac{1}{2}e^{-2t}.$$

b) $L[f(t)] = F(s)$

$$g(t) = \begin{cases} f(t-a), & t > a \\ 0 & t < a \end{cases}$$

prove that $L[g(t)] = e^{-as} F(s)$

→) we have $L[g(t)] = \int_0^\infty e^{-st} g(t)\, dt$

$$L[g(t)] = \int_0^a e^{-st} g(t)\, dt + \int_a^\infty e^{-st} g(t)\, dt$$

$$= \int_0^a e^{-st}\, 0\, dt + \int_a^\infty e^{-st} f(t-a)\, dt$$

$$L[g(t)] = 0 + \int_a^\infty e^{-st} f(t-a)\, dt$$

$$L[g(t)] = \int_a^\infty e^{-st} f(t-a)\, dt$$

put $t - a = u$

$t = u + a$

$dt = du$

if $t = a \Rightarrow u = 0$ & $t = \infty$ $u = \infty$

$$L[g(t)] = \int_0^\infty e^{-s(u+a)} f(u)\, du$$

$$= e^{-sa} \int_0^\infty e^{-su} f(u)\, du$$

$$L[g(t)] = e^{-as} F(s)$$

(1) a) $f(t) = \begin{cases} \sin t & 0 < t \le \pi/2 \\ \cos t & t > \pi/2 \end{cases}$

function interms of Hevilide function

By standard result

$f(t) = \sin t + [\cos t - \sin t]\ u(t - \pi/2)$

$\therefore L[f(t)] = L[\sin t] + L[\cos t - \sin t]\ u(t - \pi/2)$

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$L[\sin t] = \frac{1}{s^2+1}$ — ②

$L[[\cos t - \sin t]\, u(t-\pi/2)] = L[f_1(t-\pi/2)u(t-\pi/2)]$

Here $f_1(t-\pi/2) = \cos t - \sin t$

$\therefore f_1(t) = \cos(t+\pi/2) - \sin(t+\pi/2)$

$= (-\sin t) - \cos t$

$f_1(t) = -\sin t - \cos t$

$L[f_1(t)] = L[-\sin t - \cos t] = -\frac{1}{s^2+1} - \frac{s}{s^2+1}$

$= \frac{-(s+1)}{(s^2+1)}$ $F(s)$

$L[\cos t - \sin t]\, u(t-\pi/2) = e^{-as} F(s)$

$= e^{-\pi/2 s} \frac{-(s+1)}{(s^2+1)}$ — ③

$\therefore$ from ① ② & ③

$L[f(t)] = \frac{1}{s^2+1} + e^{-\pi/2 s}\left(-\frac{(s+1)}{(s^2+1)}\right)$

$L[f(t)] = \frac{1-(s+1)e^{-\pi/2 s}}{s^2+1}$

b) $\int_0^\infty t^2 e^{-t} \cos t \, dt$

we have $\int_0^\infty e^{-st} t^2 \cos t \, dt = L[t^2 \cos t]$ — ①

Consider $L[\cos t] = \frac{s}{s^2+1} = F(s)$

$L[t^2 \cos t] = (-1)^2 F''(s)$

$= \frac{d^2}{ds^2}\left(\frac{s}{s^2+1}\right)$

$= \frac{d}{ds}\left(\frac{(s^2+1) - s(2s)}{(s^2+1)^2}\right)$

$= \frac{d}{ds}\left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)$

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$$L[t^2 \cos t] = \frac{d}{ds}\left(\frac{1-s^2}{(s^2+1)^2}\right)$$

$$= \frac{(s^2+1)^2(-2s) - (1-s^2)2(s^2+1)(2s)}{(s^2+1)^4}$$

$$= \frac{(s^2+1)(-2s) - (1-s^2)4s}{(s^2+1)^3}$$

$$= \frac{-2s^3 - 2s - 4s + 4s^3}{(s^2+1)^3}$$

$$= \frac{2s^3 - 6s}{(s^2+1)^3}$$

$$L[t^2 \cos t] = \frac{2s(s^2-3)}{(s^2+1)^3} \quad \text{--- (2)}$$

from (1) & (2)

$$\int_0^\infty e^{-st}\, t^2 \cos t \, dt = \frac{2s(s^2-3)}{(s^2+1)^3}$$

Put $s = -1$

$$\int_0^\infty e^{t}\, t^2 \cos t \, dt = \frac{2(-1)(1-3)}{(1+1)^3}$$

$$= \frac{(-2)(-2)}{(2)^3}$$

$$= \frac{4}{8}$$

$$\int_0^\infty e^{t}\, t^2 \cos t \, dt = \frac{1}{2}$$

$$\therefore \int_0^\infty e^{t}\, t^2 \cos t \, dt = \frac{1}{2}$$

12) a) statement:- If $L[f(t)] = F(s)$ & $L[g(t)] = G(s)$ then $L^{-1}[F(s)\,G(s)] = \int_0^t f(u)\, g(t-u)\, du$

proof:- let $L[f(t)] = F(s)$ & $L[g(t)] = G(s)$

consider $L\left[\int_0^t f(u)\, g(t-u)\, du\right]$

$$= \int_0^\infty e^{-st} f(u)\, g(t-u)\, du\, dt \quad \text{--- (1)}$$

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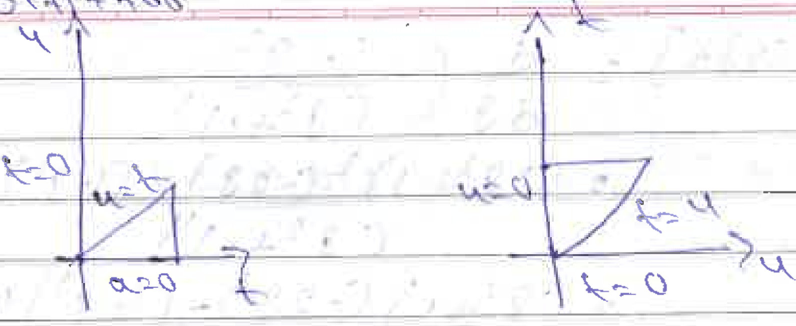

The existing region is t=0 to t=∞
u=0 to u=t

on changing the co-ordinate axes
u=0 to u=∞
t=u to t=∞
(In both the cases area remain same.)
∴ eq^n ① reduces to.

$$L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] = \int_{u=0}^{u=\infty}\int_{t=u}^{t=\infty} e^{-st} f(u)\, g(t-u)\, dt\, du$$

put $t-u=v$ ∴ $t=u+v$
∴ $dt=dv$
∴ $t=u \rightarrow v=0$ & $t=\infty \rightarrow v=\infty$

$$\therefore L\left[\int_{u=0}^{u=t} f(u)\, g(t-u)\, du\right] = \int_{u=0}^{u=\infty}\int_{v=0}^{v=\infty} e^{-s(u+v)} f(u)\, g(v)\, du\, dv.$$

$$= \int_{u=0}^{u=\infty} e^{-su} f(u)\, du \int_{v=0}^{v=\infty} e^{-sv} g(v)\, dv$$

$$= L[f(t)]\, L[g(t)]$$

$$= F(s)\, G(s)$$

$$L[f(t) * g(t)] = F(s)\, G(s)$$

$$L^{-1}[F(s)\, G(s)] = f(t) * g(t)$$

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b) $\frac{1}{(s^2+4s+1)^2}$

$s^2+4s+1+4-4$

$s^2+4s+(2)^2+1-4$

$s^2+4s+(2)^2-3$

$(s+2)^2-3$

$\frac{1}{((s+2)^2-(\sqrt{3})^2)^2}$

$F(s) = \frac{1}{(s+2)^2-(\sqrt{3})^2}$ $G(s) = \frac{1}{(s+2)^2-(\sqrt{3})^2}$

$L^{-1}[F(s)] = e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}}$ $L^{-1}[G(s)] = e^{-2t}\frac{\sinh\sqrt{3}t}{\sqrt{3}}$

$L^{-1}[F(s)\,G(s)] = f(t) * g(t)$

$= F(s) * G(s)$

$= e^{-4t}\frac{\sin^2 h\sqrt{3}t}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}\int_0^t e^{-4u}\sin^2 h\sqrt{3}(t-u)\,du$

$= \frac{1}{\sqrt{3}}\int_0^t e^{-4u}\left(\frac{e^{\sqrt{3}(t-u)} - e^{-\sqrt{3}(t-u)}}{2}\right)^2 du$

$= \frac{1}{\sqrt{3}}\int_0^t e^{-4u}\left(\frac{e^{2\sqrt{3}(t-u)} - e^{-2\sqrt{3}(t-u)}}{4}\right) du$

$= \frac{1}{4\sqrt{3}}\int_0^t \left[e^{-4u}e^{2\sqrt{3}(t-u)} - e^{-4u}e^{-2\sqrt{3}(t-u)}\right] du$

$= \frac{1}{4\sqrt{3}}\int_0^t e^{-6\sqrt{3}u+2\sqrt{3}t} - e^{-2\sqrt{3}u-2\sqrt{3}t}\,du$

$= \frac{1}{4\sqrt{3}}\left[\frac{e^{-6\sqrt{3}u+2\sqrt{3}t}}{-6\sqrt{3}} - \frac{e^{-2\sqrt{3}u-2\sqrt{3}t}}{-2\sqrt{3}}\right]_0^t$

$= \frac{1}{4\sqrt{3}}\left[\frac{e^{-4\sqrt{3}t} - e^{2\sqrt{3}t}}{-6\sqrt{3}} - \frac{e^{-4\sqrt{3}t} - e^{-2\sqrt{3}t}}{-2\sqrt{3}}\right]$

$= \frac{1}{-4\sqrt{3}}\left[\frac{e^{-4\sqrt{3}t} - e^{2\sqrt{3}t} - 3e^{-4\sqrt{3}t} - 3e^{-2\sqrt{3}t}}{6\sqrt{3}}\right]$

$= \frac{1}{-4\sqrt{3}}\left[\frac{-2e^{-4\sqrt{3}t} - e^{2\sqrt{3}t} - 3e^{-2\sqrt{3}t}}{6\sqrt{3}}\right]$

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$$L^{-1}[F(s)G(s)] = \frac{1}{-24\times3}\left[-2e^{-4\sqrt{3}t} - e^{2\sqrt{3}t} - 3e^{-2\sqrt{3}t}\right]$$

$$L^{-1}[F(s)G(s)] = \frac{2e^{-4\sqrt{3}t} + e^{2\sqrt{3}t} + 3e^{-2\sqrt{3}t}}{72}$$

PART-A

i) we have $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$\therefore L[f'(t)] = \int_0^\infty e^{-st} f'(t)\,dt$

By using integrating by parts we see

$$L[f'(t)] = \left. e^{-st} f(t)\right]_0^\infty - \int_0^\infty f(t)\, e^{-st}(-s)\,dt$$

$$= 0 - f(0) + s\int_0^\infty e^{-st} f(t)\,dt$$

$$= -f(0) + s\,F(s)$$

$$L[f'(t)] = sF(s) - f(0)$$

Year : 2016-17 onwards
Class : B.com Ist Sem

Department of Commerce

FINANCIAL ACCOUNTING VOL-I
2 MARKS QUESTIONS

1. Why is total debtors a/c prepared?
   The total Debtors Account is prepared to ascertain the amount of Credit sales or Closing balance of Debtors or Cash Collected Debtors or amount of B/R Received.
2. State any two features of single entry system.
   Both the aspects of each and every transaction are not recorded in the books
   It is unscientific unsatisfactory and inaccurate
3. How is minimum rent calculated in the event of strike or lock out?
   The minimum rent is calculated on the basis of an agreement between landlord and tenent. The agreement may be calculated in any one of the following forms
   Proporationate Decrease in minimum rent based on period of Strike or lockout or
   Actual amount of royalty earned may be treated as minimum rent.

4. What is short workings? How do you calculate it?
   Excess of minimum rent over royalty is called short working it can be calculated on the basis of minimum rent and royalty ie. Short workings = Minimum Rent – Royalty
5. Find out the sales from the following information.

Opening Stock	Rs 15000
Closing Stoch	Rs 25000
Purchases	Rs 85000

   Margin of Profit is 20% on Cost price
   Ans:

Opening Stock	Rs 15000
Purchases	Rs 85000
	100000
Less:Closing Stock	Rs 25000
Cost of goods sold	75000
Add: Profit 75000 X $^{20}/_{100}$	15000
Sales	90000

6. Why is total Creditors a/c prepared?
   The total Creditors Account is prepared to ascertain the amount of Credit Purchases or Closing balance of Creditors or Cash paid Creditors or amount of B/P Issued.
7. Give the meaning of Royalty and write the different types.
   Royalty is a consideration payable by lessee to lessor for using the rights of lessor. The main types are-

Tenent and landlord	in mining Royalty
Publisher and Author	in copyright ,,
Manufacturer and Patentee	in Patent ,,

8. What is short workings? Give example.
   Excess of minimum rent over royalty is called short working
   Eg If Royalty for a year is Rs 10000 and Minimum Rent is Rs 12000 then Short working is_
   Short workings = Minimum Rent – Royalty
   =12000 -10000 =2000

	V A		IT		CAB	ECO	IFM				
	V B		IT		ECO	GST	COST		IFM		
Saturday	I A	PM	KAN		FA	MIL	OEC				
	I B	KAN	MPA		FA	MIL					
	III A		QA		PS	IB	MIL				
	III B	QA	ENG		EDP	PS	MIL				
	V A	COST	COST		CAB	ECO	IFM				
	V B		CAB		GST	COST	M A\C		IFM		

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Commerce College, NIPANI.

9. What is the journal for short working irrecovered?

Profit & loss A/C-------------------Dr To Short workings A/C. (Being short working irrecovered).	XXX -----	-- XXX
	XXX	XXX

10. Why the trail balance can't be prepared under single entry system of book keeping?
All the ledger account balances not available in single entry system of book keeping so It is not possible to prepare trail balance with incomplete records.
11. What is the effect of strike or lock out on Royalty accounts?
The activity stops and the amount of royalty may not fetch the minimum Rent then the minimum rent is to be reduced on the following base.
Proporationate Decrease in minimum rent based on period of Strike or lockout
Actual amount of royalty earned may be treated as minimum rent.
12. Find out the cost of goods sold from the following information.

Opening Stock	Rs 5000
Closing Stoch	Rs 10000
Purchases	Rs 75000

Ans:

Opening Stock	Rs 5000
Purchases	Rs 75000
	80000
Less:Closing Stock	Rs 10000
Cost of goods sold	70000

13. Why is total Bills Receivable a/c prepared?
The total Bills Receivable Account is prepared to ascertain the amount of Bills Receivable Received from Debtors or Closing balance of Bills Receivable or Cash Received on Bills Receivable or amount of Bills Receivable Dishonoured
14. Give two difference between statement of affairs & Balance sheet.

Statement of affairs	Balance sheet.
a) It is prepared under single entry system b) Some assets are estimated c) It is prepared to ascertain the capital	a) It is prepared under double entry system b) All the assets & Lliabilities are actual Value c) It is prepared to ascertain financial position

15. Give any two difference between single entry & double entry system.

Single entry	Double entry system
It is a incomplete form of book keeping It is unscientific & unsatisfactory	It is a complete form of book keepingb) It is scientific & satisfactory

16. What is statement of affairs ?
It is a staement containing various assets & liabilities of the firm. It is just like a balance sheet but not a balance sheet because some assets are estimated.
17. Give two merits of single entry system.
Ans: The main merits of single entry system are-
a) It is easy & simple to understand
b) It is not a costly system
c) There is no delay in recording the transaction

18. State any two demerits of single entry system.

Ans: The main demerits of single entry system are-

a) It is inaccurate, unscientific & unsatisfactory.

b) It will not disclose the correct financial position & results.

19. Define single entry system.

It is a method of maintaining the accounts in a manner convenient to a particular trader where the double entry principle is not applicable to all the transactions.

20. Why the statement of affairs is prepared?

It is prepared to ascertain the opening capital of the business

21. How do you ascertain the profit under the single entry system?

The profit or loss is ascertained on the basis of opening capital, closing capital, additional capital, drawing. Expenses & incomes

22. How does the capital fund deffer from capital .

Capital fund	Capital
a)It is created by capitalising capital receipt b)It can not be withdrawn c)It is relating to non trading concern	It is contributed by owners It can be with drawn by the owner It is relating to trading concern.

23. How does the cash book differ from receipt & payment a/c

Cash book	Receipt & payment a/c
It is actual record of each entry It is recorded daily It is essential for both trading and non trading concerns	It is a summery of cash book It is recorded at the end of the year It is essential for both trading and non trading concerns

24. Give any two difference between Income & Expenditure A/C and P & L A/c

Income & Expenditure A/C	P & L A/c
It is a revenue a/c of non trading concern Closing balance of this a/c is called surplus or deficit	It is a revenue a/c of trading concern Closing balance of this a/c is called net profit or net loss

26. Define capital expenditure.

It is a non recurring expenditure incurred for acquisition of capital assets such as purchase of land machinery etc, or discharge of liability. It is a balance sheet item.

27. State any two difference between Receiprs & Payments a/c & Income & Expenditure

Receiprs & Payments	Income & Expenditure a/c
It is a real a/c It is a summery of cash transactions It includes all capital & revenue a/c	It is a nominal a/c It is a summery of Incomes & expenses It includes only revenue a/c

28. State any two difference between Capital Expenditure & Revenue Expenditure Expenditure

Capital expenditure	Revenue expenditure
It is non recurring in nature It is a real or personal account	It is recurring in nature It is a nominal account

29. What is income and expenditure account of a profession?

It is an income and expenditure account of household transactions.

30. How do you treat the wages paid to workers in kind in crop account?
   It is shown on the both side of the crop account.
31. What is farm account?
   Application of accounting principles and techniques to farming which constitutes the activities such as farming dairy farming, poultry farming
32. Give two examples of farm accounting.
   Ans: the main examples of farm accounting are-
   Agricultural farm Dairy farm
   Poultry farm Horticulture sericulture
33. What is meant by work in progress?
   Work in progress is the fees earned in respect of incomplete maters of a particular accounting year for which no bills have been rendered and are not brought into account.
34. What is Bills of Costs?
   These are the final bills prepared by lawyers and issued to their clients on completion of jobs.
35. Do you record cash sales and RDD in total debtors a/c? Give reasons.
   NO because cash sale is a cash transaction comes to cash book.
   RDD is a provision relating to P and L account
36. How do you record wages paid in kind in crop account?
   It is recorded on both side of crop account.ie expense as debit side and income as on credit side.
37. What is farm account?
   Application of accounting principles and techniques to farming which constitutes the activities such as farming dairy farming, poultry farming
38. What is meant by work in progress?
   Work in progress is the fees earned in respect of incomplete maters of a particular accounting year for which no bills have been rendered and are not brought into account.
39. What is receipts and expenditure account?
   This is a hybrid system of recording the professional accounts. Under this incomes are recorded on cash bases and expenses and on mercantile bases. It is also called highbred system
40. What do you mean by quasi single entry system?
   It is system where the trader maintains cash book , ledger and some subsudairy books but not all the books.
41. Mention the two objectives of conversion of single entry system into double entry system.
   a) Complete record of all the transations
   b) Check the arthematical accuracy
42. What do you ascertain from debtors account when opening balance, closing balance and credit sales are given?
   Cash and cheques collected or bills receivables received.
43. What is profession?
   Profession is an occupation executed by intellectual skill of a person or manual skill controlled by intellectual skill of that person.

44. What is client deposit account?
It is an account opened by professional to recored the cash advance cash received for expenses from their client for expenses

45. Mention the different kinds of farm transaction.
Ans: Cash transaction
Credit transaction
Exchange transation
Notional transation.

46. What do you mean by recoupment of short working?
It is one clase of royalty agreement under which lessee is permited to recover the shortworking of previous years of current year excess royalty over the minimum rent.

47. How do you ascertain credit sale and credit purchase?
Credit sale is ascertained by opening the debtors account
Credit purchase is ascertained by opening the Creditors account

48. Who is a chartered accountant?
A chartered accountant is person who has completed the training for a prescribed number os years and passed the chartered accountants final examination conducted by ICAI

49. Who is a lawyer?
Lawyer is qualified independent person who render legal service to his client for fees. He is a law graduate and registered his name with BAR Council.

50. When minimum rent account is prepared?
Minimum rent account is prepared in the year only where short working arises.

51. What is notional transaction?
These are the transactions made between farm and household of the farm such as consumption of farm output by household.

52. Give four examples of profession.
a. Chartered Accountant, Doctor
b. Lawyer. Architect

53. What are exchange transactions?
Exchange transactions are those transactions where to facilitate with each other. The exchange of goods and services take place between the parties.

54. What do you mean by recoupment of short working?
Recovery of short workings of preceding years out of excess royalty of current year as per agreement between lesser and lessee.

55. What is mercantile system of accounting?
Mercantile system of accounting cash as well as credit transactions are recorded in the books. Under this system transactions related to particular period are recorded without considering its receipt or payment.

56. What is the journal for short working recovered by lessee?

Lessor A/C-------------------Dr To Short workings A/C. (Being short working recovered).	XXX -----	-- XXX
	XXX	XXX

57. Write any two features of farm accounting,

Ans: Farming is like a family business

Consumption of farm produce by household

58. State Any two objectives of accounts of professionals.

To Maintain financial Records neatly.

To bifurcate household expenses from profession.

59. How minimum rent may be reduced in case of strike?

Minimum rent will be reduced according to the agreement between lessee and lessor. It may br proportionate reduction or fixed amount

60. Who is Lessee

Lessee is the person who uses the asset of the creator or the owner in lieu of a consideration for using such an asset.

Examples of Lessees include publishers, mining company, or manufacturers etc.

61. Who is Lessor

The person who creates or owns the asset and provides the right of using such an asset to the third party is known as the lessor or the landlord. Furthermore, lessor receives consideration from the third party.

Examples of lessors include owner of the mine or quarry, author of a book, artist in case of a music composition etc.

62. State Any two objectives of farm accounting

To ascertain true cost and profit

To provide authentic (reliable or believable) information

63. What is minimum rent?

This is the minimum amount payable by lessee to lessor when the royalty does not fetch the reasonable amount as per agreement.

In some years if the lessee fails to raise the reasonable quantity of output or fails to raise the reasonable turnover then the lessee is to pay minimum amount fixed as per agreement this amount is called minimum rent

64. What is Average Clause

An average clause is applied to find out the value of a claim where value of the stock on the date of fire is more than the value of insured stock. Average clause is applied by the insurance companies to discourage the under insurance of stock or any other assets.

Amount Claimed = $\frac{Policy\ Value}{Total\ Value}\ X\ Total\ Loss$

65. What is Insurance?

It is an agreement between Insured and Insurer to compensate the losses suffered due to uncertainties in future, for a consideration called premium.

66. What is meant by Fire Claims?

It is a kind of General insurance where an agreement is made between the industry (i.e., insured) and General Insurance Company (i.e., insurer) to indemnify the compensation for the loss of stock or profit due to fire accident, for a consideration called premium.

67. . Who is an Insured?

Insured is a person/industry/asset, to whom/which the insurance is made. The compensation shall be received on happening of certain event determined i.e., death of a person or destroy of asset or properties.

68. Who is Insurer?

Insurer is an insurance company which pays the losses suffered by the insured on happening of certain event estimated in advance i.e., death of a person or destroy of asset or properties.

69. What is Trading Account?

Trading Account is a ledger prepared to find out the Gross Profit of an accounting year. It includes the trading activities done by an industry during a financial year.

70. When do we have to prepare the previous year's 'trading account under insurance?

The previous year's trading account is prepared to find out the last year gross profit to help the calculation of Gross Profit during the year in which fire accident occurred, to find out the stock on the date of fire accident.

71. What is Gross Profit Ratio?

Gross Profit Ratio is a ratio which shows the relationship between the Gross Profit and Net Sales. Net Sales = Total Sales – Return inwards.

72. How do you calculate Gross Profit Ratio?

Gross Profit Ratio = Gross Profit x100 Net Sales

73. What is Memorandum Trading Account?

The Memorandum Trading Account is similar to usual trading account. It is prepared from the begin date of accounting year and till the date of fire accident. It is not prepared as per double entry system of booking.

74. What is meant by Salvage?

The value of stock saved from the fire accident is called salvaged stock. Sometimes it is also referred as scrap value or realizable value of stock. The saved stock should be deducted from the stock of the date of fire.

Year : 2017-18 onwards
Class : B.com III^rd^ Sem

Statistics-I

01. Define mutually exclusive event

Two or more events are mutually exclusive if only one of them can occur at a time

02. If a die is thrown, what is the probability of getting face with 3 dots upperface?

$n = 6 = 6 \quad m = 1$

$P(A) = {}^m/_n = {}^1/_6$

03. Define mathematical expectation.

Let X be a discrete random variable with probability mass function p(x) then, mathematical expectation of X is – $E(X) = \sum x.p(x)$

04. What is the probability that a boll drawn at random from a bag containing 3 white and 6 black balls is white

$n = 9 \qquad m = 3$

$P(A) = {}^m/_n = {}^3/_9 = {}^1/_3$

05. If S= [1 2 3 4 5 6] and A = [4 6] find P(A)

$P(A) = {}^m/_n$

$= {}^2/_6 = {}^1/_3$

06. What is probability distribution

A systematic presentation of the values taken by a random variable and the corresponding probabilities is called probability distribution

07. If E(X) = -2 find E(3X+8)

$E(aX+b) = aE(X) + b$

$= (3 X -2) + 8$

$= -6+8$

$= 2$

08. If variance of (X) = 9 and $E(X^2) = 25$ Find E(X)

$V(X) = [E(X)^2] - [E(X)]^2$

$9 = 25 - [E(X)]^2$

$[E(X)]^2 = 25-9$

$[E(X)]^2 = 16$

$E(X) = \sqrt{16} \qquad E(X) = 4$

09. Give the meaning of permutation.

Permutation is an orderly arrangement of 'r' objects from there 'n' objects.

$^nP_x = \frac{n!}{(n-x)!}$

10. Give the meaning of combination.

Combination is selection of 'r' objects from these 'n' objects.

$^nC_x = \frac{n!}{(n-x)!.x!}$

11. Define probability.

Probability of an event is a numerical measurement which indicates the chance of occurrence of an event.

12. Define an experiment.

Experiment is an operation which can produce some well-defined outcomes.

13. What are the types of experiment?

i) Deterministic experiment and

ii) Random experiment

14. Define Deterministic experiment.

A deterministic experiment is one when repeated under the same condition; it results in the same output. It has a unique outcome.

15. Define random experiment.

A random experiment is one when repeated under the same condition; it may not result in the same output. It has no unique outcome.

16. What is sample space?

The set of all possible outcomes of random experiment is called as sample space.

17. Give the meaning of an event.

Event is a subset of an experiment.

18. Define :

Null or impossible event.

It is an event which does not contain an outcome. It is denoted by Ø.

Elementary or simple event.

Elementary or simple event is an event which contains only one event.

Sure or certain event.

Sure or certain event is an event which contains all the event equal to sample space.

19. Give the meaning of complementary event.

Let A be an event. Then complementary event A is the non-occurrence of A. It is denoted by A'

20. What is union of events?

Union of two or more events is the event occurrence of at least one of these two events is called union of events.

21. Give the meaning of intersection of events.

Intersection of two or more event is the event of simultaneously of all these event is called intersection of an event.

22. Define mutually exclusive event.

Two or more events are mutually exclusive only if one of them occurs at a time is called mutually exclusive event.Mutually exclusive event cannot occur at a time.Intersection of Mutually exclusive event is null event

23. Give mathematical definition of probability.

Let a random experiment have n likely outcomes. Let m of these outcomes be favourable to an event A. Then probability of A is

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}} = \frac{m}{n}$$

24. What are the limitations of mathematical definition?

The outcomes are equally likely.

The number of outcome 'n' is finite.

25. Explain Addition theorem of any two events.

Let A and B be two events with Respective probabilities P(A) and P(B), then the occurrence of at least one of these two event is

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

26. Explain Addition theorem of mutually exclusive event.

Let A and B be two mutually exclusive events with Respective probabilities P(A) and P(B), then the occurrence of at least one of these two event is

$P(A \cup B) = P(A) + P(B)$

27. Define conditional probability.

Let A and B be twoevents. Then conditional probability of B given A is the probability of happening of B when it is known that A has already happened. On the other hand, the happening of B when nothing is known about happening of A is called as conditional probability.

$P(B/A) = P(A \cap B)/P(A)$

28. Define independent event.

TwoeventsA and B are independent if and if $P(A \cap B) = P(A).P(B)$

The occurrence or non-occurrence of one does not depend on the other.

29. Define multiplication theorem.

Let A and B be two events with respective probabilities P(A) and P(B). Let P(B/A) be the conditional probability of event B given that event A has happened. The probability of simultaneous occurrence of A and B is $P(A \cap B) = P(A).P(B/A)$

Two events A and B are independent$P(A \cap B) = P(A).P(B)$

30. If $P(A) = {}^1/_3$. Find $P(A)^1$

$P(A)^1 = 1 - P(A) \quad = 1 - {}^1/_3 \qquad = {}^2/_3$

31. A and B are mutually exclusive events with $P(A) = {}^1/_3$and $P(B) = {}^1/_5$

Find $P(A \cup B)$ and$P(A \cap B)$

$P(A \cap B) = P(A).P(B) = {}^1/_3 . {}^1/_5$

$= {}^1/_{15}$

$P(AUB) = P(A) + P(B) - P(AB)$

$= {}^1/_3 + {}^1/_5 - {}^1/_{15} = \frac{5+3-1}{15} = {}^7/_{15}$

32. If $P(A \cap B) = {}^1/_2$ and $P(B) = {}^2/_3$ find $P(A/B)$

$P(A/B) = P(A \cap B)/P(A)$

$= {}^1/_2 / {}^2/_3 = {}^1/_2 . {}^3/_2$

$= 3/4$

33. If $P(B) = {}^3/_5$ and $P(A/B) = {}^1/_3$ find $P(B \cap A)$

$P(A/B) = P(A \cap B)/P(A)$

${}^1/_3 = P(A \cap B)/{}^3/_5$

$P(A \cap B) = {}^1/_3 . {}^3/_5$

$= {}^3/_{15}$

$= {}^1/_5$

34. P(A)=find P(B/A) and P(AUB)

$P(B/A) = P(A \cap B)/P(A)$

$= {}^1/_4 \ / {}^1/_2 = {}^1/_4 . / {}^2/_1$

$= {}^1/_2$

$P(AUB) = P(A) + P(B) - P(AB)$

$= {}^1/_2 + {}^1/_3 - {}^1/_4$

$= {}^7/_{12}$

35. If $P(AUB) = {}^1/_3$, $P(A \cap B) = {}^1/_{12}$ and $P(A) = {}^1/_6$ find P(B).

$P(AUB) = P(A) + P(B) - P(A \cap B)$

${}^1/_3 = {}^1/_6 + P(B) - {}^1/_{12}$

${}^1/_3 + {}^1/_{12} - {}^1/_6 = P(B)$

$\frac{4+1-2}{12} = P(B)$

${}^3/_{12} = P(B)$

36. If P(A) =0.7 , P(B)=0.1and P(AUB)=0.7 find P(A/B): Are A and B independent events.

$P(AUB) = P(A) + P(B) - P(A \cap B)$ $0.7 = 0.7 + 0.1 - P(A \cap B)$ $0.7 = 0.8 - P(A \cap B)$ $P(A \cap B) = 0.8 - 0.7$ $P(A \cap B) = 0.1$	If A and B independent events $P(A \cap B = P(A) P(B)$ $0.1 = 0.7 \times 0.1$ $0.1 \neq 0.07$ A and B not independent events

37. Define Random variable.

Random variable is a function which assigns real number to every sample point in the sample space.

38. Define discrete random variable.

A variable x which takes value $x_1, x_2 \dots\dots\dots x_n$ with probabilities $p_1, p_2 \dots\dots\dots p_n$ is a discrete random variable.

39. Define continuous random variable.

A random variable whose range is unaccountably infinite is a continuous random variable.

40. Define probability mass function.

Let x be a discrete random variable, and let p(x) be a function such that p(x)= p=x.

Then p(x) is the probability mass function.

41. Define probability density function.

A similar function is defined for a continuous random variable x.it is called probability density function (p.d.f). it is denoted by f(x).

42. Write the mean and variance and S.D of random variable.

Mean = E(x)

$var(x) = E(x^2) - [E(x)]^2$

S.D = $\sqrt{\text{variance}}$

43. Find the A. M of the values: 70 75 80 90 110

$$\overline{X} = \frac{\sum Xi}{N} = \frac{425}{5} = 85$$

44. State any two advantages of median

i) Easy to understand and simple to calculate

ii) It can be applicable to open end classes.

45. What do you mean by dispersion?

"Dispersion is the measure of the variation of items" (A L Bowley)

46. Coefficient of variation of the price of an article is 50%, its SD is 30 find AM

$CV = {}^{SD}/_{AM}$, $AM = {}^{SD}/_{CV}$ $AM = {}^{30}/_{0.50} = 60$

47. Define range and its coefficient.

Range is the difference between the two extreme observations of the distribution.

Range = $X_{max} - X_{min}$

Coefficient of Range= $\frac{X_{max} - X_{min}}{X_{max} + X_{min}}$

48. State any two merits of SD.

i) It is rigidly defined

ii) It is used for further mathematical treatments.

49. What do you mean by quartiles?

The values which divide the given data into four equal parts are known as "Quartiles"

50. Define discrete and continuous variables.

The variables which cannot take all the possible values within a given specified range are called Discrete variable Eg the Marks in a test of a group of students

The variables which can take all the possible values in a given specified range are called continuous variable Eg age of students in a school.

51. State two functions of statistics

Present general statement in precise and definite form

Simplifying and condensing huge amount of quantitative information

52. What are the partition values?

The values which divide the series into a number of equal parts are called the partition values.

53. Calculate the Median for the values: 12 09 19 10 26 25

Ans: 09 10 12 19 25 26

Median = AM of two middle terms

= ½(12+19)

= ½(31) =15.5

54. Find coefficient of range for the following values.

X:	10	11	12	13	14	15	16	17	18	19
F:	04	06	10	12	13	05	06	07	08	04

L = 19, S = 10

Coefficient of range = $\frac{L-S}{L+S}$ = $\frac{19-10}{19+10}$ = $^{9}/_{29}$ = 0.31

55. Define quartile deviation and coefficient of quartile deviation.

It is a measure of dispersion based on the upper quartile and lower quartile.

$QD = \frac{Q3-Q1}{2}$

Coefficient of QD = $\frac{Q3-Q1}{Q3+Q1}$

56. Calculate SD for the values: 55 35 85 40

X^2 = 3025, 1225, 7225, 1600

$\sum X^2$ = 13075

$\sum Xi$ = 215

$SD = \sqrt{\frac{\sum X2}{N} - \left(\frac{\sum X}{N}\right)}$

$SD = \sqrt{\frac{13075}{4} - \left(\frac{215}{4}\right)}$

$SD = \sqrt{3268.75 - (53.75)}$

$SD = \sqrt{3268.75 - 2889.06}$

$SD = \sqrt{379.6875}$SD = 19.4855

57. State two limitations of statistics

Statistics does not study qualitative phenomenon

Statistical laws are not exact

58. Distinguish between inclusive and exclusive methods of classification.

Inclusive	Exclusive

i) Both upper limit and lower limits values are included in same class ii) Fractional values cannot be accountable	Upper limit value of a class is considered in next class Fractional values can be accountable

59. What is the use of coefficient of variation?
    i) To compare two or more groups of similar data with respect to stability.
    ii) To study the variations in the price of shares of different companies.
60. Name the graphs which are used to locate median and mode
    For Median-Cumulative frequency curves that is more than and less than Ogive curve
    For Mode-Histogram
61. Name the graphs which used to locate mode.
    Histogram
62. Find median of 40.5 62.8 10.5 25.6 35.3 47.0 65.4
    Ans: 10.5 25.6 35.3 40.5 47.0 62.8 65.4
    Median = $\left[\frac{N+1}{2}\right]^{th}$ item $\therefore median = 4thiten = 40.5$
63. What is mean deviation?
    The average that is taken of the scatter is an arithmetic mean which accounts for the fact that this measure is often called the mean deviation.
64. For a frequency distribution Mean =52 and median =55 find mode
    Mean -Mode = 3(Mean- Median)
    52 - Mode= 3(52-55)
    52 – Mode = 3(-3)
    52- Mode = -9
    52+9 = mode
    Mode =61
65. Shoe size of 10 students are as : 5, 6, 7, 8, 9, 10, 6, 8, 8 find suitable average
    Suitable Average is mode. Therefore mode is 8
66. Give the formula for coefficient variance.
    $CV = {}^{SD}/_{AM}$
67. For a symmetric distribution Q1= 25 and Q3 = 65 find the median.
    Median $= \frac{Q_1+Q_3}{2} = \frac{25+65}{2} = {}^{90}/_{2} = 45$
68. What do Y, X, a and b signify in a fitting a straight line trend of the form Y = a = bX by least square method.
    Y = Dependable variable(Value) X = Independent variable (Time)a and b are parameters
69. Mention the uses of quartile deviation.
    It can used to measure the dispersion in case of open end class
    It can used to measure the Skewness
70. Find QD if Q_1 =44.5, Q_2 =50.5 and Q_3 = 55.5
    $QD = \frac{Q_3-Q_1}{2} = \frac{55.5-44.5}{2} = {}^{110}/_{2} = 55$
71. If the straight line trend predicting the sales of the company taking 1994 as origin of

Y = 150 +10X then predict the possible sales for the year 2005

Ans: X = 0,1,2,3,4,5,6,7,8,9,10 & 11

Y = 150 +10(11)

Y = 150 + 110

Y = 260 so predicted sales for 2005 is 260

72. The average profit of 8 industries is Rs 11.85 crore. Find the total profits.

$X = \frac{\sum Xi}{N}$ $\quad 11.5 = \frac{\sum Xi}{8}$ $\quad 11.5 \times 8 = \sum Xi = 92$ Crores

73. State any two demerits of median

It is not based on each and every item of distribution.

It is not useful for further mathematical treatments.

74. Distinguish between Absolute and Relative measures of dispersion.

Absolute measure of dispersion	Relative measure of dispersion
Dispersion expressed in original unit of series is called Absolute measure of dispersion	Dispersion expressed in ratio or % is called relative measure of dispersion

75. Mention any two examples of trend

Population of a country in different years

Sales of a departmental store in different months.

76. Mention the components of time series.

Secular trend :

Periodic Movement

Irregular variations.

77. Define any two components of time series

Secular Trend:

This trend may show the growth or decline in a time series over a long period. This is the type of tendency which continues to persist for a very long period. Prices, export and imports data,

Seasonal Trend:

These are short term movements occurring in a data due to seasonal factors. The short term is generally considered as a period in which changes occur in a time series with variations in weather or festivities..

78. Define secular trend.

The general tendency of time series data to increase or decrease or stagnate during along period of time is called secular trend.

79. Mention any two uses of range.

The range is useful when you wish to evaluate the whole of a dataset.

The range is useful for showing the spread within a data set and for comparing the spread between similar datasets.

80. Write any two properties of symmetrical distribution.

1.The mean, median and mode are all equal.

2.The distance between the Q_2& Q_1 is same as Q_3& Q_2

82. Define primary data.

The data which are originally collected by an investigator or an agency for the first time for statistical investigation and are used in statistical analysis are termed as primary data.

83. Calculate mode for the following data.

29 30 36 29 30 34 29 30 17 30

Ans: The variable which repeats for maximum times is the mode ie 30

84. . Define Bowley's coefficient of skewness.

Bowley's coefficient of skewness $= \frac{Mean - mode}{Standerd\ deviation}$ or $\frac{3(Mean - Median)}{Standerd\ deviation}$

85. Write the formula for coefficient variance.

Coefficient variance $= \frac{Standerd\ deviation}{Mean} X100$

86. Name two types of diagrammatic representation.

One dimensional bar diagrams

Two dimensional bar diagrams

87. What are the various sources of secondary data?

The main sources of secondary data are:

a. Official publications of govt agencies.
b. Official publications of the foreign govt
c. Reports and publications of commercial and trade associations.

88. Define standard deviation

90. Name any two relative measures of dispersion.

Absolute measure

Relative measure

91. Define time series.

92. CalculateQ_2 from the following data.

30 19 18 28 10 12

Q_2= median

Rearrange the data in ascending order

10 12 18 19 28 30

$Q_2 = \frac{18+19}{2} = \frac{37}{2} = 18.5$

93. Mention any two uses of time series.

i) Yield valid statistical inferences,

ii) To Know the future

94. Define Statistics.

According to Bowley "statistics are numerical statements of facts in any department of enquiry placed in relation with each other."

95. What is a statistical investigation?

The word investigation may mean exploration in pursuit of a inquisitive fact. Statistical investigation may be conducted to study any problem by means of statistical methods.

96. Give an example where a statistical investigation can be conducted

97. Who is an investigator?
In a statistical investigation enquiry, the person who conducts the enquiry is known as investigator.

98. Mention the points in stage of planning a statistical enquiry.
The main points in stage of planning a statistical enquiry are:
a. Objectives of the enquiry
b. Scope of the enquiry
c. Statistical units to be used.

99. Mention the two sources of a statistical data
Primary data
Secondary data

100. What is a secondary data?
The required data is already collected and processed by some agency or person. Such data are called secondary data.

101. Define a population.
Population may be defined as "The aggregate of individual items, whether composed of people or things, which are subjected to a statistical investigation".

102. Write any two merits of censes.
a. It helps collect more reliable data.
b. The study is extensive

103. Write any two demerits of census.
a. It is an expensive method
b. The census cannot be conducted for infinite population.

104. What is sampling?
Sample is a part of a population which is selected to study the population.

105. Mention the principles on which the logic of sampling is based.
a. Law of statistical regularity and
b. Law of inertia of large numbers.

106. Write a demerit of stratified sampling method.
a. It is a subject method.
b. The personal prejudice or bias of the investigator.

107. What are the cases of sampling error?
The errors arising due to wrong procedure, inadequate sample size, improper selection of the sample units are called sampling errors.

Year: 2017-18 onwards Goods & Service Tax - II
Class - B.com VI th Sem
B. G. Kankanawadi

1. What is exempt supply?
   Exempt supply is defined in section 2(47) of GST Act.
   "Exempt supply" means supply of any goods or services or both which attracts nil rate of tax or which may be wholly exempt from tax under section 11, or under section 6 of the Integrated Goods and Services Tax(IGST) Act, and includes non-taxable supply.
2. What is value of supply?
   Value of supply in simple terms, the amount paid by the recipient of supply to the supplier as consideration for supply.
3. What is transaction value?
   "Transaction value "means the price actually paid or payable for the goods when sold and includes in addition to the amount charged as price any amount that the buyer is liable to pay to or on behalf of the assessee by reason of or in connection with the sale whether payable at the time of sale or at any other time.
4. Who is distinct person?
   Section 25(4) of CGST Act 2017 provides that, A person who has obtained or is required to obtain more than one registration, whether in one state or union territory or more than one state or union territory shall in respect of each such registration, be treated as distinct persons for the purpose of this Act.
5. What are the types of consideration?
   There are three types of consideration in valuation under GSTie.
   a) Consideration received full in money.
   b) Consideration received not in money.
   c) Consideration received partly in money.
6. What is input tax credit?
   Section 2(63) of CGST Act 2017 defines Input Tax credit. Input Tax Credit means the credit of input tax.In general input tax credit means, at the time of paying tax on output dealer can reduce the tax that he has already paid oninputs.
7. What is debit note?
   Debit notes are defined u/s 2(38) of the CGST Act:
   Debit notes can be raised in GST under two situations:
   When there is an amendment in the amount of taxablevalue of the goods after issuance of invoice
   When there is a change in the tax after issuance of invoice
8. What is tax invoice?
   An invoice is a commercial document issued by a seller to a buyer, relating to a sale transaction and indicating the products quantities, agreed prices for products, discounts if any or services the seller had provided to the buyer.
9. Mention the modes of payment in GST.
   The following are the modes of payment in GST model; Payment of taxes, interest, penalty, fees any other amount by internet banking through authorized banks.
   Over the counter payment
   Payment through NEFT or RTGS (Real-Time Gross Settlement) from any bank
   Payment through Debit card or Credit card.
10. What do you mean return in GST?
    Return is a statement of specified particulars, relating to business activity undertaken by the taxable person during a prescribed period. A return is a document containing details of income which a taxpayer is required to file with the tax administrative authorities.
11. Expand CIN, CPIN, and RTGS,
    CIN stands for Challan Identification Number.**RTGS**, Real-Time Gross Settlement
    CPIN stands for Common Portal Identification Number
12. What is refund under GST?
    GST Refund refers to process of claiming of net amount due to the taxpayer from the tax administration, when the GST paid is more than the actual GST liability.
13. Mention any two situations in which refund can beclaimed.

The liabilities of a taxpayer under GST are maintained in electronic liability register. In the electronic liability register the tax due on filing a GST return, interest, penalty and demands are maintained.

54. Mention the modes of payment in GST.
The following are the modes of payment in GST model; Payment of taxes, interest, penalty, fees any other amount by internet banking through authorized banks.
Over the counter payment
Payment through NEFT or RTGS (Real-Time Gross Settlement) from any bank
Payment through Debit card or Credit card.

55. State the features of GST payment process. The payment process under GST regions has the following
features:
Under GST regime, taxes, penalty, late fees are paid through GST portal.
The tax due can be paid either by cash or through utilization of Input Tax Credit.

56. What do you mean return in GST?
Return is a statement of specified particulars, relating to business activity undertaken by the taxable person during a prescribed period. A return is a document containing details of income which a taxpayer is required to file with the tax administrative authorities.

57. What are the types of return under GST?
GSTR-1 Details of outward supplies
GSTR-2 Details of Inward supplies.
GSTR-3 Monthly return
GSTR-4 Return for compounding taxable person.

58. What is CPIN?
CPIN stands for Common Portal Identification Number (CPIN) given at the time of generation of challan. It is a 14-digit unique number to identify the challan. The CPIN remains valid for a period of 15 days.

59. Expand CIN, BRN and E-FPB
CIN stands for Challan Identification Number. BRN stands for Bank Reference Number E-FPB stands for Electronic Focal Point Branch.

60. What are the major and minor heads in GST tax payment?
Major heads are IGST, CGST and SGST Minor heads include tax, interest, fees and penalty.

61. What is electronic credit ledger?
Electronic credit ledger is maintained in GST portal. All the approved claims for input tax credit are credited to the electronic credit ledger under the appropriate head of CGST. SGST or IGST, UTGST and GST cess.

62. What is refund under GST?
GST Refund refers to process of claiming of net amount due to the taxpayer from the tax administration, when the GST paid is more than the actual GST liability.

63. Mention any two situations in which refund can be claimed.

Export of goods or services
Supplies to SEZs units and developers
Deemed exports of unjust enrichment?

64. What is doctrine
The doctrine of unjust ...

Export of goods or services
Supplies to SEZs units and developers
Deemed exports of unjust enrichment?

14. What is Revenue Neutral Rate?

The Revenue Neutral Rate is a structure of different rates established in order to match the current revenue generation with revenue under new tax regime (GST).

15. Mention any two items of lower rate of tax

Ans: the main items covered under lower rate of tax are

- Condensed milk.
- Refined sugar and sugar cubes.
- Pasta.
- Diabetic food.
- Medicinal grade oxygen.

16. What is Zero rated supply?

According to section 16(1) IGST Act 2017, "zero rated supply" means any of the following supplies of goods or services or both, namely; export of goods or services or both; or Supply of goods or services or both to a Special Economic Zone developer or a Special Economic Zone unit.

17. What is exempt supply?

Exempt supply is defined in section 2(47) of GST Act.
"Exempt supply" means supply of any goods or services or both which attracts nil rate of tax or which may be wholly exempt from tax under section 11, or under section 6 of the Integrated Goods and Services Tax(IGST) Act, and includes non-taxable supply.

18. What is nil rated supply?

Goods or services on which GST rate of 0 % is applicable are called NIL rated goods or services. Example of Nil rated supplies is salt, jaggery, cereals, Live sheep and goats, Birds' eggs, Natural Honey etc.

19. What is non-taxable supply?

Non- taxable supply is the supply of goods or services or both on which GST is not leviable. Non Taxable supplies are those goods or services kept out of the purview of GST.

20. What is abatement?

Abatement means reduction, exemption and concessions in taxable value of supply.

21. What is Harmonized System of Nomenclature (HSN)?

It is a six digit uniform code that classifies products and is accepted worldwide. These set of defined rules is used for taxation purposes in identifying the rate of tax applicable to a product in a country.

22. What do you mean by Services Accounting Codes (SAC) codes)?

Services Accounting Codes (SAC codes) are codes issued by CBEC to uniformly classify each service under GST. Each service has a unique SAC

23. What is Integrated GST?

b) If there is no supply of goods or services in return for the payment.

42. What is input tax credit?
Section 2(63) of CGST Act 2017 defines Input Tax credit. Input Tax Credit means the credit of input tax.In general input tax credit means, at the time of paying tax on output dealer can reduce the tax that he has already paid on inputs.

43. What is input?
Input means any goods other than capital goods used or intend to be used by a supplier in the course or furtherance of business.

44. Define capital goods.
Capital goods means goods, the value of which is capitalized in the books of account of the person claiming the input tax credit and which are used or intended to be used in the course or furtherance of business.

45. What is taxable supply?
Taxable supply means a supply of goods or services or both which is leviable to tax under GST.

46. Mention the documents required to claim ITC.
aAn invoice issued by the supplier of goods or services or both.
b. Invoice issued by recipient if receiving goods or services from unregistered supplier along with proof of payment of tax (in case of reverse charge).
c. A debit note issued by a supplier

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Debit notes can be raised in GST under two situations:
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Credit notes can be issued in GST under the following
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- When the goods supplied are of inferior quality, and the same are returned to the supplier.

49. What is supplementary tax invoice?
Supplementary tax invoice is a type of invoice that is issued by a taxable person in case where any deficiency is found in a tax invoice already issued by a taxable person.

50. What is tax invoice?
An invoice is a commercial document issued by a seller to a buyer, relating to a sale transaction and indicating the products quantities, agreed prices for products, discounts if any or services the seller had provided to the buyer.

51. What is bill of supply?
Bill of supply is a document to be issued by a registered person supplying exempted goods or services or both or paying tax under the provisions of section 10 (composition scheme) instead of a tax invoice.

Integrated Goods and Services Tax (IGST): IGST refers to tax charged under the IGST Act 2017 on inter-state supply of goods or services or both other than supply of alcoholic liquor for human consumption. The rate of IGST cannot exceed 40%.

24. What is value of supply?

Value of supply in simple terms, the amount paid by the recipient of supply to the supplier as consideration for supply.

25. What is transaction value?

"Transaction value "means the price actually paid or payable for the goods when sold and includes in addition to the amount charged as price any amount that the buyer is liable to pay to or on behalf of the assessee by reason of or in connection with the sale whether payable at the time of sale or at any other time.

26. Who is related person?

A person who is under influence of another person is called as related person like member of the same family or subsidiaries of a group of company etc.

27. 4Who is distinct person?

Section 25(4) of CGST Act 2017 provides that, A person who has obtained or is required to obtain more than one registration, whether in one state or union territory or more than one state or union territory shall in respect of each such registration, be treated as distinct persons for the purpose of this Act.

28. What are the types of consideration?

There are three types of consideration in valuation under GST.

They are,

a) Consideration received full in money.
b) Consideration received not in money.
c) Consideration received partly in money.

29. What is open market value?

Open market value of a supply of goods or services or both means the full value in money, excluding the integrated tax, central tax, State tax, Union territory tax and the cess payable by a person in a transaction, where the supplier and the recipient of the supply are not related and price is the sole consideration.

Open market value is a fair market value. In other words open market value will be the amount which is fairly available in open market.

30. State any two inclusions to transaction value.

a) Taxes, duties, cesses, fees and charges except CGST, SGST, UTGST and GST compensation cess:
b) Amount incurred by recipient on behalf of the supplier

31. How is GST levied on imported goods?

In this case GST rate will be levied on the sum total custom value of imported goods and import duty paid thereon

32. Who is recipient of goods or services?

Recipient is one who is liable to pay consideration to the supplier for the supplies or services received by him.

They are,

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33. Who is supplier?

Supplier in relation to goods or services or both, shall mean .the person supplying the said goods or services or both, and

Shall include an agent acting as such on behalf of such supplier in relation to the goods or services or both supplied.

34. When general valuation rules are applicable?

General valuation rules can be applied in cases where, price is sole consideration. That means, this rule is applicable in cases where supply of goods or services or both will be made against consideration wholly in terms of money and money only.

35. When special valuation rules are applicable?

Special valuation rules can be applied in cases where, the price is not sole consideration or consideration received not full in money or supply between related or distinct persons.

36. What do you mean by supply of like kind and quality?

Supply of like kind & quality means any other supply made under similar circumstances, is same or closely resembles in respect of characteristics, quality, quantity, functionality, reputation to the supply being valued.

37. Who is pure agent in GST?

Pure Agent means a person who enters into a contractual agreement with the recipient of supply to act on their behalf and incur expenditure or costs in the course of supply of goods or services or both.

38. How do you treat discount in GST?

Discounts given before or at the time of supply will be allowed as deduction from transaction value. Discounts given after supply will be allowed only if certain conditions are satisfied.

39. How do you treat cost of durable and returnable package while computing transaction value of supply?

Cost of durable and returnable ...

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39. How do you treat cost of durable and returnable package while computing transaction value of supply?
Cost of durable and returnable packing if it is included in the selling price, shall be excluded, as it is sent back to the supplier.

40. What is consideration?
Consideration is the concept of value offered and accepted by people or organizations entering into contract. Anything of value promised by one party to the other when making a contract can be treated as consideration.

41. State the payments which are not treated as consideration.
a) If there is no direct link between the payment and the supply
b) If there is no supply of goods or services in return for the payment.

42. What is input tax credit?
Section 2(63) of CGST Act 2017 defines Input Tax credit. Input Tax Credit means the credit of input tax.In general input tax credit means, at the time of paying tax on output dealer can reduce the tax that he has already paid on inputs.

43. What is input?

16. What is Zero rated supply?

According to section 16(1) IGST Act 2017, "zero rated supply" means any of the following supplies of goods or services or both, namely; export of goods or services or both; or Supply of goods or services or both to a Special Economic Zone developer or a Special Economic Zone unit.

17. What is exempt supply?

Exempt supply is defined in section 2(47) of GST Act.

"Exempt supply" means supply of any goods or services or both which attracts nil rate of tax or which may be wholly exempt from tax under section 11, or under section 6 of the Integrated Goods and Services Tax(IGST) Act, and includes non-taxable supply.

18. What is nil rated supply?

Goods or services on which GST rate of 0 % is applicable are called NIL rated goods or services. Example of Nil rated supplies is salt, jaggery, cereals, Live sheep and goats, Birds' eggs, Natural Honey etc.

19. What is non-taxable supply?

Non- taxable supply is the supply of goods or services or both on which GST is not leviable. Non Taxable supplies are those goods or services kept out of the purview of GST.

20. What is abatement?

Abatement means reduction, exemption and concessions in taxable value of supply.

21. What is Harmonized System of Nomenclature (HSN)?

It is a six digit uniform code that classifies products and is accepted worldwide. These set of defined rules is used for taxation purposes in identifying the rate of tax applicable to a product in a country.

22. What do you mean by Services Accounting Codes (SAC) codes)?

Services Accounting Codes (SAC codes) are codes issued by CBEC to uniformly classify each service under GST. Each service has a unique SAC

23. What is Integrated GST?

Integrated Goods and Services Tax (IGST): IGST refers to tax charged under the IGST Act 2017 on inter-state supply of goods or services or both other than supply of alcoholic liquor for human consumption. The rate of IGST cannot exceed 40%.

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26. Who is related person?

A person who is under influence of another person is called as related person like member of the same family or subsidiaries of a group of company etc.

Input means any goods other than capital goods used or intend to be used by a supplier in the course or furtherance of business.

44. Define capital goods.

Capital goods means goods, the value of which is capitalized in the books of account of the person claiming the input tax credit and which are used or intended to be used in the course or furtherance of business.

45. What is taxable supply?

Taxable supply means a supply of goods or services or both which is leviable to tax under GST.

46. Mention the documents required to claim ITC.

aAn invoice issued by the supplier of goods or services or both.

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Bill of supply is a document to be issued by a registered person supplying exempted goods or services or both or paying tax under the provisions of section 10 (composition scheme) instead of a tax invoice.

52. What are the ledgers maintained by taxpayer in GST?

The 3 ledgers are:

Electronic tax liability register

Electronic cash ledgers

Electronic credit ledgers

01. What are the documents required to claim refund ?

02. What are the features of payment Process?

03. Write a shart note on GSTR-1.

04. Mr.Bandepp is an air travel agent. From the following information, compute the value of supply of services and GST@ 18%.

Particulars	Basic fare
a) domestic travel	4,00,000
b) International travel	10,00,000

05.Kitur Ltd purchased goodS for 2,70,000 and Sold goods for Rs 3,00,000 within the Karnataka State. GST rate on purchase is at 12%. and On Sales is at 18%.
Compute input tax Input Credit and balance of GST Payable.

06.Umesh ltd.Belagavi purchased goods for Rs2,60,000 and sold goods for € 4.80,000. ArunUd, vijayapur. GST rate on purchase is @ 12% and on sales @ 18%.
Compute output tax, input tax credit and balance of GST payable.

01. Explain zero rated supply, nil rated supply and Nontaxable supply with examples.

02. Ganesh Ltd sold a product of Shree Ltd Kolhapur for Rs4,24,800 (including GST @ 18%)
The above selling price is not included the following.

a) Normal secondary packing cost Rs40,000
b) Cost of special packing Rs. 10,000
c) Durable and returnable packing cost Rs20,000
d) Freight Cost Rs10,000
e) Insurance Charges Rs. 30,000
f) Advertisement Rs50,000
g) Weightment charges Rs 1600
h) Free sample Rs 30,000
i) Loading Charges Rs12,000
j) Trade discount (normal practice)Rs 12,000

Compute transaction value and amount of GST Payable.

03. Ganesh Ltd sold a product of Shree Ltd Kolhapur for Rs5,60,000 (including GST @ 12%)
The above selling price is not included the following.

a) Normal secondary packing cost Rs3,000
b) Cost of special packing Rs. 2,000
c) Durable and returnable packing cost Rs7,000
d) Freight Cost Rs 1,500
e) Insurance Charges Rs. 2,500
f) Weightment charges Rs500
g) Trade discount (normal practice)Rs 8,000
j) Compute transaction value and amount of GST Payable.

04. Explain the different types of return

05. Explain the procedure and provisions for claiming return under GST

53. What is electronic liability register?

The liabilities of a taxpayer under GST are maintained in electronic liability register. In the electronic liability register the tax due on filing a GST return, interest, penalty and demands are maintained.

54. Mention the modes of payment in GST.

The following are the modes of payment in GST model; Payment of taxes, interest, penalty, fees any other amount by internet banking through authorized banks.

Over the counter payment

Payment through NEFT or RTGS (Real-Time Gross Settlement) from any bank

Payment through Debit card or Credit card.

55. State the features of GST payment process. The payment process under GST regions has the following

features:

Under GST regime, taxes, penalty, late fees are paid through GST portal.

The tax due can be paid either by cash or through utilization of Input Tax Credit.

56. What do you mean return in GST?

Return is a statement of specified particulars, relating to business activity undertaken by the taxable person during a prescribed period. A return is a document containing details of income which a taxpayer is required to file with the tax administrative authorities.

57. What are the types of return under GST?

GSTR-1 Details of outward supplies

GSTR-2 Details of Inward supplies.

GSTR-3 Monthly return

GSTR-4 Return for compounding taxable person.

58. What is CPIN?

CPIN stands for Common Portal Identification Number (CPIN) given at the time of generation of challan. It is a 14-digit unique number to identify the challan. The CPIN remains valid for a period of 15 days.

59. Expand CIN, BRN and E-FPB

CIN stands for Challan Identification Number. BRN stands for Bank Reference Number E-FPB stands for Electronic Focal Point Branch.

60. What are the major and minor heads in GST tax payment?

Major heads are IGST, CGST and SGST Minor heads include tax, interest, fees and penalty.

61. What is electronic credit ledger?

Electronic credit ledger is maintained in GST portal. All the approved claims for input tax credit are credited to the electronic credit ledger under the appropriate head of CGST. SGST or IGST, UTGST and GST cess.

62. What is refund under GST?

GST Refund refers to process of claiming of net amount due to the taxpayer from the tax administration, when the GST paid is more than the actual GST liability.

63. Mention any two situations in which refund can be claimed.

1. What is exempt supply?
Exempt supply is defined in section 2(47) of GST Act.
"Exempt supply" means supply of any goods or services or both which attracts nil rate of tax or which may be wholly exempt from tax under section 11, or under section 6 of the Integrated Goods and Services Tax(IGST) Act, and includes non-taxable supply.
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The following are the modes of payment in GST model; Payment of taxes, interest, penalty, fees any other amount by internet banking through authorized banks.
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Payment through NEFT or RTGS (Real-Time Gross Settlement) from any bank
Payment through Debit card or Credit card.
10. What do you mean return in GST?
Return is a statement of specified particulars, relating to business activity undertaken by the taxable person during a prescribed period. A return is a document containing details of income which a taxpayer is required to file with the tax administrative authorities.
11. Expand CIN, CPIN, and RTGS,
CIN stands for Challan Identification Number.**RTGS**, Real-Time Gross Settlement
CPIN stands for Common Portal Identification Number
12. What is refund under GST?
GST Refund refers to process of claiming of net amount due to the taxpayer from the tax administration, when the GST paid is more than the actual GST liability.
13. Mention any two situations in which refund can beclaimed.
Export of goods or services
Supplies to SEZs units and developers
Deemed exports of unjust enrichment?
14. What is Revenue Neutral Rate?
The Revenue Neutral Rate is a structure of different rates established in order to match the current revenue generation with revenue under new tax regime (GST).
15. Mention any two items of lower rate of tax

Export of goods or services
Supplies to SEZs units and developers
Deemed exports of unjust enrichment?

64. What is doctrine

The doctrine of unjust enrichment means no person can be unjustly enriched at expense of another person.

65. What is the time limit to claim refund?

Any person claiming the refund of GST tax should make an application within a period of 2 years from the relevant date.

66. What is the relevant date for excess payment of GST due to mistake?

The relevant date for excess payment of GST due to mistake is date of payment.

BGK GIBCN Year: 2016-17 onwards
class : B.com VI th Sem

INCOME TAX QUESTIONS

APRIL 1997.

01. Give two examples of capital receipts ?
Two examples are-a)Saleproceed of a machinery by a textile manufacturing Co.
b)Insurance claim received for damage caused to building.

02. What are long term capital assets ?
Shares held in a Co. ,any other listed securities and a unit of mutual fund specified u/s 10(23D). In case of these assets if they are held by an assesses for more than 12 months immediately preceding the date of their transfer is called as long term capital asset. for other capital assets held for more than 36 months is called long term-capital asset.

03. Who can claimdeduction u/s 80 E ?
This deduction is claimed by an individual who makes repayment of loan in the previous year taken by him from any financial institution or approved charitable institution for the purpose of pursing his higher education ,or interest on such loan should be made out of the taxable income of the assesses.

04. What is listed security ?
This is the security listed in the stock exchanges. Security means a guarantee in respect of payment of principal amount and interest. These are the securities listed in the stock exchange price list.

05. .Mention the maximum amount deductible u/s 80DD ?Who can claim it ?
The deductible amount is a sum of Rs75,000 ror normal disability and125000 for severedisability for any amount of expenditure incurred for the medical treatment of a handicapped dependent or for any amount paid or deposited in the approved scheme of LIC or UTI. It can be claimed by an assesses for medical treatment of handicapped dependent.

06. What is amount deduction available u/s 80U ?Who can claim it.
The amount of deduction available is Rs75,000 from the gross total income of the assesses. It can claimed by an individual resident ,who at the end of the previous year suffering from a permanent disability. For sever disability Rs. 125000

07. .How do you set off long term capital losses?
The amount of long term capital loss should be set off only against long term capital gain in the same assessment year. If not possible that loss can be carry forward for next 8 AY

08. What is indexed cost of acquisition?
Indexed cost of acquisition means an amount which bears to the cost of acquisition , the same proportion as the 'cost inflation index' for the year in which the asset is transferred bears to the 'cost inflation index' for the first year in which the asset was held by the assesses or for the year 2001-02 whichever is later.

$$\text{Indexed cost of} = \text{cost of acquisition} \times \frac{\text{cost inflation index for Current Previous year}}{\text{Cost inflation index of the year of acquisition}}$$

09. What is the maximum limit in respect of salary commission etc to working partners of a professional firm u/s 40(b) ?

The maximum limit in respect of salary , commission etc to working partners of a professional firm ,

1)on the first Rs3,00,000 of the book-profit or in case of a loss	-------- Rs1,50,000 or at 90% of the book-profit whichever is more.
2) on the balance of the book Profit.	------- at60% of the book profit.

10. .Mention the amount of deduction available to the assesses u/s 80 GG.
The deduction amount available to the assesses u/s 80GG is as under,
    1. actual rent paid by him in excess of 10% of his 'Total income' or,
    2. 25% of his 'Total income 'or,
    3. Rs 5000 per month
    whichever is less is allowable deduction

11. What is the deduction allowable u/s 80DDB ? Who can claim it ?
A deduction of a fixed amount of Rs 40,000 is allowed for that previous year. An assesses actually incurred any expenditure for the medical treatment of the specified disease or ailment in the previous year. for senior citizen maximum amount is Rs.60000

12. What are the provisions of sec 80D under income tax act.
The individual out of his taxable income makes payment, to the insurance premium on his own or his dependent health under an approved scheme known as 'med claim' is eligible for deduction . The amount of deduction would be the actual premium paid or Rs25,000 whichever is less.(for senior citizen Rs. 50,000)

13. What is book-profit ?
Book –profit means the net profit as shown in the profit and loss account in the manner as laid down u/s 28 to 44D of the act .The salary ,bonus payable to partners, if debit to profit and loss account should be added back to the net profit.

14. .Give 4 examples of capital receipts ?
The 4 examples are,
    i. Sale proceeds of a machinery by a textile manufacturing company.
    ii. Insurance claim received for damage caused to building
    iii. Compensation received by the bus driver taken over of the buses.
    iv. Sale proceeds of shares held by a person as an investment.

15. What is slump sale ?

"slump sale" means the transfer of one or more undertakings as a result of the sale for a lump sum consideration without values being assigned to the individual assets and liabilities in such sales.

16. What is casual income?
The casual income is a receipt which is of both the casual and non-recurring in nature. It is receipt which is unforeseen and unexpected. It is in the nature of windfall.
winning from lotteries betting race etc.

17. State any four deductions from GTI of an individual
    1. Deductions in respect of premium paid on Life Insurance policies and contribution to PF etc. 80C
    2. Deductions in respect of premium paid on specified pension fund of LIC – 80 CCC.
    3. Deductions in respect of premium paid on MediclaimInsurance Policy. Sec. 80 D
    4. Deductions in respect of maintenance and medical treatment of a dependent person with disability. Sec – 80 DD

18. State the index number for the year 2001-02 and 2019-20
The Cost index number for the year 2001-02 is 100 and for the year 2019-20 is 289

19. Give four examples of income taxable under the head other source
    1. Any fees, commission or remuneration received by employees from other than his employer.
    2. Directors fees
    3. Remuneration or fees received for examination work
    4. Interest on bank deposits and loans

20. What do you mean by security
A Word security means a guarantee in respect of payment of principal amount and interest. These securities can be issued by central or state or local authorities or a company or corporation established by state or central act.

21. What do you mean by Commercial security
These are 'issued by some company or corporation and the government securities are issued by the government. From tax point of view there is no difference between the two.

22. How do you treat a dividend received from co-operative society
It is taxable income under the head income from other source

23. Who is an individual?
An individual means a person, a human being either male or female, minor or a person of unsound mind.

24. State the types of capital gain
There are two types capital gain namely_
Short term Capital Gain
Long term Capital

25. What is the maximum limit for deduction u/s 80C
    The maximum limit for deduction u/s 80C is Rs.1,50,000

26. Who is a working partner?
    A partner who is taking active part in conducting the affairs of the business or profession is called working partner.Any salary, bonus or commission payable to working partner is allowed subject to the provision of section 40(b)
27. How do you treat a dividend received from a company?
    Dividend received may be from domestic company and/or foreign company
    Domestic Company: Dividend received from domestic company is exempted u/s 10 (15)
    Foreign Company: Dividend received from foreign company is taxable under the head other source.
28. write any two donations 100%of which is allowed as deduction u/s 80G
    a. Chief Minister's Earthquake Relief Fund, Maharashtra
    b. Andra Pradesh Chief Minister's Cyclone Relief Fund
    c. National Trust for Welfare of persons with Autism, Cerebral palsy, Mental Retardation and Multiple Disabilities
    d. National or state Blood Transfusion Council

29. State the rate of income tax on in case of winning from lottery and give the equation for its gross up
    The rate of income tax on in case of winning from lottery is 30% it is grossed up by the following equation.
    Gross winning from lottery = $Lottery\ amount\ received\ X\ \frac{100}{70}$

30. What is Short term capital gain?
    Short term capital asset are asset as follows,
    1)shares held in a company any other listed security ,a unit of UTI and unit of mutual fund specified u/s 10(23D) ,if there assets are held by the assesses for not more than 12 months immediately preceding the date of their transfer.
    2)other capital asset :- if they are held by an assess for not more than36 months immediately preceding the date of their transfer.

31. What do you mean by Set of loss?
    Set off means adjusting the losses of one source against the income under othe sources in the same assessment year.
32. Who is a non-working partner
    Partner who is not taking active part in conducting the affairs of business or profession of the firm is called non-working partner. Remuneration given to non-working partner is not allowed

Department of Commerce

2019-20 Batch

K.L.E. Society's

G. I. BAGEWADI ARTS, SCIENCE AND COMMERCE COLLEGE, NIPANI - 591 237.

Accredited at 'A' Level by NAAC with CGPA 3.35

College with Potential for Excellence (CPE)

EXAMINATION

Class : B.Com 1st sem **Subject** : Business Environment

Roll No. : **Date** :

Register No. : C1930645 **Test** : I ☐ II ☐

Marks Scored : ☐ Name : Komal Gadakari

Signature of Student **Signature of Valuer** **Signature of the Invigilator with Date**

Section - A

1.

a) Micro environment is specific & particular to each firm Conditions of micro environment factors differ from business to business

b) A modern business management is more than a concept, it's an integrated approach to everything an organization

c) It is a mix of capitalism & communism business under the system like business under complete ownership of govt. Business managed by govt & private sector also

d) Giving more opportunities & importance for business in private sector. Expanding the area of operations for business in private sector.

OR

The transfer of ownership, property or business from the government to the private sector is termed as privatisation

(P.T.O)

f) Economic policies gives direction to conduct business activities Promotion & regulation of business activity is monitored through measures like monetary, fiscal, Trade & industrial policy

h) a) i) CRR - Cash Reserve Ratio
ii) SLR - Statutory Liquidity Ratio

i) It is application of goods code of conduct it include truth & honesty, loyalty, values & principle that will excepted by the society

j) * Long term profit
* To provide competitive advantages

k) * Easy faster & effective communication in business
* less wastage
* More innovative approach

l) A systematic application of scientific or other organized knowledge to practical tasks.

e) Small scale industries are those industries in which the manufacturing, production & the service are run by the small scale company.

a) Political Environment is the state, govt, & its institutions & legislations & the public & private stakeholders who operate & interact with or influence the system

(P.T.O)

Sec - B

2) -: 5 features of Modern Business :-

* Large size in Modern Business houses are of large size. The study made by forbes says that 200 companies control 30% of world economic activities. There are firms whose revenue exeedes the GDP of certain nation.

* Market driven in Modern Business Organization are market driven & not government regulated. The universal liberalization process has eased Government restrictions. These business houses are operating as per the condition of the market.

* Technology Based in Business firms of 21st century are technology based. They are installing heavy & latest plant & machinery there is more stress on mechanization & computerization.

* Modern in Business is becoming more modern & new types of business are entering the market. leading business sector of 21st century are IT, BT. & other

* Diversification in Modern Business houses are entering into different areas of business. They are not restricting to particular type of business only.

(P.T.O)

4) Components of political environment in

1.) Political System :-

A. Democratic System ; This system of political governance is elected by the people. People elect govt. under this system.

B. Autocratic system : Government under this system is controlled by King or queen. They derive their power to rule the country & control the govt.

2) The constitution :-

Constitution of a country states rights & duties of the citizen including that of cooperative citizen. It defines powers, priviledges, rights & duties available to citizens including business organization.

3) Political Parties & their ideologies in

Political environment of a nation & its effects on business is influenced by political parties & their ideologies. Every political party has its own economic & social objectives.

4) Nature of Government in

Government decides the economic & business policies. The nature & composition of govt. has significant impact on business environment.

(P.T.O)

R.NO - 148

5) : 5 Favourable arguments of state intervention in business :

i) Planned Economic Development :-

State intervention in business ensures planned economic development. Economic policies of the government, which gives direction to business activities are framed keeping into requirements of the nation.

ii) To ensure growth & Development in

Economic growth is measured in terms of GDP i.e volume of goods and services produced in a year. Development is measured in terms of development in all sectors of economy.

iii) Development of Core - industry :-

The development of certain nation depends on core industries like steel, cement, power, transportation e.t.c. These industries requires heavy investments.

iv) Market failures :- Market is always Imperfect. Market may not understand the Sentiment & feeling of people. Govt. can regulate through fixing minimum price

v) Protection of Environment :-

Every business, particularly industrial activities exploit natural resources Unrestricted business activity leads to over exploitation of resources & it will create ecological imbalance.

(P.T.O)

Section-C

7) Introduction :- Macro Environment factor are large & affect every business The factors are universally applicable to every firm

* Macro - Environment types:

i) Economic Environment : Business is an economic activity. Economic factors has strong influence on the business. Economic variables like demand, supply, Price e.t.c determine functioning business.

* There are 2 types;

i) a) National Environment ; Economic environment of a country has significant influence on business environment. It was divided into 3 types.

Capitalist :-

* Government intervention in business is minimum.
* Restrictions & regulations are minimum
* Market determines the business condition
  Ex: USA, Europe.

Communist :-

* Controlled by government
* Role of market is minimum
* factors of production owned & control by Govt Ex- China.

Mixed economy :-

* Business which is under control or complete ownership of govt.
* Business which is control & managed by both Public & private.
  Ex :- India

B. Non - Economic Environment :-

Non economic factors have economic content similarly economic variable have social concern in them.

i) Political Environment :-

Business activities in a country are determined by political system. Political organization of a country is defined by its constitution.

ii) Legal Environment :- Activities of business are regulated by various laws. Provisions of these laws dictate the method procedure & formalities to be completed

iii) Sociological Factor :- "Keith Devis" 'Says Business is social mission & having a social mission & having a broad influence in the way people live'

iv) Demegraphic Environment :- Demographic factor of business environment pertain to population. Size & trend of population growth rate, age, cast, life style, culture e.t.c.

v) Technological environment :- Development in the field of science & technology will increase competitiveness of business. It helps to innovate new product

vi) Physical Factor :- Survival of business depends on physical resources it receives from nature like raw material, water e.t.c

(P.T.O)

vii) Cultural environment :- Culture can be defined as living styles & habits of people. Rituals, customs, traditions, festivals, family life, dress code, food habits e.t.c.

viii) Sectoral Environment :-
Firms which are doing similar business belong to a particular sector or industry.
Ex: Transport industry

8) Economic Condition in
Introduction :- Growth of a business depends on prevailing or supportive economic conditions. Economic conditions should be helpful for the growth of business.

i) Agriculture :- It is the basic industry. It gives food to the people & also raw materials to other industries. Growth of nation depends on agriculture as it feeds the people & also industry

ii) Infrastructure :- Infrastructure facilities like good roads, transportation system, communication facilities, powers e.t.c. are essential for the growth of business.

iii) Population :- population & it's composition gives support business. Growing population with more young & educated people will provide large customers

(P.T.O)

or can be

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Date: / /
Page No.:

4. Income level :- Survival & growth of business is depends on customer support. People can become customers, if they have adequate income. If the income level is low it will help for the growth of business.

5. Saving & investment :- Business requires capital, capital can be raised either from banks or directly from investors through equity shares or debentures.

6. Distribution of income :- There should be equitable distribution of income. Distribution of income levels i.e small portion of population being rich & large majority poor will not help the growth of business.

7. Economic Reforms :- Development of business requires helpful business environment. Economic reforms measures like LPG i.e liberlization Privatization & Globalization.

8. Standard of living :- Standard of living is measured interms of consumption of confirms luxuries & quality life. People must have the income to buy the goods & services.

9. Financial System :- Growth of business depends on availability of finance at favourable terms & conditions. Business also depends on various financial services like payments, cheque e.t.c

(P.T.O)

Date : / /
Page No.:

10) Level of economic development :-
Developed economies provide better opportunities of business growth by well developed market easy availability of finance, better infrastructure & people who spend on variety of goods & services

Section - D

11)
a) List of Business ethics like-
* Honesty
* Integrity
* Promise keeping & Truthworthiness
* Loyalty
* Fairness
* Concern for others
* Respect for others
* Law abiding
* Commitment to Excellence
* Leadership
* Accountability

b) List of social responsibility :-
* Having a moral compass
* Being a part of your faith community
* Growing your own garden
* Towards investors like safety & protection of investment
* Towards employee to pay better wages & incentives
* Providing healthy working condition
* Business should deliver goods at resonable price
* Business should regularly pay taxes that are due
* Provide employment to local people

Date: / /
Page No.:

c) * Enhance business reputation
* Positive work environment
* Improves customer happiness
* Retain Good Employees
* Builds investor loyalty
* Avoid legal problems
* These are the benefits of business ethics.

Thank You.

2019-20 Batch

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College with Potential for Excellence (CPE)

EXAMINATION

Class : B.Com

Subject : Financial Accounting-I

Roll No. :

Date :

Register No. : C1930622

Test : I ☐ II ☐

Marks Scored : ☐

Name : Asmita Mengare

Signature of Student

Signature of Valuer

Signature of the Invigilator with Date

Section - A

1.

a) The features of singal entry below:-

1) It is an unsystematic method of recording transaction.

2) under this system only one cash book is maintained which mixes up both the private and business transaction.

b) The Debtors account is prepared to ascertain the amount of credit sales or closing balance of Debtors or cash collected Debtors or amount of B/R Received.

c) The main merits of single entry system are:

1) It is easy and simple to understand.

2) It is not a costly system

3) There is no delay in recording the transaction.

d) Lawyer is qualified independent person who render legal service to his client for fees.

He is a law graduate and registered his na-
-me with BAR Council.

e) Work in progress is the fees earned in respect of incomplete maters of a particular accounting year for which no bills have been rendered and are not brought into account.

f) The different kinds of farm transaction are:-
1) Cash transaction
2) Credit transaction.
3) Exchange transaction
4) Notional transaction

g) Two features of farm accounting:-
1) The farming is a type of Family Business.
2) The family members put their on labour along with [illegible] ...
3) The [illegible] may made in Cash or [illegible]

J) i) Excess [illegible] over is called
ii) Short [illegible] calculated on the basis [illegible] and royalty

Short [illegible] Royalty.

K) The [illegible] on the basis of an [illegible] landlord and tenent.
The [illegible] stated in any One [illegible]

I'

4) Lessee is the Person who uses the asset of the creator or the owner in lieu of a Consideration for using such an asset.

Ex:- Mining company : manufactionurers.

Section - B.

2)

objectives of maintaining the books of accounts by a Profession is below:

1. To record all financial transaction of Profession.

2. To find out surplus or deficit of Profession

3. To find out the currect Profit or loss of the Profession by making the necessary adjustment to a Propotionate Common Expenses and house hold transaction.

4. To ascertain the share of Profit, drowings and capital in case of Partnership Firm.

5. To find out the currect financial Position that is the value of assets and liabilities or the Profession of a Partnership Particular Period -

6. To maintain the Books of Account as a

Specified under the Income tax Act rules.

3)

Debtors Account

Dr. Cr.

Particular	₹	Particular	₹
To opening Debtor	5000	By Cash / Bank *	20000
To credit sales	30000	By Bills Receivable	10000
To Bills Receivable [Dishonoured]	2000	By Bad Debt	500
		By Discount allowed	1500
		By Sales Return	1000
		By closing Debtor	4000
	37000		37000

credit sales = Total sales − Cash sales = 35000 − 5000 = 30000

5)

Crop Account

Dr. Cr.

Particular		₹	Particular	₹
To opening stock		150000	By wages in kind.	20000
Wheat	100000			
Seeds	20000			
fertilizers	30000		By sales (wheat)	800000
To Purchase		30000	By wheat Consumed by Propritor	5000
Seeds	12000			
fertilizers	18000			
			By wheat distroyed by Rodents	8000
To wages		200000		
in cash	180000			
in kind	20000			

Dr				Cr
To Depreciation on Farm equipment	20000			
To General P&L A/c [Profit transferred]	433000			
	833000			833000

6)

Royalty chart.

Year	Output	Rate	Royalty	Min-Rent	Excess	S.W	S.W.R	S.W.I.R	Payment
2013	1600	2	3200	8000	—	4800	—	-	8000
2014	2000	2	4000	8000	—	4000	—	—	8000
2015	7000	2	14000	8000	6000	-	6000	2800	8000
2016	12000	2	24000	8000	16000	-	—	—	24000
2017	16000	2	32000	8000	24000	-	—	—	32000

P.T.O

Section - C

9) In the Books of Green farms

Crop Account

Dr. | | | | Cr.

Particular	₹	Particular	₹
To Opening Stock (1.1.10)	17000	By Sales	
Fertiliser 1250		Paddy	53000
Seeds 750		By Drawing	
Paddy 15000		Paddy	2375
To Purchase	5450	By closing stock	9050
Fertilizer 4000		Paddy ~~7500~~	
Seeds 1450		Fertilizer 875	
To crop exp	12600	Seeds 675	
To Gross Profit * [Transferd to P&L]	29375		
	64425		64425

livestock Account

Dr. | | | | Cr.

Particular	₹	Particular	₹
To Opening stock	40250	By Sales	
livestock 37500		livestock	12000
Cattle feed 2750		Milk	39250
		By Drawing	
To Purchase	22500	Milk	1500
livestock 7500		By closing stock	
Cattle Feed 15000			

To live stock EXP	3125	livestock	33000	
To Gross Profit *	21375	Cattle Feed	1500	34500
[Transferd to P&L A/c]				
	87250			87250

Profit and loss A/c
For the year ending 31.3.2018

DR | | | CR

Particular	₹	Particular	₹
To Repair & maint-ance	3000	By Gross Profit	
		Crop	29375
To Deprecation on Farm equipment @10%	3750	livestock	21375
To Genral EXP	2925		
to Depreciation on land and Building 2%	1375		
To Net Profit * [transferd to B/S]	39700		
	50750		50750

Balancesheet
as on 31.3.2018

DR | | | CR

Particular	₹	Particular	₹
Capital 1,10450		Farm equipment 37500	33750
Add NetProfit 39700		Less Dep. @10% 3750	
150150		[37500 × 10/100]	

less - Drawing 3875 Paddy 2375 Milk 1500	148275	Land & Building 68750 less Depreciation 1375 @ 2% [68750×2/100]		67375
Sundry creditor	6500	Sundry Debtors		5250
o/s expenness	1100	Cash in hand		1825
		Cash at Bank		2125
		Clossing stock		
		livestock		33000
		Paddy		7500
		Cattlefeed		1500
		Fertilizer		875
		Seeds		675
	153875			153875

10)

In the Books of Mr. Ishwar

Total Debtors A/c

DE | | | CE

Particular	₹	Particular	₹
To Balance b/d	20000	By Cash A/c	60400
To Credit sales	91000	By Discount allowed	1000
		By Return inword	1200
		By Bills Recivable	1400
		By Bader debt	1000
		By Balance c/d *	46000
	111000		111000

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EXAMINATION

Class : ______________ Subject : ______________

Roll No. : 120 Date : ______________

Register No. : C2032120 Test : I ☐ II ☐

Marks Scored : ☐

Signature of Student Signature of Valuer Signature of the Invigilator with Date

Bills Recivable Account

DE CE

Particular	₹	Particular	₹
To Balance b/d	1000	By Cash A/C	600
To Debtors A/c	1400	By Balance c/d *	1800
	2400		2400

Creditors Account

DE CE

Particular	₹	Particular	₹
To Discount earned	2000	By Balance b/d	36000
To Cash A/C	35600	By Credit Purchase	40000
To Bills Payable	1000		
To Balance c/d *	33400		
	76000		76000

Bills Payable Account

DE CE

Particular	₹	Particular	₹
To Cash	2000	By Balance b/d	3600
To Balance c/d *	5600	By creditors	4000
	7600		7600

Trading and Profit and loss A/c
For the year ending 31.12.17

Dr. Particular	₹	Particular	₹ Cr.
Opening stock	30000	By Sales	109800
To Purchase	43000	Cash 20000	
cash 4000		credit 91000	
credit 40000		less Return 1200	
less - Return 1000			
To wages	10000	By Closing stock	41000
To carrage inword	3000		
To Duty & octori	1000		
To Gross Profit *			
	150800		150800
To office exp	3000	By Gross Profit	63800
To salory	2400	By Discount	2000
To Printing	1600		
To Bad Debts	1000		
To Discount	1000		
To R.D.D.	2300		
To Depericiation on furniture	400		
To Dep. on Building	1600		
To Dep. on Land Plant	4000		
To Net Profit *	48500		
	65800		6580

Balance sheet
For the Year Ending 31.12.2017

Liability		₹	Asset		₹
Capital	89400		Debtors	46000	43700
Add Net Profit	48500		less R.D.D. 5%	2300	
	137900		Plant	40000	36000
less Drawing	16000	121900	less Depriciation	4000	
			@10% [40000 × 10/100]		
Creditors		33400	Furniture	4000	3600
Bills Payable		5600	less Depriciation	400	
			@10% [4000 × 10/100]		
			Building	32000	30400
			less Depriciation	1600	
			@ 5% [32000 × 5/100]		
			Cash		4400
			Stock		41000
			Bills Recivable		1800
		160900			160900

Assignments 2019-20

Advance learners V Sem

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EXAMINATION

Class : Department of Botany | Subject : Botany P-II

Roll No. : [redacted] 149 | Date :

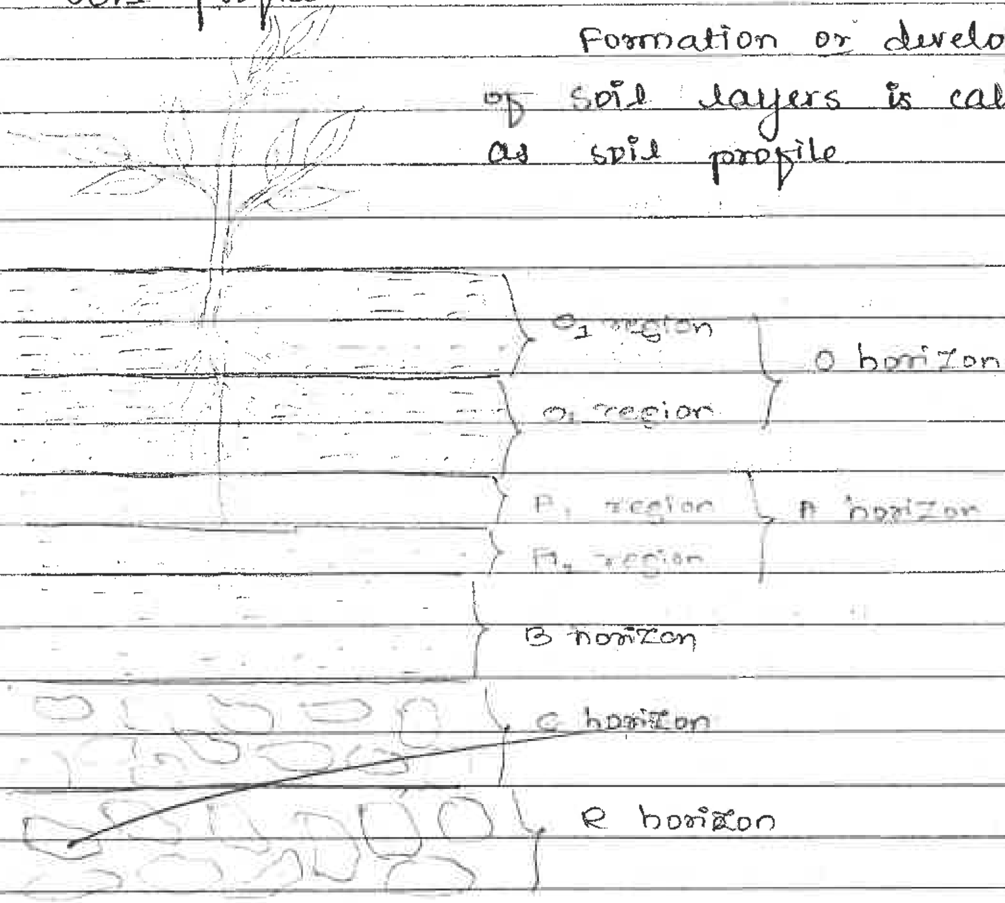
Marks Scored : | Test :

Reg NO - S1919424

Signature of Valuer | Signature of the Invigilator with date

1) Explain soil profile.

Formation or development of soil layers is called as soil profile



During pedogenesis formation of soil takes. The fully developed or development of soil with organic matter is called as soil profile.

The development of soil profile includes certain following steps. they are

1) O horizon
2) A horizon
3) B horizon

4) C horizon
5) R horizon.

1) O horizon – It is the first layer of the soil profile formation it is formed by leaves, branches, flower, dead organic matter etc. In has another two parts or region
i) O_1 region
ii) O_2 region

i) O_1 region – It is the 1st region of the O horizon which includes dead organic or animal matter, leaves, flower, branches etc. which is mainly seen in forest area.

ii) O_2 region – It is the 2nd region of the O horizon. In which the dead organic matter & other substances are decomposed by micro-organisms, bacteria, fungi etc.

2) A horizon – It is second layer of the soil profile. It is consist of 2 regions they are
i) A_1 region
ii) A_2 region.

i) A_1 region – It is first region of the A horizon. Which dark region. It consist of dark dead organic matter which is of dark black or brown in color is called as humus. It is rich with minerals & nutrients.
ii) A_2 region – It is second region of the A horizon. It is some what light in color. It is not so much of rich with nutrients & minerals.

3) B horizon – It is the third layer of the soil profile. It found after the A_2 region.

It is of some what found with dark soil particles.

4) C horizon – It is 4^{th} most layer of the soil profile. It is found after the B horizon. It consist of rocky beds. means fully with rocks

5) R horizon – It is final layer or last layer of the soil profile. It is found after the C horizon It consist of parent rocks with collected water.

(5) Explain morphological adaptations in xerophytes.

→ Habit – These are fully grown in desert areas without water supply.

→ Root – In xerophytes the roots are very thick, deep. It consist of root hair & root cap. these are hard & strong towards light.

→ Stem – In xerophytic plant the stem is very hard, thick & strong cuticle development.
- Bark is very thick
- water collected in Epider.
- It consist of chlorenchyma & sclerenchyma tissue.
- xerophytic plants have no so much of leaves.
- The stomata opens at night
- They avoid the high transpiration rate.

→ Leaves – In xerophytic plant the leaves are some what long shaped, they are very thick & hard. they stored H_2O without transpiration.
- The spines are modified into stipules
- The leaves consist of watery fluid

eg- opuntia.

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EXAMINATION

Class : BSc Vth Sem   Subject : Botany

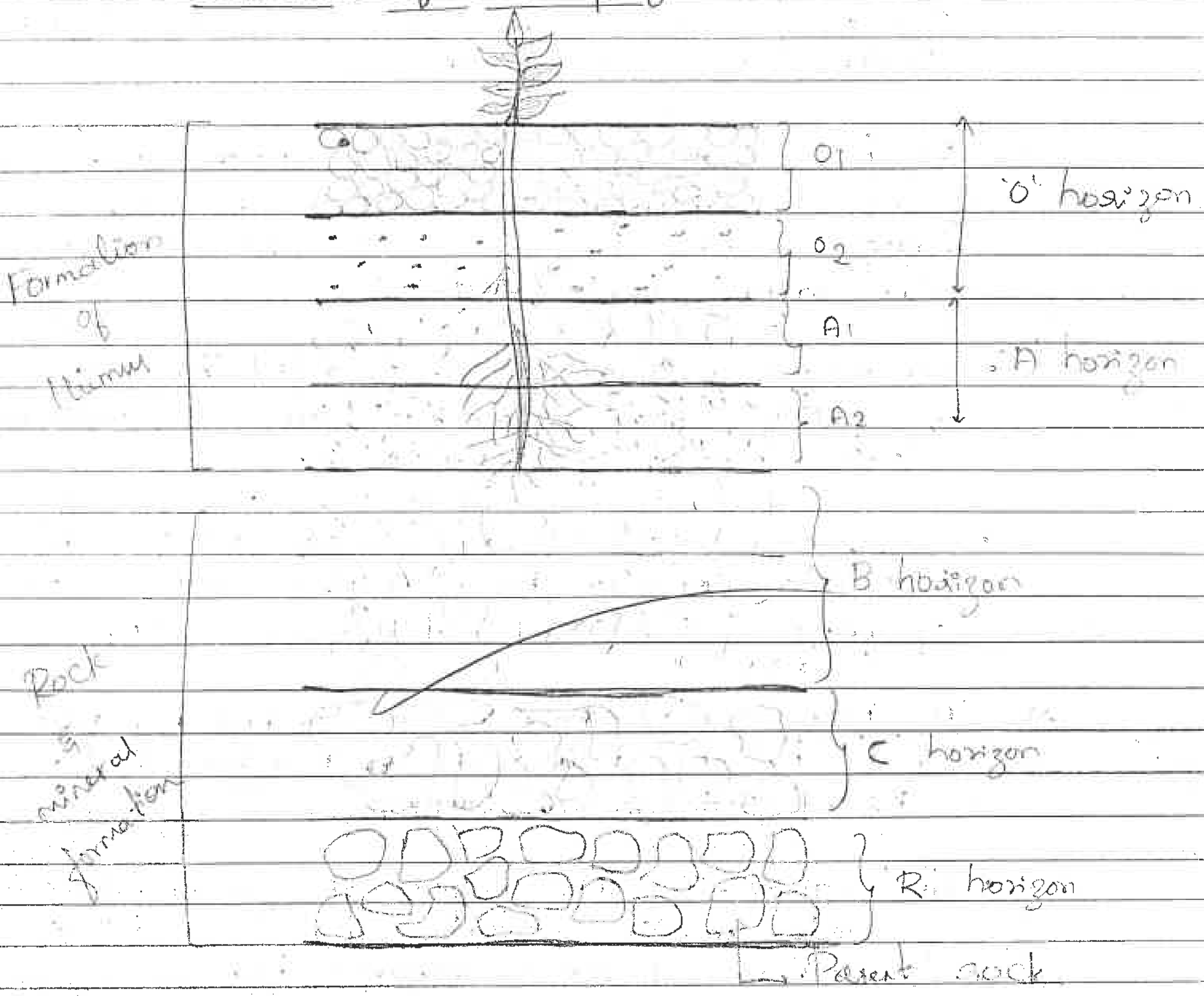
Roll No. : 122   Date :

Marks Scored :   Test :

Reg- NO - S1919487

Signature of Valuer   Signature of the Invigilator with date

1) Structure of soil profile.



Due to the interaction of various factors in the environment influences / results in the formation of soil or the process called pedogenesis. There are diff types of soil. The factors that influences the soil formation are climatic, topographic & biological factors.

Soil profile - It is defined as "the arrangement of soil & soil constituents with mineral matter in the different layers is called soil profile." Soil profile constitute the 5 main different layers / zones. They are.

1) 'O' horizon - It is the first uppermost horizon of the soil profile. It constitute the various matter on the surface & is divided into 2 sub layers / zones / regions.

- O_1 region - O_1 region is marked by the presence of fallen leaves, flowers, fruits & various dead matter of organisms.

- O_2 region - It is the region present just below the O_1 region. In this region there is presence of fungi, Bacteria & viruses is seen.

2) 'A' horizon - 'A' horizon is the mineral horizon & is present below the O horizon & is divided into 2 sub layers.

- A1 region - In this region, the decomposition of dead matter occurs. & various minerals are present & dark coloured humus is formed with rich mineral & salts.

- A2 region - This region is light in colour followed by the A1 region & less amount of

Salts & minerals are observed.

3) 'B' horizon - It is the horizon present below the 'A' horizon. It is seen with such mineral & salts like manganese, Calcium, Potassium etc.

4) 'C' horizon - This horizon followed by the the 'B' horizon. In this incompletely weathered rocks are present.

5) 'R' horizon - 'R' horizon is present below the 'C' horizon. In this completely weathered rock bed is seen upon which collected water is there.

2) Morphological adaptation in Xerophytes.

Xerophytes adapt morphologically & anatomically to dry conditions where less availability of water is there. They adopt morphologically as follows.

- → Root adaptations — Roots of the Xerophytes long, extensive, some times longer than the shoots. & present upto the deeper layers.
  → Tap root system is observed. ~~Spr~~ Roots spread into wide areas
  → Root are present with well developed root hairs & root caps. at the root apex.

- Stem → Stem in Xerophytes are stunted, woody, bark is present with thick layer.
  → Bark is dark & covered with oils & resins.
  → Eg — Opuntia - Stem is fleshy, green, leaf like covered with spines.

→ There is presence of wax coat.

→

Adaptation in leaf

→ leaf is reduced, scale-like, with coating of wax.

→ leaves are rolled & reduce the transpiration.

→ leaves are always thickened with storage of water

→ Stomata are sunken to reduce the transpiration

→ Water is stored in the leaf in the Xerophytes.

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EXAMINATION

Class : Bsc V sem

Subject : Botany.

Roll No. : 156

Date :

Marks Scored :

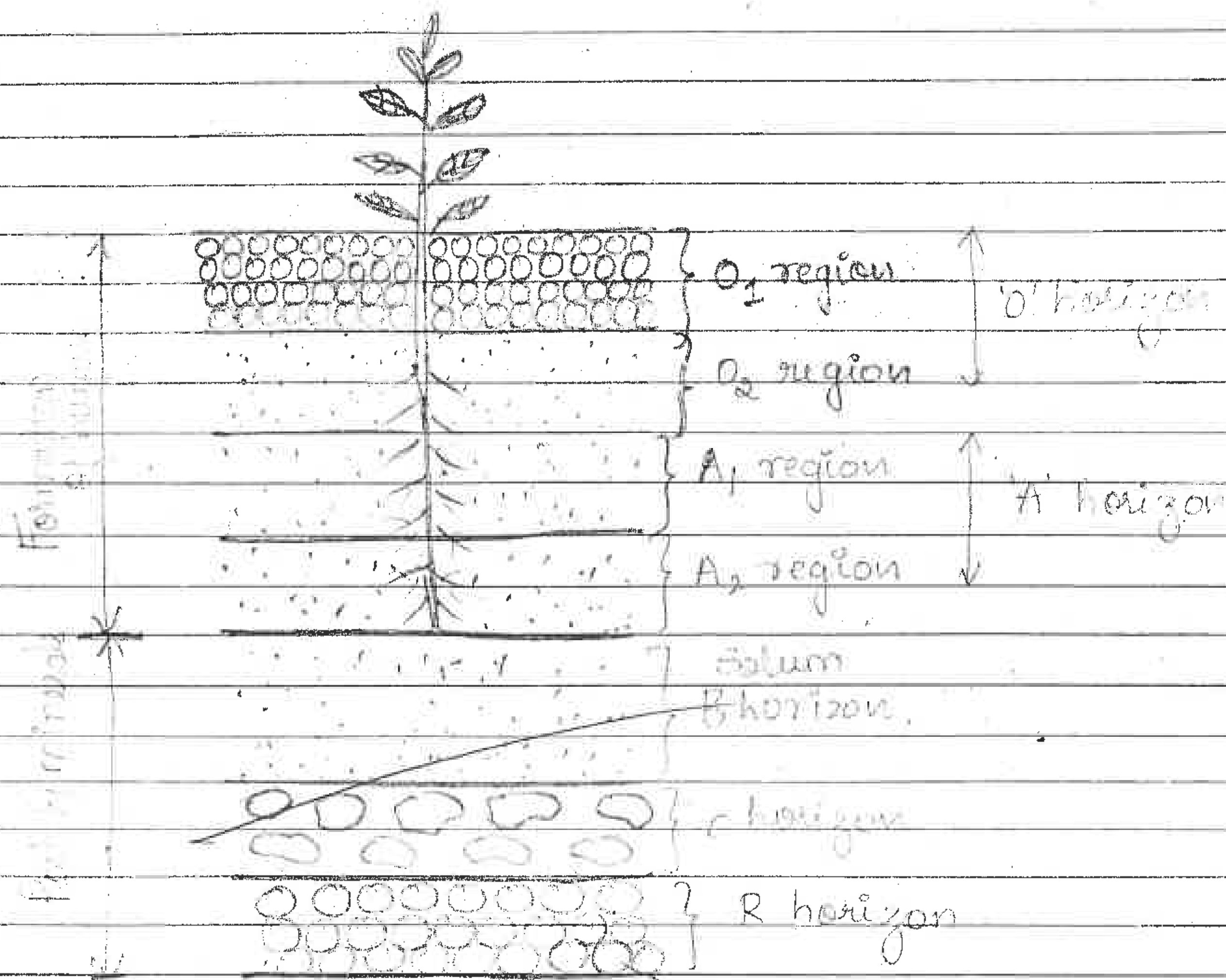
Test :

Reg NO - S1919507

Signature of Valuer

Signature of the Invigilator with date

① Explain the Structure of Soil profile.



Due to the interaction of Various factors influences or resulted in the soil formation or Paedeogenesis. There are Various types of soil. The factors that influences are climatic, topographic & Biological factors that tells the which kind of soil present in an particular area.

Soil profile : The soil formed are seen in the form of no. of layers called as soil profile.

The soil profile termed from the Russian technology called as ABC technology but in general the soil profile is of 5 main horizons.

(1) 'O' horizon : The 'O' horizon is the uppermost layer of soil profile & 'O' horizon is divided into 2 types.

(*) O_1 region → Here, In the O_1 region layer constitute the presence of leaves, dust, flowers, fruits etc. i.e of Organic matter.

(*) O_2 region → followed by O_1 region there is a O_2 region where presence of fungi, Bacteria microorganisms are observed.

(2) 'A' horizon : It the second layer of soil profile & types are as follows.

(*) A_1 region → A_1 region has dark colour with abundant no. of salt & minerals. Usually black or brown in colour & called humic region.

(*) A_2 region → The soil in this region is of light colour & little amount of minerals are seen.

(3) B horizon → 3rd horizon of soil profile & rich in minerals such as calcium, Iron, Magnesium, Magnese etc & clay.

(4) C horizon → 4th layer of soil profile & has Incompletly weathered soil.

(5) 'R' horizon: The lowermost layer of soil profile. There is a present of parent rocks where water is present. i.e Underground water is available.

(2) Morphological adaptations in Xerophytes.

→ The morphological adaptation in Xerophytes are as follows.

(1) Adaptations in root: The roots of Xerophytes are longer than the shoots, slender, sharper extend into the deeper soil for absorption of water & minerals.

* Roots spread over wide area.
* Roots are usually tap roots.
* Root hairs & Root caps are present.

(2) Adaptations in Stem: Stems are usually stunted, woody, dry

* Covered with dark bark.
* Example is Opuntia becomes fleshy, green, leaf like covered with spines.
* There is a presence of wax coat on the stem so to reduce transpiration.

(3) Adaptations in leaf: leaf has waxy coating, & reduced in size & fleshy

* leaves always thick, fleshy & rolled to reduce transpiration.
* Sunken Stomata are present.
* Water is stored in the leaf of Xerophytes

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EXAMINATION

Class : BSc V th sem

Subject : Botany

Roll No. : 132

Date :

Marks Scored :

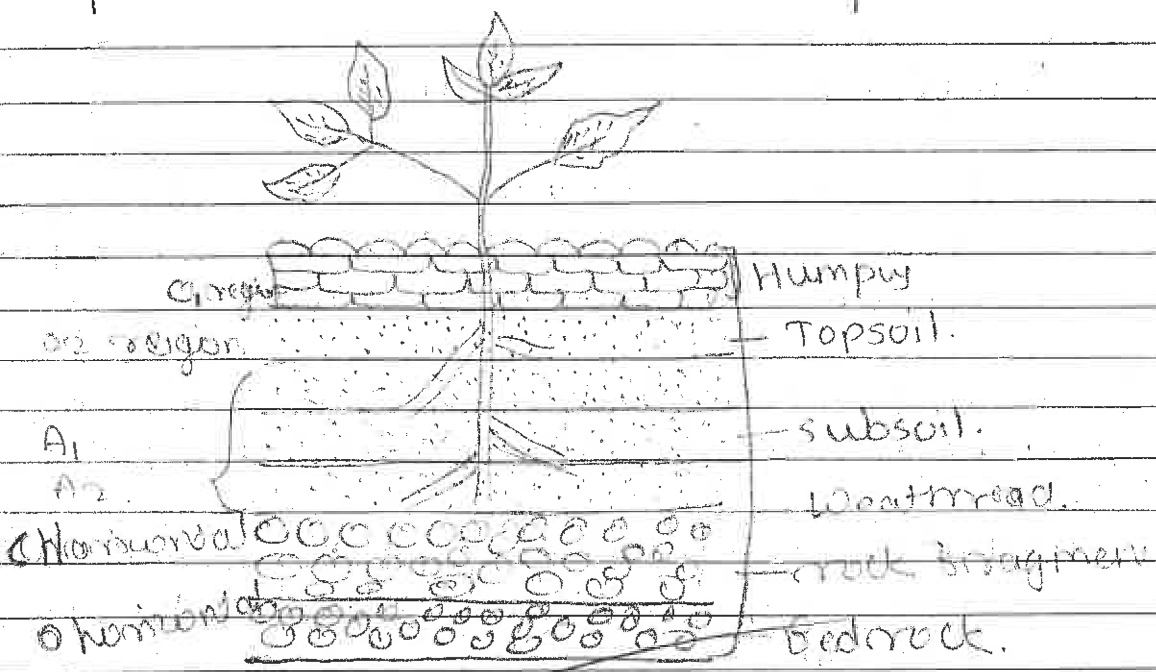
Test :

Reg. No - S1919553

Signature of Valuer

Signature of the Invigilator with date

1) Explain the structure of soil profile.



Thus during Pedogenesis Hetro added organism of organic compounds dead organic matured matter. leaving of mineralization of dead organic matter here minerals gradually added to different layer of developing soil. The soil. The soil when fully developed can be seen leaving a no of layers of horm of a soil is known as soil profile.

* Soil Profile :- due to the interaction intersity of various factors influsing the process of pedogenesis here developement of variety of soil type. It depends upon the nature of Parent matter and other factors. i.e Biological factors which type of soil will developed in Area.

The different horizontal of soil Profile Historically in Russian Terminology, ~~However~~ where classified into ABC terminology. However according to the present any ingeneral are the various conclusion. The soil profile Perfect ~~in~~ 5 various Types

3) Components of ecosystem.

Biotic components are the living things that shape an ecosystem for example Biotic compound and abiotic compounds.

1] Biotic compounds:- This are the compound include plant, Animal, fungi and Bacteria.

2] Abiotic compounds:- This are the compound Non living compounds. That influence an ecosystem.

example:- abiotic factor are temperature presence, air, current and minerals.

An ecosystem is a community of living organisms in conjuction with the non living components of their environment interacting as a system. These biotic and abiotic component linked together. Through Nutrient cycles and energy flows.

K.L.E. Society's

G. I. BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPANI - 591 237.

(Re-accredited at 'A' Level by NAAC with CGPA 3.35)

College with Potential for Excellence

EXAMINATION

Class : Bsc VIb | Subject : Botany

Roll No. : 124 | Date :

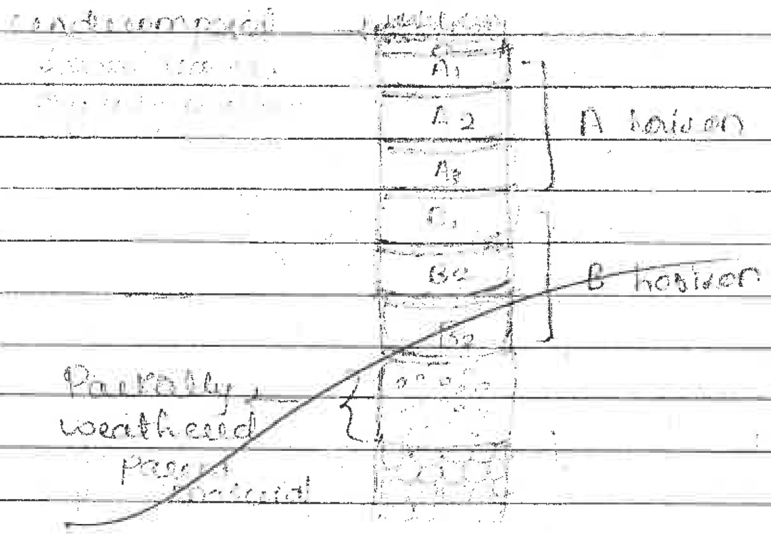
Marks Scored : | Test :

Reg NO - S1919575

Signature of Valuer | Signature of the Invigilator with date

① Explain the structure of soil profile.

Soil profile may be defined as verticle section through a soil. It represent the sequence of horizons or layers differen-tiated from one another but genetically related and included to the parent material from which are soil profile is developed.



O-horizon - Dominated by organic matter, leaf & stem litter.

→ Present in dense forests & in isolated patches elsewhere.

A-horizon :- zone of accumulation of organic matter and nutrients (most roots occur here)

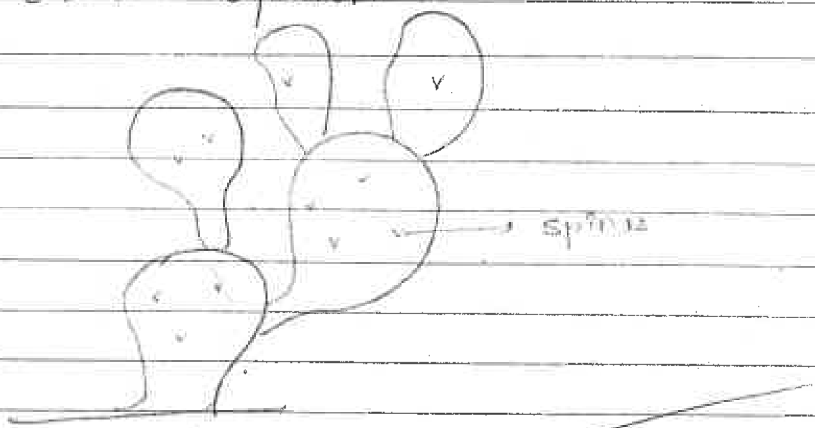
B- horizon- zone of illuviation. Here accumulation of clay takes place.

C- horizon - Here parent material & rocks are present.

② Explain the xerophytic adaptation in plants.

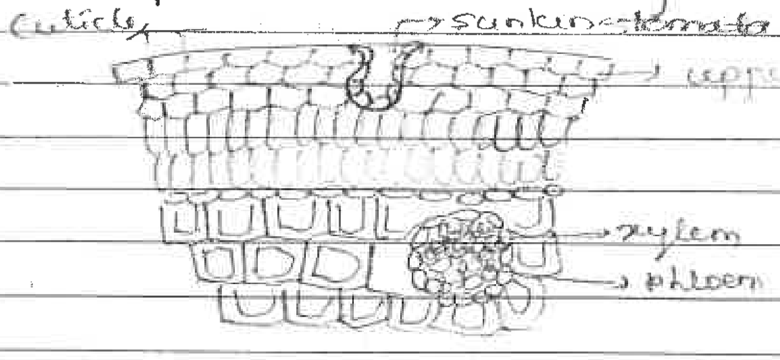
xerophytes are those plants which are habitat of desert area are called xerophytic plants

Ex :- Opuntia.



The opuntia can servive in the desert area because they posses adaptations that keep them from drying out.

The first adaptation are the ~~sto~~ presence of stomata, which are microscopic openings in plant leaves that releases water vapour. In xerophytes sunken stomata are present. They ~~prevent~~ reduces rate of transpiration in day time.



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College with Potential for Excellence

EXAMINATION

Class : B.Sc Vth Sem

Subject : Botany

Roll No. : 125

Date :

Marks Scored :

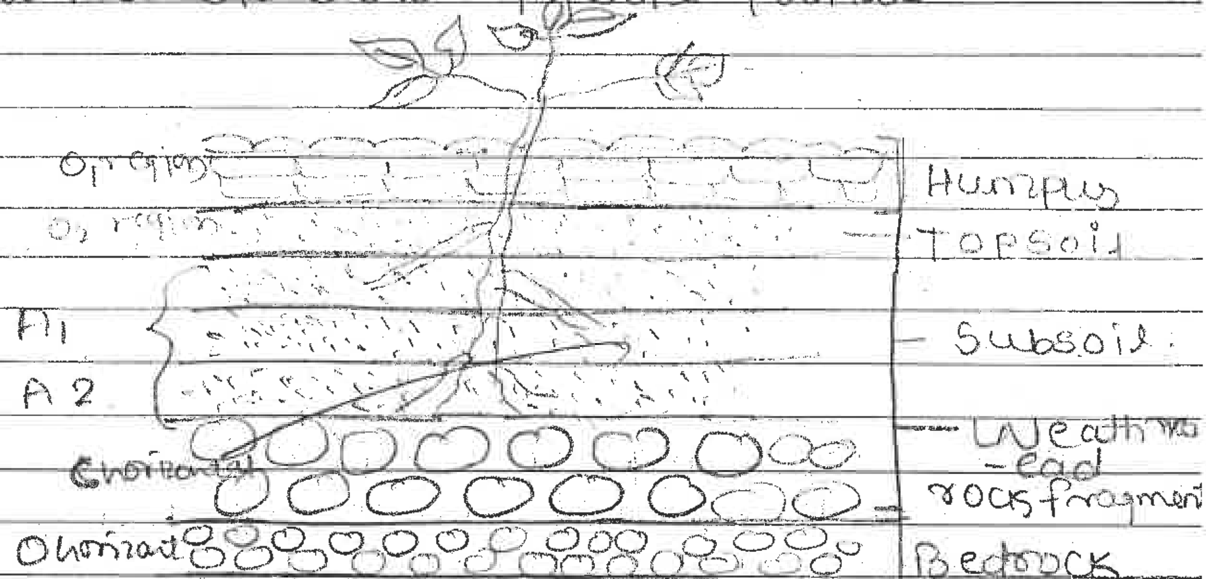
Test :

Reg No - S1919613

Signature of Valuer

Signature of the Invigilator with date

1) Explain the structure of soil profile.



Thus during pedogenesis there added organisms of organic compounds dead organic material matter leaving of mineralization of dead organic matter. Here mineral are gradually added to different layer of developing soil. The soil when fully developed can be seen leaving a No of layers of horn of a soil is known as soil profile.

Soil profile - due to the intensity of various factors influencing the process of petogenesis there development of vality of soil type. It depends upon the nature of parent matter and other factors. i.e Biological factors which type of soil will developed in area.

The different horizontal of soil Profile Historically in Russian Terminology, where classified into ABC Terminology. However according to the present any in general are the various conclusion The soil profile present in 5 various Types.

3) components of ecosystem.

Biotic components are the living things that shape an ecosystem. for example Biotic compound ad abiotic compounds.

1] Biotic compounds - This are the compound include plant, Animals, fungi and Bacteria

2] Abiotic compounds - This are the Non living compounds. That influence an ecosystem
examples abiotic factor are Temperature Pressure, air, current and minerals.

An ecosystem is a community of living organisms in conjuction with the Non living component of Their environment interacting as a system. These biotic and abiotic component linked together through Nutrient cycles and energy Flows.

K.L.E. Society's

G. I. BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPANI - 591 237.

(Re-accredited at 'A' Level by NAAC with CGPA 3.35)

College with Potential for Excellence

EXAMINATION

Class : Bsc Vth sem

Subject : Botany

Roll No. : 234

Date :

Marks Scored :

Test :

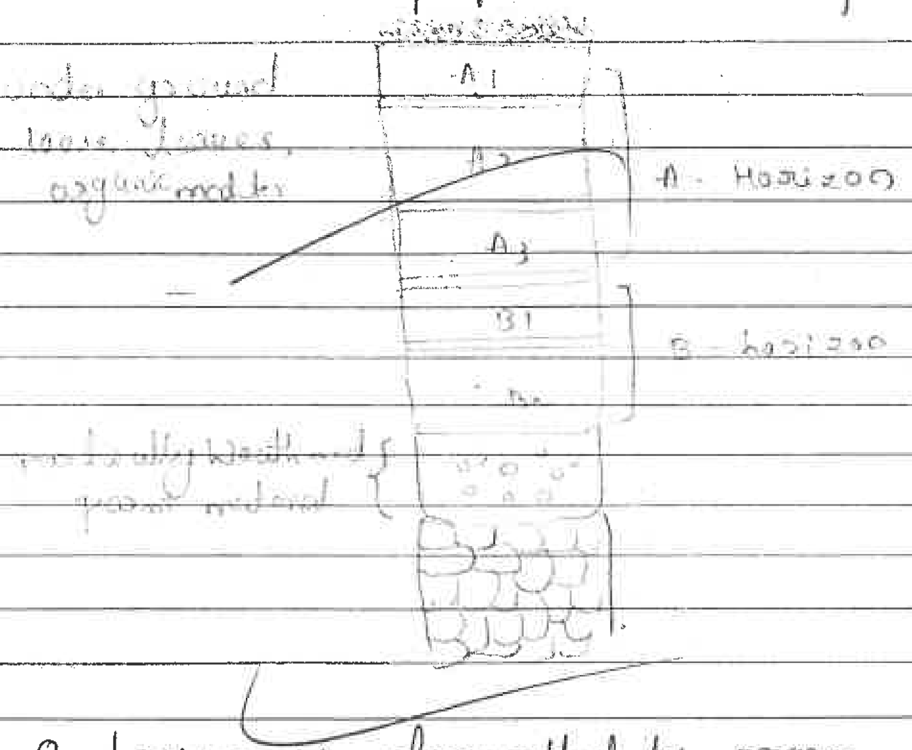
Reg NO - S1919583

Signature of Valuer

Signature of the Invigilator with date

I) Explain the structure of soil profile

→ soil profile may defined as vertical section through a soil. It represents sequence of horizons or layers differentiated from one another but genetically related and included to the parent material from which the soil profile is developed.

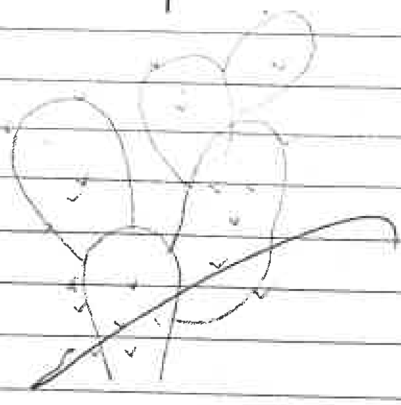


O - horizon is dominated by organic matter leaf & stem litter present in the dense forests & in isolated patches elsewhere.

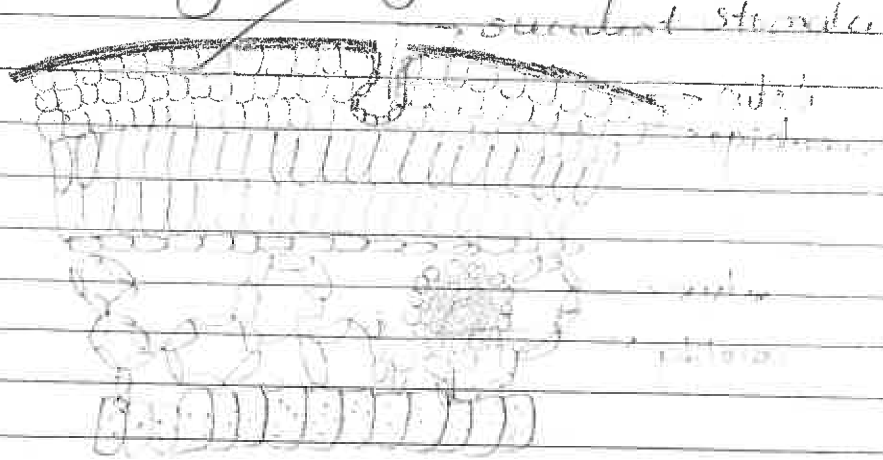
A - horizon - Zone of accumulation of organic matter and nutrients

B horizon- Zone of illuviation where accumulation of clay take place.
C- horizon- Here plant material & rooks are present.

2) Explain the adaptation in xerophyte
→ Xerophytes are those plants which are habitat of desert area are called the xerophytes. Ex- Opuntia



They can survive in these areas because they possess adaptations that keep them from drying out. The first adaption of is the presence of stomata, which are microscopic openings in plant leaves that releases water vapour, think of them as tiny pores.
To prevent the transpiration the stomata closed in the day time and it opens during the night



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EXAMINATION

Class : BSC-II^th sem.   Subject : Botany

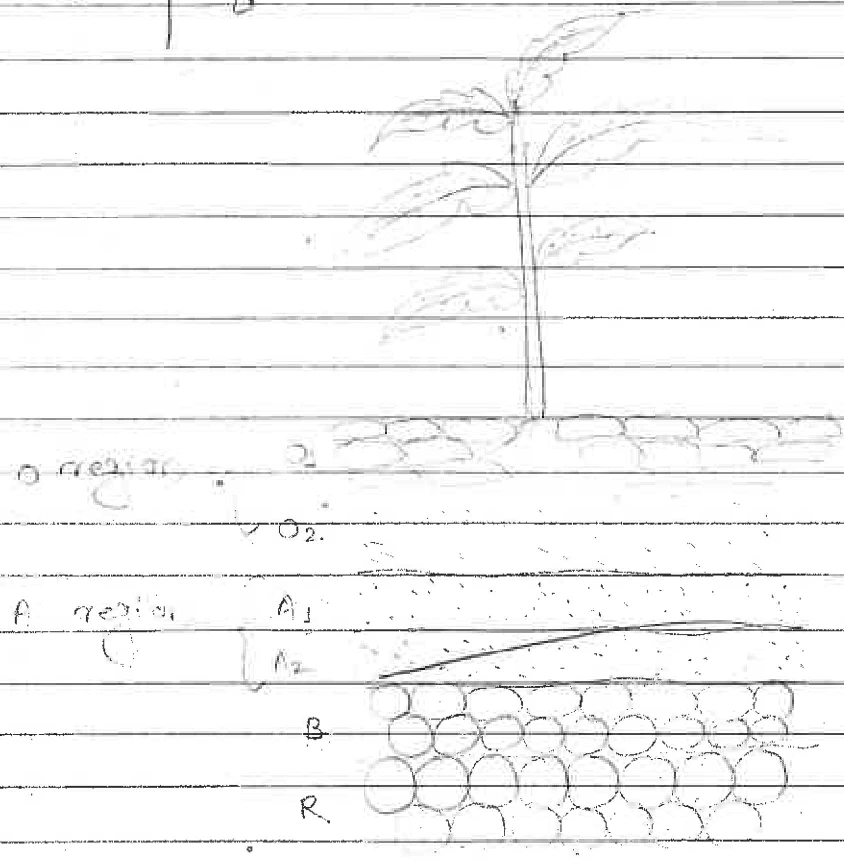
Roll No. : 148   Date :

Marks Scored : ______   Test :

Reg NO - S1919619

Signature of Valuer   Signature of the Invigilator with date

1] Soil profile ⟶



* O_1 region ⟶ It is the upper most layer of the surface contain a dead leaves. stem, root & also contain the dead animals.

* O_2 region ⟶ It is below the O_1 region. It is the second layer of the surface. In this layer decompose the organic material.

* A region ⟶ It is just below the O_2 region. In this region the bacteria, minerals are present. In A region, there are A_1 & A_2 two regions are found. A_1 is darker

region & A_2 is a lighter region.

* A_1 region → In this region, amorphous decomposed into the organic matter & this organic matter mix with mineral matter & form humus. ~~Organic matter mix with mineral & that~~ is brown, black in colour. A_1 region is a darker region.

* A_2 region → A_2 region is a lighter region. It just below the A_1 region. In this layer mineral particles are present in large amount.

* B region → It is below the A_2 region. In this region B_1, B_2 & B_3 regions are present. In this region soil development is formed.

* R region → It is the lowest region of the surface. In this region sandy rocks are present.

2) Ecosystem →

Ecosystem is the basic fundamental unit of functional ecology. It consists percentage of living organism (biotic) & non-living organism (abiotic) substances.

Abiotic factor → This is the non-living substance of environment.

Ex. water, soil, Air, light, etc.

Biotic factor → This is the living substance of environment.

Ex. plant, animal, bacteria, etc.

~~It is an~~

It is an interaction system where biotic & abiotic factor intract to produce an exchange of material between living & non-living substance.

Ecosystem is based on.

Producer

1° consumer

2° consumer

3° consumer

Decomposer.

Producer → All plants

It is prepared there own food with the help of photosynthesis.

Ref. No. *Date :*

vi. Distributed books to toppers

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DISTRIBUTION OF BOOKS TO TOPPERS, 2019-20, ON THE EVE OF DR.PRABHAKAR KORE BIRTHDAY CELEBRATION, 1.8.2019

K. L. E. Society's

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vii. Felicitation to Advance Learners

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FALICITATION TO UNIVERSITY RANK HOLDERS OF 2019-20, VALEDICTORY FUNCTION,9.9.2021

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CASH PRIZE AWARDS AND CERTIFICATES TO TOPPERS 2019-20, VALEDICTORY FUNCTION,9.9.2021

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TOPPERS AS BEST BOY AND BEST GIRL FOR THE YEAR 2019-20, VALEDICTORY FUNCTION,

K. L. E. Society's

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Affiliated to Rani Channamma University, Belagavi, Karnataka, India

Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

Ref. No. *Date :*

viii. Outcome: Ranks and Centum scorers

K.L. E. Society's

G. I. Bagewadi Arts, Science & Commerce College , Nipani

Department Of Mathematics

CASH PRIZE AWARDS FOR THE YEAR 2019-20

- Every year Cash prize of Rs. 1000/- and Rs.500 is instituted by Dr.(Smt) M. M. Shankrikopp Dept. of Mathematics, in the memory of her late father Shri Mahalingappa Shankrikopp of Adur, for students of B.Sc. who secured 100% and more than 95% and stood first in Mathematics at their respective classes and also some special cash prizes for securing I[st] in Maths Aptitude Test.

CASH PRIZE WINNERS

S.No.	Name of the student	Highest in	Amount	Sign.
1.	Miss Shruti Kumbar of B.Sc IV sem.	B.Sc. II Sem. Apr. 19 with 99 %	Rs. 500/-	S. R. Kumbar
2.	Miss Umeshelma Mulla of B. Sc. VI Sem.	B.Sc. IV Sem. Apr. 19 with 100% in M_1 and M_2	Rs. 1000/-	Mulla
3.	Miss Sangita More of M.Sc. II sem.	B.Sc. V I Sem. Apr. 19 with 97.3%	Rs. 500/-	[illegible]
4.	Mr.Sukshsay Padre of B. Sc. II Sem.	B.Sc I Sem. Dec. 19 with 99%	Rs. 500/-	[illegible]
5.	Miss Pranali Karape of B.Sc. IV Sem.	B.Sc III Sem. Dec. 19 with 97.5%	Rs. 500/-	PKharape
6.	Miss Sonali Bharde of B.Sc. VI Sem.	B.Sc I Sem. Dec. 19 with 96.3%	Rs. 500/-	Sonali

- Special cash prize of Rs. 500/- is awarded to Sri Mr.Abhishek Madiwal of B.Sc. V Sem. for securing first Prize in **Maths. Aptitude Test April – 2020.**

HOD
Head
Department of Mathematics
K L E's G. I. B. College, Nipani.

Principal
PRINCIPAL
K. L. E. Society's
G. I. Bagewadi College, Nipani

K. L. E. Society's

G. I. Bagewadi Arts, Science & Commerce College , Nipani

Department Of Mathematics

CASH PRIZE AWARDS FOR THE YEAR 2019-20

To encourage students to excel in Mathematics, the Department of Mathematics is instituted Cash prize of Rs. 200/- each to students of B.Sc. and M.Sc. who scored 100 out of 100 in Mathematics at their semester examinations.

Cash prize winners for the even semester 2018-19 & odd semester 2019-20 are

S. No.	Name	Paper	for the class	Amount	Sign.
Even semester 2018-19					
1.	Shruti Kumbar	M_2	B.Sc. II Sem.	200/-	S R. Kumbar
2.	Shubhangi Shendage	M_2		200/-	[illegible]
3.	Pradnya Bhivase	M_2	B.Sc IV Sem.	200/-	[illegible]
4.	Muskana Shekhaji	M_2		200/-	[illegible]
5.	Ummesalma Mulla	M_1 & M_2		400/-	[illegible]
6.	Sonali Barde	M_2		200/-	Sonali.
7.	Aishwarya Padre	M_2		200/-	[illegible]
8.	Dilshad Mulla	M_2		200/-	[illegible]
9.	Laxmi Samsuddi	M_2		200/-	[illegible]
10.	Laxmi Khot	M_2		200/-	AKhot
11.	Sangita More	M_1 and M_3	B.Sc VI Sem	400/-	[illegible]
12.	Samina Mulla	M_1		200/-	[illegible]
13.	Anuja Patil	M_3		200/-	[illegible]
14.	Anita Hamidwade	M_3		200/-	[illegible]
15.	Snehal Jadhav	M_3		200/-	SnJadhav
Odd Semester 2019-20					
16.	Sri Sukshay Padre	M_2	B.Sc I Sem.	200/-	[illegible]
17.	Miss Ashwini Rangapure	M_2	B.Sc. I Sem.	200/-	
18.	Miss Pranali Kharape	M_2	B.Sc. III Sem.	200/-	PKharape
19.	Miss Sukanya Chougale	M_1		200/-	[illegible]
20.	Miss Shruti Yalagoudanavar	M_1		200/-	Shruti
21.	Miss Shubhangi Shendage	M_1		200/-	[illegible]

22.	Miss Dilshad Mulla	M_1 & M_2	B.Sc. V Sem.	400/-	DMulla
23.	Miss Laxmi Sanmsuddi	M_1		200/-	Ladd
24.	Aishwarya Padre	M_1		200/-	Asp
25.	Sonali Barade	M_2		200/-	Sonali.
26.	Tejaswhini Patil	M_1		200/-	TPatil

Totally 29 students scored 100 out 100 in Mathematics

HOD
Head
Department of Mathematics
K.L.E's G. I. B. College, Nipani.

Principal
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G. I. Bagewadi College, Nipani.

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Website: www.klegibnpn.org.in **E-mail:** klegib_npn@yahoo.co.in **Ph.: 08338-220116**

CERTIFICATES AND CASH PRIZES TO TOPPERS FOR THE YEAR 2019-20, NATIONAL PI DAY CELEBRATION, 12.3.2020

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CERTIFICATES AND CASH PRIZES TO TOPPERS FOR THE YEAR 2019-20, NATIONAL PI DAY CELEBRATION, 12.3.2020

K. L. E. Society's

G. I. Bagewadi Arts, Science and Commerce College,
Nipani - 591237

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Affiliated to Rani Channamma University, Belagavi, Karnataka, India

Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

Ref. No. *Date :*

viii. Outcome: Ranks and Centum scorers

ರಾಣಿ ಚನ್ನಮ್ಮ ವಿಶ್ವವಿದ್ಯಾಲಯ,
ರಾ. ಹೆ. – 04 "ವಿದ್ಯಾಸಂಗಮ" – ಬೆಳಗಾವಿ – 591156
ಪರೀಕ್ಷಾ ವಿಭಾಗ"

RANI CHANNAMMA UNIVERSITY,

N.H. - 04 "VIDYASANGAMA"- BELAGAVI - 591156
"EXAMINATION SECTION"

Web Site: www.rcub.ac.in **E-mail: rcuexam@gmail.com**
PH No: 0831-2565237, 2565235 **Fax No: 0831-2565240**

Ref No: ರಾಚವಿ/ಬೆಳಗಾವಿ/ಪರೀಕ್ಷಾ ವಿಭಾಗ/2021-22/ 631 Date: 26/04/2021

PROVISIONAL RANK LIST OF B.Sc COURSE FOR THE ACADEMIC YEAR 2019-20

The Principals of all Degree colleges requested to go through the provisional rank holders list of B.SC September-2020 Examination.

Sl No	Reg. No	Name	Max. Marks	Sec. Marks	%	Rank No	Caste	College
1	S1731032	SAVITRI HIREMATH	3800	3635	95.66	1	IIIB	6260-TUNGAL SCHOOL OF BASIC & APPLIED SCIENCES, JAMKHANDI
2	S1717692	PRADNYA BHIVASHE	3800	3623	95.34	2	IIIB	4288-G I B ARTS, SCIENCE & COMMERCE COLLEGE, NIPPANI
3	S1710415	ARATI MUGALI	3800	3601	94.76	3	IIIB	4015-GOVT. FIRST GRADE COLLEGE, GOKAK
4	S1729679	BHOOMIKA CHINCHAKHANDI	3800	3601	94.76	3	IIA	6222-BLDEA`s COMMERCE, BHS ARTS & TGP SCIENCE COLLEGE, JAMAKHANDI
5	S1731013	DARSHANA MALAGANVI	3800	3586	94.37	4	IIIB	6260-TUNGAL SCHOOL OF BASIC & APPLIED SCIENCES, JAMKHANDI
6	S1733165	SAHANA NAAZ JUNNEDI	3800	3584	94.32	5	IIB	5206-ANJUMAN ARTS, SCIENCE AND COMMERCE COLLEGE, VIJAYAPURA
7	S1729988	SHRUTI BIRADAR	3800	3581	94.24	6	GM	6222-BLDEA`s COMMERCE, BHS ARTS & TGP SCIENCE COLLEGE, JAMAKHANDI
8	S1717737	SAVITA PATHADE	3800	3580	94.21	7	IIIB	4288-G I B ARTS, SCIENCE & COMMERCE COLLEGE, NIPPANI
9	S1716201	SHILPA NIDONI	3800	3578	94.16	8	IIA	4261-J S S ARTS, SCIENCE & COMMERCE COLLEGE, GOKAK
10	S1717672	MUSKAN SHEKHAJI	3800	3565	93.82	9	IIB	4288-G I B ARTS, SCIENCE & COMMERCE COLLEGE, NIPPANI
11	S1731026	NINGAVVA MARICHANDI	3800	3560	93.68	10	IIA	6260-TUNGAL SCHOOL OF BASIC & APPLIED SCIENCES, JAMKHANDI

Note: If any discrepancies are found intimate the University within 10th May 2021.

Registrar (Evaluation)
Rani Channamma University, Belagavi

Copy To,

1. Registrar, Rani Channamma University, Belagavi.
2. Finance Officer, Rani Channamma University, Belagavi.
3. P.S Vice-Chancellor, Rani Channamma University, Belagavi.
4. The Principals of all Colleges affiliated to Rani Channamma University, Belagavi
5. Web Site Copy
6. Office Copy

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani.

ರಾಣಿ ಚನ್ನಮ್ಮ ವಿಶ್ವವಿದ್ಯಾಲಯ,
ರಾ. ಹೆ. – 04 "ವಿದ್ಯಾಸಂಗಮ" – ಬೆಳಗಾವಿ – 591156
ಪರೀಕ್ಷಾ ವಿಭಾಗ"

RANI CHANNAMMA UNIVERSITY,

N.H. - 04 "VIDYASANGAMA"- BELAGAVI - 591156
"EXAMINATION SECTION"

Web Site: www.rcub.ac.in
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E-mail: rcuexam@gmail.com
Fax No: 0831-2565240

Ref No: ರಾಚವಿ/ಬೆಳಗಾವಿ/ಪರೀಕ್ಷಾ ವಿಭಾಗ/2021-22/531

Date: 26/04/2021

PROVISIONAL RANK LIST OF B.COM COURSE FOR THE ACADEMIC YEAR 2019-20

The Principals of all Degree colleges requested to go through the provisional rank holders list of B.COM September-2020 Examination.

Sl No	Reg. No	Name	Max. Marks	Sec. Marks	%	Rank No	Caste	College
1	C1719225	AKSHATA BALAKRISHNA BHAT	3700	3531	95.43	1	GM	4219-KLS GOGTE COLLEGE OF COMMERCE, BELAGAVI
2	C1766270	DANESH SHIVANAND SHIRAHATTI	3700	3507	94.78	2	IIIB	6205-BASAVESHWAR COMMERCE & B.B.A. COLLEGE, BAGALKOT
3	C1719468	PRATIK JOSHI	3700	3501	94.62	3	GM	4219-KLS GOGTE COLLEGE OF COMMERCE, BELAGAVI
4	C1722695	REKHA CHOUGALE	3700	3442	93.03	4	IIIB	4241-JAIN COLLEGE OF BBA, BCA & COMMERCE, BELAGAVI
5	C1767142	SAHANA SADALAGI	3700	3442	93.03	4	IIA	6211-JSS S T C ARTS & COMMERCE COLLEGE, BANAHATTI
6	C1719228	AKSHATA RAJESH BONGALE	3700	3437	92.89	5	IIA	4219-KLS GOGTE COLLEGE OF COMMERCE, BELAGAVI
7	C1735838	SHIVARANJANI PATIL	3700	3437	92.89	5	GM	4364-KLE SOCIETY'S COMMERCE COLLEGE, KHANAPUR
8	C1730212	APOORVA RAJENDRA KAMATE	3700	3431	92.73	6	IIIB	4288-G I B ARTS, SCIENCE & COMMERCE COLLEGE, NIPPANI
9	C1774639	SUSHMITA ALAGUR	3700	3431	92.73	6	IIIB	6259-S D M TRUST'S DANIGOND COLLEGE OF COMMERCE, TERDAL
10	C1719595	SHREESHAIL NAIK	3700	3429	92.68	7	IIIA	4219-KLS GOGTE COLLEGE OF COMMERCE, BELAGAVI
11	C1751868	SAGAR MAKHIJA	3700	3425	92.57	8	IIIB	5216-KUMUD BEN DARBAR COLLEGE OF BUSINESS ADMINISTRATION, VIJAYAPURA
12	C1719268	ASHA NEVAGI	3700	3423	92.51	9	GM	4219-KLS GOGTE COLLEGE OF COMMERCE, BELAGAVI
13	C1722696	RESHMA DESAI	3700	3422	92.49	10	IIIB	4241-JAIN COLLEGE OF BBA, BCA & COMMERCE, BELAGAVI

Note: If any discrepancies are found intimate the University within 10th May 2021

Registrar (Evaluation)
Rani Channamma University, Belagavi

Copy To,

1. Registrar, Rani Channamma University, Belagavi.
2. Finance Officer, Rani Channamma University, Belagavi.
3. P.S Vice-Chancellor, Rani Channamma University, Belagavi.
4. The Principals of all Colleges affiliated to Rani Channamma University, Belagavi
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K.L.E. Society's
G. I. Bagewadi College, Nipani.

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani.

ರಾಣಿ ಚನ್ನಮ್ಮ ವಿಶ್ವವಿದ್ಯಾಲಯ,
ರಾ. ಹೆ. – 04 "ವಿದ್ಯಾಸಂಗಮ" – ಬೆಳಗಾವಿ – 591156
ಪರೀಕ್ಷಾ ವಿಭಾಗ"

RANI CHANNAMMA UNIVERSITY,

N.H. - 04 "VIDYASANGAMA"- BELAGAVI - 591156
"EXAMINATION SECTION"

Web Site: www.rcub.ac.in **E-mail: rcuexam@gmail.com**
PH No: 0831-2565237, 2565235 **Fax No: 0831-2565240**

Ref No: ರಾಚವಿ/ಬೆಳಗಾವಿ/ಪರೀಕ್ಷಾ ವಿಭಾಗ/2021-22/ 531 Date: 26/04/2021

PROVISIONAL RANK LIST OF M.SC IN MATHEMATICS COURSE FOR THE ACADEMIC YEAR 2019-20

The Head of the Department and Principals of all PG colleges requested to go through the provisional rank holders list of **M.SC IN MATHEMATICS** September -2020 Examination.

Sl No	Reg. No	Name	Max. Marks	Sec. Marks	C.G.P.A	Rank No	Caste	College
1	MT181204	JYOTI ARUN CHAVAN	2400	1950	8.12	1	GM	9423-KLES G.I. BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPPANI
2	MT181008	ARATI MAGADUM	2400	1904	7.93	2	IIIB	9401-9401- RANI CHANNAMMA UNIVERSITY P.G. CAMPUS, VIDYASANGAMA, BELAGAVI
3	MT181212	PRIYA VEERANAGOUDA PATIL	2400	1899	7.91	3	IIIB	9423-KLES G.I. BAGEWADI ARTS, SCIENCE & COMMERCE COLLEGE, NIPPANI

Note: If any discrepancies are found intimate the University within 10th May 2021.

Registrar (Evaluation)
Rani Channamma University, Belagavi

Copy To,

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PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

Co-ordinator IQAC
K. L. E. Society's
G. I. Bagewadi College, Nipani.

K.L.E. Society's
G.I.Bagewadi Arts, Science and Commerce College, Nipani-591237
Accredited at 'A' level by NAAC with CGPA 3.35
(Affiliated to Rani Channamma University, Belagavi, Karnataka, India)
Website: www.klegibnpn.edu.in **E-mail:** klegib_npn@yahoo.co.in **Ph.: 08338-220116**

Centum Scorers

Odd Semester 2019-20

S. No.	Name	Class	Semester	Subject
1	Ashwini Rangapure	B.Sc	**I Semester**	Mathematics II
2	Sushay Padre		**I Semester**	Mathematics II
3	Pranali Kharape		III Semester	Mathematics II
4	Shrikant Mali		III Semester	Chemistry
5	Shruti Yalagoudanavar		III Semester	Mathematics I
6	Shubhangi Shendage		III Semester	Mathematics I
7	Sukanya Chougale		III Semester	Mathematics I
8	Yashodha Khajave		III Semester	Chemistry
9	Aishwarya Padre		**V Semester**	Mathematics I
10	Dilshad Mulla		**V Semester**	Mathematics I
11	Dilshad Mulla		**V Semester**	Mathematics II
12	Laxmi Sansuddi		**V Semester**	Mathematics I
13	Sonali Barade		**V Semester**	Mathematics II
14	Tejaswhini Patil		**V Semester**	Mathematics I
15	Asmita Mengane	B.Com	I Semester	Secretarial Practice
16	Asmita Mengane		I Semester	Financial Accounting
17	Komal Gadakari		I Semester	Financial Accounting
18	Sahil Shrikhande		I Semester	Financial Accounting
19	Ashwini Halagadagi		III Semester	PED
20	Shubham Thane		III Semester	Business Statistics
21	Apoorva Kamate		V Semester	Indian Financial Market
22	Asmita Gumathannavar		V Semester	Goods and Service Tax
23	Snehal Patil		V Semester	Management A/C
24	Veena Alatagi		**V Semester**	Income Tax
Even Semester 2019-20				
25	Ashwini Nasalapure	B.Com	VI Semester	Goods and Service Tax
26	Ashwini Nasalapure		VI Semester	Income Tax
27	Apoorva Kamate		VI Semester	Goods and Service Tax
28	Apoorva Kamate		VI Semester	Auditing
29	Apoorva Kamate		VI Semester	Income Tax
30	Ashwini Malage		VI Semester	Goods and Service Tax
31	Darshan Dandage		VI Semester	Goods and Service Tax
32	Snehal Patil		VI Semester	Goods and Service Tax
33	Swati Hokale		VI Semester	Goods and Service Tax
34	Veena Alatagi		VI Semester	Income Tax

IQAC Co-ordinator
K.L.E's G. I. B. College, Nipani.

K.L.E's G.I.BAGEWADI ARTS SCI & COM. COLLEGE NIPANI

PRINCIPAL
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G. I. Bagewadi College, Nipani.

K. L. E. Society's

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Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

Ref. No. *Date :*

2.2.1: Slow learners and Remedial Programs Organized for them

i. List of slow learners subject wise

ii. Practice Test

iii. Book facility from departments

K. L. E. Society's
G. I. Bagewadi Arts, Science and Commerce College,
Nipani - 591237
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Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

Ref. No. *Date :*

i. List of Slow Learners Subject Wise

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Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116

Number of Slow Learners Subject wise and Class wise in the Year 2019-20

Subject	Class	No. of slow learners
Physics	B.Sc I	16
	B.Sc II	14
	B.Sc III	20
	Total	**50**
Chemistry	B.Sc I	61
	B.Sc II	13
	B.Sc III	08
	Total	**82**
Mathematics	B.Sc I	41
	B.Sc II	32
	B.Sc III	20
	Total	**93**
Botany	B.Sc I	18
	B.Sc II	03
	B.Sc III	05
	Total	**26**
Zoology	B.Sc I	16
	B.Sc II	03
	B.Sc III	05
	Total	**24**
Commerce	B.Com I	16
	B.Com II	33
	B.Com III	14
	Total	**63**
Economics	BA I	08
	BA II	01
	BA III	02
	Total	**11**
	Grand Total	**349**

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

K. L.E. Society's

G.I. Bagewadi Arts, Science & Commerce College, Nipani

DEPARTMENT OF PHYSICS

Time table for Remedial Coaching Classes at UG level for the year 2019-20

Days	Time	Class	Name of the Staff
Tuesday	8.15 am to 9.15 am	B.Sc I & II Sem.	RGK,VNC & GMM
Friday	8.15 am to 9.15 am	B.Sc III & IV Sem	DSK & AYS
Tuesday	4.00pm to 5.00pm	B.Sc V & VI Sem	ADT ,DAC & RSC
Each teacher will engage the class alternate week for two months Aug. & Sept. I term and Feb. & March II term.			

Name	Signature
ADT- Prof. A. D. Tigadi	
RGK- Dr.R.G.Kharabe	
GMM- Smt G.M.Madanalli	
AYS - Miss.A.Y.Sanadi	
DSK-Miss.D.S.Koppal	
VNC-Smt V.N.Chogale	
DAC-Smt.D.A.Chimney	
RSC-Miss.R.S.Chougale	

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

HOD

Head

Department of Physics

K.L.E's G. I. B. College, Nipani

K. L. E. Society's
G. I. Bagewadi Arts, Science and Commerce College, Nipani-591237
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Department of Physics

Remedial Classes for Slow learners

Time table for 2019-20
B.Sc. classes

Class	Subject	Day	Time
B.Sc.I and II sem	Physics	Tuesday	8.15 am to 9.15 am
B.Sc.III and IV sem	Physics	Friday	8.15 am to 9.15 am
B.Sc.V andVI sem	Physics	Tuesday	4.00pm to 5.00pm

HOD
Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE * NIPANI

Principal
PRINCIPAL
G.I. Bagewadi Arts, Science &
Commerce College, NIPANI

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DEPARTMENT OF PHYSICS

B. Sc.I & II Semester

Acadamic growth of slow learners after extra tests,assignments and guidens for the year 2019-20 (Result Awaited)

Sl.No.	Reg. No.	Student Name	Marks Entry level	I SEM		II SEM		Exit level	Growth
				T_1	T_2	T_1	T_2		
1	S1919422	Aniket Jadhav	52	5	8	7	6	Promoted	-----
2	S1919434	Bhakti Karambale	46	4	6	6	3	Promoted	-----
3	S1919437	Gajendra Savant	59	3	4	6	5	Promoted	-----
4	S1919441	Girija Kulkarni	51	8	5	7	6	Promoted	-----
5	S1919442	Gouri Nesare	56	4	7	6	5	Promoted	-----
6	S1919458	Madhuri Patil	51	7	5	7	6	Promoted	-----
7	S1919460	Mahesh sankaje	53	5	3	5	4	Promoted	-----
8	S1919471	Namrata Lokande	54	6	6	5	7	Promoted	-----
9	S1919475	Nikhil Kademani	46	4	5	4	5	Promoted	-----
10	S1919476	Nikhil Nimbalkar	57	3	6	7	8	Promoted	-----
11	S1919477	Nikhita Wadakar	51	7	5	6	7	Promoted	-----
12	S1919480	Nilesh Gavade	51	8	7	5	6	Promoted	-----
13	S1919483	Omesh Injal	48	6	5	4	5	Promoted	-----
14	S1919484	Omkar Mahajan	42	3	5	6	7	Promoted	-----
15	S1919489	Pallavi Chougale	49	5	6	7	8	Promoted	-----
16	S1919494	Pallavi Shedbale	50	6	7	7	8	Promoted	-----

IQAC CO-ORDINATOR

HOD
Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

PRINCIPAL
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G.I. Bagewadi Arts, Science & Commerce College, NIPANI

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DEPARTMENT OF PHYSICS

B. Sc.III & IV Semester

Acadamic growth of slow learners after extra tests,assignments and guidens for the year 2019-20 (Result Awaited)

Sl.No.	Reg. No.	Student Name	Marks Entry level	III SEM		IV SEM		Exit level	Growth
				T_1	T_2	T_1	T_2		
1	S1819434	Chaitali Khot	47	5	6	7	8	Promoted	-----
2	S1819442	Dundappa Hawaldar	47	4	5	6	7	Promoted	-----
3	S1819458	Kiran Manakale	47	5	6	7	7	Promoted	-----
4	S1819480	Muskan Inamdar	48	7	8	7	6	Promoted	-----
5	S1819489	Ummesalma Tamboli	49	8	7	6	7	Promoted	-----
6	S1819497	Pooja Patil	50	6	5	5	7	Promoted	-----
7	S1819510	Praveen Dodamani	48	5	4	4	5	Promoted	-----
8	S1819511	Preeti Mattiwade	47	5	6	5	6	Promoted	-----
9	S1819514	Priyanka Hajare	41	3	4	4	5	Promoted	-----
10	S1819522	Reshma Khot	47	4	5	6	7	Promoted	-----
11	S1819528	Rohini Khot	FAIL	3	4	4	5	Promoted	-----
12	S1819536	Rutuja Malaba	48	6	5	6	7	Promoted	-----
13	S1819537	Rutuja Narasagoudar	FAIL	3	4	5	6	Promoted	-----
14	S1819540	Sachin Khot	40	4	5	5	7	Promoted	-----

IQAC CO-ORDINATOR **HOD** **PRINCIPAL**

Head
Department of Physics
K.L.E's G. I. B. College, Nipani

PRINCIPAL
G.I. Bagewadi Arts, Science & Commerce College, NIPANI

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

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DEPARTMENT OF PHYSICS

Class : B.Sc - V^{th} & VI^{th} Sem

Acadamic growth of slow learners after extra tests,assignments and guidens for the year 2019-20 (Result Awaited)

Sl.No.	Reg. No.	Student Name	%Entry level	V SEM		VI SEM		%Exit level	Growth
				T_1	T_2	T_1	T_2		
1	S1717601	Ayushi Kadam	63.3	5	4	8	9	64	YES
2	S1717603	Abhinandann Rayagonnavar	57	4	5	6	4	65.6	YES
3	S1717612	Akash Sindhe	61	6	7	4	9	75	YES
4	S1717613	Akshay Avade	64	4	ab	ab	5	Fail	NO
5	S1717615	Amarjit Sindhe	61	5	6	5	7	74.3	YES
6	S1717620	Arun Kotiwale	52.6	ab	ab	ab	ab	AB	NO
7	S1717623	Asmita Kamble	54.6	5	8	6	4	75	YES
8	S1717624	Basavaraj Shanvagave	48.3	5	6	5	6	75	YES
9	S1717641	Hemant Sajane	68.3	7	5	8	8	80.3	YES
10	S1717642	Hrishikesh Davare	63.3	6	8	5	7	73.6	YES
11	S1717679	Nikita Nadage	60.3	5	8	7	6	81.6	YES
12	S1717706	Priyanka Kesarkar	69.3	5	8	7	6	80.6	YES
13	S1717710	Prutviraj Patil	69.6	4	5	9	7	87	YES
14	S1717716	Rajashri Khot	72.3	5	6	4	7	92.6	YES
15	S1717720	Rohan Devkate	64.3	4	5	8	7	85.6	YES
16	S1717728	Samarth Shiragave	69	5	6	5	7	83.3	YES
17	S1717742	Shebarani Nagannavar	69.6	4	5	8	7	89	YES
18	S1717745	Shivani Patil	64.6	5	6	7	7	77.6	YES
19	S1717750	Shweta Patil	62	5	6	7	7	83.6	YES
20	S1717774	Vidhyasagar Chogale	60	5	5	7	7	77	YES

Growth Rate = $\frac{18}{20}$ **x100 =90%**

IQAC Co-ordinator

HOD
Head
Department of Physics
K.L.E's G. I. B. College, Nipani.

Principal
PRINCIPAL
G.I. Bagewadi Arts, Science & Commerce College, NIPANI

G.I.BAGEWADI ARTS, SCI & COM COLLEGE NIPANI K.L.E's

K.L.E. Society's

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Ph: 08338-220116, 220119 E-mail: klegib npn@yahoo.co.in

SLOW LEARNERS

KEY STEPS:

1. Failures and below 55% scored in each class were considered as slow learners.
2. Slow learner students are advised to meet the concerned staff and H.O.D. to discuss the issues related to subject.
3. Students are provided with text books from departmental library as well as central library and study materials for reference.
4. Students areasked to solve IA and previous year question papers and evaluated by the concerned staff and H.O.D.
5. Performance of such students is monitored by concerned staff and H.O.D.

CONVENOR **H.O.D.** **PRINCIPAL**

Head
Department of Chemistry
K.L.E's G. I. B. College, Nipani.

PRINCIPAL
K.L.E. Society's
G. I. Bagewadi College, Nipani.

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E-mail: klegib_npn@yahoo.co.in

TIME TABLE FOR REMEDIAL COACHING CLASSES AT UG LEVEL FOR THE YEAR 2019-20

DAYS	TIME	CLASS	NAME OF THE STAFF	Signature
Wednesday	8:15am to 9:15am	B.Sc. I and II Sem	Smt. D.D. Bhoite Mr. P.T. Narwade	
Saturday	8:15am to 9:15am	B.Sc. III and IV Sem	Smt. R. R. Mane Smt. G.B. Chendake	
Wednesday	8:15am to 9:15am	B.Sc. V and VI Sem	Dr. A. A. Kambale Smt. D. S. Kanagali	

Each teacher will engage the class alternate week for two months.

First term August and September

Second term February and March.

H.O.D.
Head
Department of Chemistry
K.L.E's G. I. B. College, Nipani.

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

PRINCIPAL
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G. I. Bagewadi College, Nipani.

K.L.E. Society's

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NIPANI- 591237 Dist:- Belgavi

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e-mail- kle_gibnpn.yahoo.in.

IQAC INITIATIVE

Slow Learner FOR I & II SEMESTER B.SC. - NOVEMBER 2019-20

si.no	Register no	Name of the student	% at entry (1st sem)	% at exit (2nd sem)	Growth
1	S1919403	ABHISHEK AMBI	43%	Promoted	--
2	S1919406	ADITYA SIDDANNAVAR	56%	Promoted	--
3	S1919409	AISHWARYA PATIL	50%	Promoted	--
4	S1919410	AISHWARYA PATIL	46%	Promoted	--
5	S1919415	AKSHATA YARANAL	53%	Promoted	--
6	S1919416	AKSHAY AKKOLTE	55%	Promoted	--
7	S1919417	AMAN MULLA	43%	Promoted	--
8	S1919419	AMBIKA KAMATE	56%	Promoted	--
9	S1919423	ANIL SHENDRE	50%	Promoted	--
10	S1919425	ANVITA KURBE	55%	Promoted	--
11	S1919428	ASHRAFI ISARAZA	47%	Promoted	--
12	S1919431	ASHWINI SUTAR	48%	Promoted	--
13	S1919439	GAYATRI CHAVAN	51%	Promoted	--
14	S1919443	HARIPRASAD NALAWADE	50%	Promoted	--
15	S1919452	KRUSHNAT HIRIKUDE	53%	Promoted	--
16	S1919463	MANISH PATIL	54%	Promoted	--
17	S1919475	NIKHIL KADEMANI	37%	Promoted	--
18	S1919478	NIKITA ARAGE	44%	Promoted	--
19	S1919484	OMKAR MAHAJAN	48%	Promoted	--
20	S1919485	OMKAR PATIL	47%	Promoted	--
21	S1919486	OMKAR SANKAPAL	44%	Promoted	--
22	S1919495	PRADEEP KHOT	46%	Promoted	--
23	S1919498	PRAJAKTA KURANE	48%	Promoted	--
24	S1919499	PRAJWAL KOTHIWALE	42%	Promoted	--
25	S1919500	PRAJWAL URAMANATTI	49%	Promoted	--
26	S1919503	PRATHAMESH DONGARE	50%	Promoted	--
27	S1919515	RADHIKA CHALAKE	54%	Promoted	--
28	S1919516	RAHUL DINAKAR	54%	Promoted	--
29	S1919517	RAHUL PATIL	40%	Promoted	--
30	S1919519	RAJSHEKHAR MATHAPATI	52%	Promoted	--
31	S1919521	RAYANNA CHIKKODI	44%	Promoted	--
32	S1919522	REVANNA BANNE	41%	Promoted	--
33	S1919523	RITU PATIL	46%	Promoted	--
34	S1919533	RUTUJA NARE	42%	Promoted	--
35	S1919536	RUTUJA UGARE	54%	Promoted	--
36	S1919537	RUTUJA VATHARE	55%	Promoted	--
37	S1919539	SAGAR KOLI	50%	Promoted	--
38	S1919542	SANJAY RUDRAGOUDAR	42%	Promoted	--

K.L.E.'s G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

39	S1919544	SANTOSH KORE	44%	Promoted	--
40	S1919546	SATWIK DESAI	38%	Promoted	--
41	S1919549	SHANKARGOUDA PATIL	54%	Promoted	--
42	S1919550	SHEKHAR SHENDRE	52%	Promoted	--
43	S1919551	SHIDHARTH BELAVI	42%	Promoted	--
44	S1919555	SHRADDHA BANAJAWAD	53%	Promoted	--
45	S1919557	SHREEDEVI BYAKUDE	46%	Promoted	--
46	S1919559	SHRISHAIL KANADE	37%	Promoted	--
47	S1919561	SHRUTI SOLAPURE	52%	Promoted	--
48	S1919565	SHWETA JADHAV	34%	Promoted	--
49	S1919566	SHWETA KONE	52%	Promoted	--
50	S1919571	SNEHAL MANGASULE	42%	Promoted	--
51	S1919574	SOMANATH KHOT	52%	Promoted	--
52	S1919578	SOORAJ SOUDE	50%	Promoted	--
53	S1919580	SOURABH BHADARGADE	53%	Promoted	--
54	S1919585	SOURABH PATIL	50%	Promoted	--
55	S1919595	SUNIL SHENDRE	44%	Promoted	--
56	S1919597	SUSHANT HIREKUDI	43%	Promoted	--
57	S1919600	SUSHANT PATIL	49%	Promoted	--
58	S1919607	TUSHAR PATIL	50%	Promoted	--
59	S1919608	VAIBHAV MALI	50%	Promoted	--
60	S1919618	YUVARAJ SUKHASARE	51%	Promoted	--
61	S1919620	ZUBER MADARKHAN	49%	Promoted	--

As per RCU direction B.Sc - 2nd Sem students are promoted to higher class

CONVENOR

H.O.D

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NIPANI- 591237 Dist:- Belgavi

[Reaccredited at 'A' level by NAAC with CGPA-3.35] Ph-08338220116

e-mail- kle_gibnpn.yahoo.in.

IQAC INITIATIVE

Slow Learner FOR III & IV SEMESTER B.SC. - NOVEMBER 2019-20

si.no	Register no	Name of the student	% at entry (1st sem)	% at exit (2nd sem)	growth
1	S1819404	ADESH PATIL	51%	Promoted	--
2	S1819411	AKSHATA KULKARNI	55%	Promoted	--
3	S1819417	ANIKET SADALAGE	54%	Promoted	--
4	S1819418	ANIL KHILARE	51%	Promoted	--
5	S1819439	DHANASHRI PATIL	54%	Promoted	--
6	S1819456	KIRAN IROLE	48%	Promoted	--
7	S1819457	KIRAN KARIBALAPPAGOL	51%	Promoted	--
8	S1819494	PAVAN JADHAV	48%	Promoted	--
9	S1819498	POORNANAND NIRMALE	44%	Promoted	--
10	S1819507	PRATIK JARALI	43%	Promoted	--
11	S1819529	RUDRA MAGADUM	54%	Promoted	--
12	S1819584	SHUBHANGI BHOJEPATIL	51%	Promoted	--
13	S1819590	SNEHA AMBI	42%	Promoted	--

As per RCU direction B.Sc - 4th Sem students are promoted to higher class

CONVENOR

H.O.D

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IQAC INITIATIVE

Slow Learner FOR V & VI SEMESTER B.SC. - NOVEMBER 2019-20

si.no	Register no	Name of the student	% at entry (5th sem)	% at exit (6th sem)	growth
1	S1717681	PARSHWAJEET PATIL	52%	65%	YES
2	S1717699	PRASAD ARALIKATTI	30%	Fail	NO
3	S1717705	PRITHVIRAJ NARAYANK	51%	68%	YES
4	S1717717	RAMAKRISHNA GUDENI	40%	60%	YES
5	S1717718	RAMIZRAZA MAKANDA	47%	63%	YES
6	S1717761	SUMIT CHOUGULE	47%	72%	YES
7	S1717763	SUSHANT LANGOTE	51%	67%	YES
8	S1717776	VISHAL MOKASHI	52%	66%	YES

CONVENOR

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LIBRARY, Department of .... Remedial Class

Year - 2019-20

Date of Issue	Acc. No. Class	Author	Title	Signature of Staff / Student	Date of Return	Signature of Staff in Charge	Remarks
07/07/2019	B.Sc-I sem	S. chand	Environmental chem	Abhishek Ambi	01/03/2020		
15/07/2019	B.Sc-I "	Vaishnavi Publication	Textbook of	Aman Mulla	01/03/2020		
22/07/2019	B.Sc-I "	Kona Publication	chemistry	Ambika Kamate	04/03/2020		
22/07/2019	B.Sc-I "	S. chand	Inorganic chemistry	Rahul Patil	04/03/2020	[signature]	Received
28/07/2019	B.Sc-I "	Vaishnavi Publication		Ritu Patil	04/03/2020		
28/07/2019	B.Sc-I "	Vaishnavi Publication	Textbook of	Sagar Koli			
16/08/2019	B.Sc-I "	Kona Publication	chemistry	Revanna Banne	11/03/2020		
16/09/2019	B.Sc-I "	Kona Publication		Rahul Dinakar			
16/09/2019	B.Sc-I "	Kona Publication		Somanath Khot			
16/09/2019	B.Sc-I "	Kona Publication		Sourabh Patil			
19/02/2019	B.Sc II sem	Arun Bahl	Elementary	Ambika kamate			
04/03/2020	B.Sc II sem	B. S. Bahl	organic chem	Sagar koli	15/03/2020	[signature]	Received
11/03/2020	B.Sc II sem	B.K. Sharma	organic chem V-1	Abhishek Ambi			
11/03/2020	B.Sc II sem	B.K Sharma	organic chem V-2	Aman Mulla			
07/07/2019	B.Sc-III sem	Puri and sharma	Physical chemistry	Adesh Patil			
15/07/2019	B.Sc-III "	Bahl and Bahl	organic chemistry	Kiran Irole			
13/07/2019	B.Sc-III "	Puri sharma	Principle of In. chem	Aniket Sadalage	10/12/2019	[signature]	Received
15/07/2019	B.Sc-III "	Vaishnavi Publication	Text book of chem	Pavan Jadhav			
22/07/2019	B.Sc-III "	Kona Publication	Text book of chem	Anil khilare			
28/07/2019	B.Sc-III "	Kona Publication	Text book of chem	Akshata Kulkarni			
04/01/2020	B.Sc IV sem	Dr. O.P. Agarwal	org. Chem	Aniket sadalage			
11/01/2020	B.Sc IV sem	R.L. Madan	org. chem	Kiran Irole	15/3/2020	[signature]	
11/01/2020	B.Sc IV sem	"	org. Chem	Pavan Jadhav			
15/07/2019	B.Sc-V sem	Peter Sykes	Mech. of org. chem	Parshwajeet Patil			
22/07/2019	B.Sc-V "	N.K. Vishnoi	organic chem	Prasad Aralikatti	11/12/2019	[signature]	Received
29/07/2019	B.Sc-V "	Arun Bahl, B.S. Bahl	organic chem	Vishal mokashi			
18/08/2019	B.Sc-V "	Vogel	Practicle org. Chem	Ramizraza M.			
04/03/2020	B.Sc VI sem	I. L. Finar	Organic	Anil kakade			
05/03/2020	B.Sc VI sem	N.K. Vishnoi	chemistry	Rutuja Patil	11/12/2020	[signature]	Received
05/03/2020	B.Sc VI sem	Kona Publication		Prasad Aralikatti			

CONVENOR - [signature]

[signature]
Head
Department of Chemistry

K. L.E. Society's
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DEPARTMENT OF MATHEMATICS
Programmes arranged for slow learners for the year 2019-20

At the beginning of the academic year we prepare the list of the slow learners will be prepared by considering their previous semester result, usually below 60% we consider as slow learners. For those students we conduct following programmes once or twice in fifteen days.

- Extra classes
- Practicing important questions
- Advanced learners help them in solving old question papers
- Counseling
- Extra assignments
- Providing them solved question banks
- In case of less attendance ask them to bring parents
- Encourage students to write important theorems twice.
- During class hours, encourage them to solve examples on board.

Time table:

Class	Day	Time
B.Sc I	Monday	3.00 pm to 4.00 pm
B.Sc II	Friday	3.00 pm to 4.00 pm
B.Sc III	Tuesday	3.00 pm to 4.00 pm

According to the above time table engage classes/counsel students/ Solve examples, theorem etc. once in 15 days or according to the students convenience.

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DEPARTMENT OF MATHEMATICS

Time table for Remedial Coaching Classes at UG level for the year 2019-20

Days	Time	Class	Name of the Staff
Monday	8.15 am to 9.15 am	B.Sc I & II Sem.	GLK and JNM
Thursday	8.15 am to 9.15 am	B.Sc III & IV Sem.	MMS and GLK
Monday	4.00pm to 5.00pm	B.Sc V & VI Sem.	GLK and JNM
Each teacher will engage the class alternate week for two months Aug. & Sept. I term and Feb. & March II term.			

Name Signature

MMS- Dr.(Smt.) M. M. Shankrikopp

GLK - Miss G.L. Karaguppi

JNM – Sri J.N Magadum

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DEPARTMENT OF MATHEMATICS

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

B. Sc. I & II Semester

Criteria: Students with less than 60% are considered as slow learners

Sr. No.	R.No.	Reg No	Name of the student	% at Entry	I Sem		II Sem		% at Exit	Growth
					T_1	T_2	T_1	T_2		
1	137	S1919404	Abhishek Gajanan Bhoi	59.23	6	7	6	8	Due to COVID-19 No Exams	All are Promoted
2	37	S1919409	Aishwarya a Patil	58.83	1	5	5	5		
3	183	S1919417	Aman Mulla	52.6	4	5	6	7		
4	140	S1919428	Ashrafi Isaraza	53.83	4	4	5	6		
5	170	S1919434	Bhakti M Karambale	58.33	9	8	9	8		
6	184	S1919438	Gangambika V Hiremath	53.3	5	6	5	6		
7	171	S1919443	Hariprasad Nalawade	58	2	5	2	5		
8	187	S1919452	Krushnat Hirikude	54	5	5	5	5		
9	145	S1919454	Kushal Mudhole	56.5	6	8	6	8		
10	58	S1919456	Madhavi V Hirekude	59.83	5	7	4	7		
11	60	S1919458	Madhuri Patil	58.68	7	6	8	5		
12	146	S1919463	Manish Prabhakar Patil	53.38	8	9	9	9		
13	139	S1919419	Miss Ambika P.Kamate	55.3	6	8	7	8		
14	147	S1919464	Miss Manjula V Bhanase	59.83	8	8	5	8		
15	152	S1919491	Miss Pallavi S Kadalage	54.33	7	9	7	8		
16	87	S1919514	Miss Priyanka Ajitkumar Dhale	57.16	9	9	10	9		
17	159	S1919570	Miss Sneha J Suryawanshi	50.5	8	8	5	7		
18	172	S1919469	Mithil Sanjay Nagaonkar	53	5	7	6	6		
19	73	S1919480	Mr Nilesh Gavade	59.84	4	4	5	6		
20	160	S1919578	Mr Sooraj Soude	55.55	4	5	4	5		
21	165	S1919595	Mr Sunil S Shendre	47.33	5	6	7	7		
22	180	S1919599	Mr Sushant Patil	56	5	5	6	7		
23	133	S1919616	Mr yallapa H Helavar	58.5	4	6	4	5		

24	148	S1919470	Muskan M Nargund	59.66	5	5	4	7		
25	149	S1919484	Omkar Mahajan	56	0	Ab	0	Ab		
26	174	S1919496	Pradnya Ghosarawade	54.33	7	8	7	7		
27	154	S1919513	Priyanka Appasab Dattawade	59	8	10	9	10		
28	191	S1919534	Rutuja Sankapal	59.5	6	8	7	8		
29	176	S1919547	Sejal Havale	57.66	6	7	5	7		
30	108	S1919548	Shailaja Vinayak Powar	54.5	8	8	8	9		
31	157	S1919549	Shankargouda B Patil	55.5	6	7	5	7		
32	177	S1919555	Shraddha J Banajawad	58.16	8	6	9	8		
33	193	S1919559	Shrishail Kanade	56.67	4	5	5	5		
34	179	S1919571	Snehal Satyappamangasule	56	7	6	7	9		
35	118	S1919574	Somanath Khot	57.33	7	8	7	9		
36	162	S1919582	Sourabh Hinglaje	50.77	6	5	7	7		
37	163	S1919585	Sourabh Kiran patil	49.54	7	9	8	8		
38	196	S1919597	Sushant Hirekudi	57.16	5	6	5	4		
39	167	S1919602	Swaraj Balasaheb Belekar	58	6	7	8	8		
40	169	S1919608	Vaibhav Mali	54.83	5	6	4	7		
41	181	S1919620	Zuber Madarkhan	57.33	5	6	6	5		

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DEPARTMENT OF MATHEMATICS

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

B. Sc.III & IV Semester

Criteria: Students with less than 55% are considered as slow learners

Sr. No.	R. No.	Reg. No.	Name of the student	% at Entry	III Sem		IV Sem		% at Exit	Growth
					T_1	T_2	T_1	T_2		
1	201	S1819401	Abhishek Madiwal	F	4	5	7	8	Due to COVID -19 NO Exams	All are Promoted
2	227	S1819418	Anil Khilare	F	1	4	5	7		
3	83	S1819427	Ashika Shivapure	54.5	5	7	7	8		
4	87	S1819431	Avinash Ambi	49.00	4	4	3	6		
5	202	S1819434	Chaitali Khot	45.5	6	8	9	10		
6	100	S1819452	Kapil Magadum	F	4	4	5	5		
7	104	S1819458	Kiran Manakale	F	3	4	6	5		
8	105	S1819460	Krishna Patil	F	4	6	7	7		
9	205	S1819480	Muskan Inamdar	F	4	3	4	4		
10	206	S1819486	Ningappa Talwar	F	3	6	7	6		
11	121	S1819488	Nividita nadage	52.00	5	7	8	8		
12	128	S1819498	Poornanand Nirmale	F	0	ab	ab	ab		
13	133	S1819506	Pratik Dixit	F	2	5	4	4		
14	228	S1819507	Pratik Jirale	F	Ab	ab	2	3		
15	210	S1819511	Preeti Mattiwade	47.00	7	9	10	9		
16	224	S1819512	Prerana Nandane	44.00	7	8	9	8		
17	142	S1819522	Reshma Khot	42.00	6	7	10	9		
18	146	S1819528	Rohini Khot	F	8	10	9	9		
19	226	S1819532	Rupesh Naik	F	4	6	5	7		
20	212	S1819537	Rutuja Narasagoudar	F	3	6	6	5		
21	225	S1819540	Sachin Khot	F	5	7	5	06		
22	150	S1819541	Sachin Nerale	F	5	5	6	5		
23	151	S1819545	Sameer Pisale	F	Ab	2	3	ab		
24	213	S1819546	Sammed Rooge	F	3	5	4	4		
25	152	S1819549	Sandesh Mali	45.00	4	5	6	6		
26	214	S1819551	Santosh Hanji	F	4	5	6	4		
27	157	S1819556	Sharad Patil	44.00	5	7	8	8		
28	173	S1819581	Shubham Kamate	F	5	7	4	5		
29	217	S1819582	Shubham Kasar	F	4	5	6	5		
30	175	S1819585	Shubhangi Jarag	49.00	6	9	10	9		
31	189	S1819613	Vaibhavi Parit	F	6	8	9	9		
32	223	S1819626	Vivek Gadakar	F	4	6	ab	ab		

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DEPARTMENT OF MATHEMATICS

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

B. Sc. V & VI Semester

Criteria: Students with less than 55% are considered as slow learners

Sr. No.	R. No.	Reg. No.	Name of the student	% at Entry	V Sem		VI Sem		% at Exit	Growth
					T_1	T_2	T_1	T_2		
1	01	S1717601	Ayushi Kadam	53.00	5	4	4	8	F	NO
2	02	S1717603	Abhinandan Raygonnavar	41.00	4	2	1	9	F	NO
3	04	S1717607	Aiaswarya Murabatte	46.00	6	4	8	ab	F	NO
4	07	S1717611	Akash Kamble	40.00	3	2	6	8	59.66	YES
5	08	S1717612	Akash Shinde	F	2	2	2	8	F	NO
6	10	S1717615	Amarjeet Shinde	48.00	2	ab	0	8	F	NO
7	14	S1717620	Arun Kothiwale	F	4	ab	ab	8	A	A
8	18	S1717624	Basavraj sanvaganv	F	2	6	4	9	F	NO
9	28	S1717638	Ganghadhar Kone	50.00	3	3	2	9	52.66	YES
10	31	S1717642	Hrishikesh Davare	54.5	6	ab	1	9	F	NO
11	37	S1717653	Komal Badake	53.00	7	6	6	8	56.66	YES
12	57	S1717684	Pooja Chilami	53.4	6	9	5	9	F	NO
13	66	S1717699	Prasad Aralikatti	F	0	ab	ab	8	A	A
14	67	S1717701	Pratiksha Patil	48.00	3	8	6	9	58.33	YES
15	74	S1717715	Rahul Tashildar	F	3	3	4	8	54	YES
16	76	S1717717	Ramkrishna Gudennavar	F	ab	2	2	8	F	NO
17	95	S1717754	Sonali Jain	F	8	8	5	9	54	YES
18	101	S1717763	Sushant Langote	F	2	4	4	8	F	NO
19	110	S1717774	Vidyasagar Chougale	F	3	5	4	8	49.33	YES
20	112	S1717776	Vishal Mokashi	F	4	4	4	9	F	NO

Growth Rate= $\frac{7}{20} \times 100 = 35.00\%$

HOD

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Department of Mathematics

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DEPARTMENT OF BOTANY

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

Criteria: Students with below 60% are consider as slow learners (depends on % scored in class)

Sr. No.	REG.NO	Name of the student	% at Entry	I Sem		II Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	S1919406	Aditya Siddannavar	48.00	7	8	6	7	No Exams Due to Covid-19	
2	S1919449	Kiran Waghamode	46.00	8	7	7	8		
3	S1919455	Laxmi Patil	42.00	7	8	7	6		
4	S1919478	Nikita Arage	55.00	7	8	6	7		
5	S1919479	Nikita Chougule	52.00	7	7	8	8		
6	S1919511	Pritam Chougule	49.00	6	6	7	8		
7	S1919517	Rahul Patil	44.00	7	7	8	6		
8	S1919519	Rajashekhar Mathapati	45.00	8	7	8	7		
9	S1919521	Rayanna Chikkodi	49.00	7	8	7	8		
10	S1919546	Satwik S Desai	48.00	8	7	7	7		
11	S1919607	Tushar Patil	43.00	6	7	6	7		
12	S1919615	Vinaykumar Patil	48.00	8	7	7	8		
13	S1919523	Ritu Patil	49.00	8	7	7	8		
11	S1919568	Siddharth Varale	55.00	7	8	8	8		
12	S1919450	Kiran Walaki	56.00	7	8	7	8		
13	S1919565	Shweta Jadhav	46.00	6	8	7	7		
14	S1919614	Vinayak Naik	47.00	7	8	7	9		
15	S1919533	Rutuja Nare	56.00	8	8	7	8		
16	S1919499	Prajwal Kotiwale	55.00	8	7	7	8		
17	S1919403	Abhishek Ambi	54.00	7	6	7	6		
18	S1919572	Snehal Shende	55.00	7	8	8	8		

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Sr. No.	REG.NO	Name of the student	% at Entry	III Sem		IV Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	S1819404	Adesh Patil	40.00	7	6	7	8	No Exams Due to Covid-19	
2	S1819590	Sneha B Ambi	47.00	8	7	6	7		
3	S1819523	Rishikesh Patil	36.00	6	7	6	7		

Sr. No.	REG.NO	Name of the student	% at Entry	V Sem		VI Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	S1717705	Prithviraj Narayankar	44.00	6	7	8	7	45.00	Yes
2	S1717718	Ramizraja Makandar	45.00	6	7	8	9	47.00	Yes
3	S1717761	Sumit Chougule	40.00	7	8	8	9	65.00	Yes
4	S1717777	Yogesh Pujari	46.00	7	7	7	8	70.00	Yes
5	S1717735	Santosh Adake	42.00	7	8	6	6	38.00	No

Growth rate = 80%

HOD
Head
Department of Botany
K.L.E's G. I. B. College, Nipani.

IQAC-Coordinator
IQAC Co-ordinator
K.L.E's G. I. B. College, Nipani.

Principal
Principal,
G. I. Bagewadi Arts, Science &
Commerce College, NIPANI.

K.L.E's G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE * NIPANI

K. L.E. Society's

G.I. Bagewadi Arts, Science & Commerce College, Nipani

DEPARTMENT OF ZOOLOGY

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

B. Sc. I & II Semester

Criteria: Students with below 60% are consider as slow learners (depends on % scored in class)

Sr. No.	REG.NO	Name of the student	% at Entry	I Sem		II Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	S1919406	Aditya K Siddannavar	48.00	4	5	6	8	No	Exams
2	S1919449	Kiran PrakashWaghamode	46.00	5	4	8	9	Due to	Covid-19
3	S1919455	Laxmi Patil	42.00	7	8	8	7		
4	S1919478	Nikita Arage	55.00	5	8	6	4		
5	S1919479	Nikita Tukaram Chougule	52.00	7	8	8	8		
6	S1919511	Pritam Chougule	49.00	6	4	4	5		
7	S1919517	Rahul Patil	44.00	8	4	8	2		
8	S1919519	Rajashekhar Mathapati	45.00	5	4	8	4		
9	S1919521	Rayanna Chikkodi	49.00	4	5	9	8		
10	S1919546	Satwik S Desai	48.00	9	8	5	4		
11	S1919607	Tushar Patil	43.00	6	8	6	5		
12	S1919615	Vinaykumar Patil	48.00	8	4	5	8		
13	S1919523	Ritu Shiavaji Patil	49.00	5	8	9	4		
11	S1919568	Siddharth Varale	55.00	8	9	6	2		
12	S1919450	Kiran Walaki	56.00	5	8	5	8		
13	S1919565	Shweta Annasaheb Jadhav	46.00	6	9	7	8		
14	S1919614	Vinayak S Naik	47.00	5	6	5	4		
15	S1919533	Rutuja Mahadev Nare	56.00	8	5	6	4		
16	S1919499	Prajwal Kotiwale	55.00	9	8	5	4		

Sr. No.	REG.NO	Name of the student	% at Entry	III Sem		IV Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	S1819404	Adesh Patil	45.00	7	6	7	8	No	Exams
2	S1819590	Sneha B Ambi	50.00	8	7	6	7	Due to	Covid-19
3	S1819523	Rishikesh Patil	45.00	6	7	6	7		

Sr. No.	REG.NO	Name of the student	% at Entry	V Sem		VI Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	S1717705	Prithviraj Narayankar	45.00	6	7	8	7	47.00	YES
2	S1717718	Ramizraja Makandar	48.00	6	7	8	9	50.00	YES
3	S1717761	Sumit Chougule	42.00	7	8	8	9	55.00	YES
4	S1717777	Yogesh Pujari	55.00	7	7	7	8	70.00	YES
5	S1717735	Santosh Adake	49.00	7	8	6	6	38.00	NO

Growth rate = 80%

HOD
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Department of Zoology
K.L.E's G. I. B. College, Nipani

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IQAC Co-ordinator
K.L.E's G. I. B. College, Nipani.

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DEPARTMENT OF COMMERCE

PROGRAMMES ARRANGED FOR SLOW LEARNERS

At the beginning of the academic year we prepare the list of slow learners by considering their previous semester result, usually below 60% we consider as slow learners. For those students we conduct following programmes once or twice in fifteen days.

- Extra classes
- Practicing important questions
- Advanced learners help them in solving old question papers
- Counseling
- Extra assignments
- Providing them solved question banks
- In case of less attendance ask them to bring parents
- During class hours, encourage them to solve examples on board.
- According to time table we will engage classes, counsel students to practice examples, solve problems etc. once in 15 days or according to students convenient.

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Department of Commerce
Slow Learners
Time Table for the year 2019-20

Days	Class	2 – 3 pm	3 - 4 pm
Monday	B.Com II Sem	Financial Accounting II	
	B.Com IV Sem	-	Corporate Accounting II
	B.Com VI Sem	-	Cost Accounting II
Tuesday	B.Com II Sem	-	-
	B.Com IV Sem	-	Business Stat / Commercial Arithmetic
	B.Com VI Sem	-	-
Wednesday	B.Com II Sem	-	-
	B.Com IV Sem	-	Financial Management
	B.Com VI Sem	Income Tax II	-
Thursday	B.Com II Sem	-	-
	B.Com IV Sem	-	-
	B.Com VI Sem	-	Goods and Service Tax II
Friday	B.Com II Sem	-	-
	B.Com IV Sem	-	-
	B.Com VI Sem	-	-
Saturday	B.Com II Sem	-	-
	B.Com IV Sem	-	-
	B.Com VI Sem	-	-

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Department of Commerce

Slow Learners

Time Table for the year 2019-20

Days	Class	2 – 3 pm	3 - 4 pm
Monday	B.Com I Sem	Financial Accounting I	-
	B.Com III Sem	-	Corporate Accounting I
	B.Com V Sem	-	Income Tax I
Tuesday	B.Com I Sem	Secretarial Practice	
	B.Com III Sem	-	Business Stat / Commercial Arithmetic
	B.Com V Sem	-	Management Accounting
Wednesday	B.Com I Sem	-	-
	B.Com III Sem	-	-
	B.Com V Sem	-	Cost Accounting I
Thursday	B.Com I Sem	-	-
	B.Com III Sem	-	-
	B.Com V Sem	-	Goods and Service Tax I
Friday	B.Com I Sem	-	-
	B.Com III Sem	-	-
	B.Com V Sem	-	-
Saturday	B.Com I Sem	-	-
	B.Com III Sem	-	-
	B.Com V Sem	-	-

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Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116

DEPARTMENT OF COMMERCE

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

B. Com I & II Semester

Sr. No.	Register No.	*Name of the student*	% at Entry (PUC II)	I Sem		II Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	C1930640	Arjun Kamble	47.53	6	7	6	7	77.29	Yes
2	C1930628	Deepali Chougule	59.33	7	8	7	8	84.00	Yes
3	C1930641	Karishma A Borgave	47.00	7	8	7	8	81.14	Yes
4	C1930653	Mayuri Mathapati	51.07	8	7	8	8	82.29	Yes
5	C1930656	Nandini Shimpukude	53.00	7	8	7	7	77.29	Yes
6	C1930658	Nikhil Umaje	53.16	7	6	7	7	72.43	Yes
7	C1930659	Nisha Bagi	59.33	7	8	8	7	78.57	Yes
8	C1930661	Om Chandrakude	57.33	6	7	7	8	79.57	Yes
9	C1930674	Praveen P Gourai	55.00	5	6	7	6	67.86	Yes
10	C1930678	Reshma Kamble	57.83	6	7	8	9	82.71	Yes
11	C1930689	Samarth Kamble	51.33	6	7	8	8	79.00	Yes
12	C1930698	Shashikala S Vibhute	57.66	7	8	8	9	85.00	Yes
13	C1930707	Shubham Kamate	51.00	7	6	7	7	65.71	Yes
14	C1930732	Vinayak Kurani	54.50	7	8	7	9	83.00	Yes
15	C1930654	Mayur Kalisinge	57.00	7	8	7	9	75.00	Yes
16	C1930712	Sourabh Patravali	48.33	8	8	7	8	79.29	Yes

Growth Rate = 16/16 * 100 = 100%

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DEPARTMENT OF COMMERCE

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20
B. Com III & IV Semester

Sr. No.	Register No.	*Name of the student*	% at Entry	III Sem		IV Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	C1830201	Abhishek tandale	F	6	6	6	7	72.37	Yes
2	C1830204	Aishwarya Jabade	F	7	8	8	9	85.37	Yes
3	C1830205	Akshay Patil	F	6	7	8	8	77.50	Yes
4	C1830212	Asmita Magadum	F	8	7	8	9	80.62	Yes
5	C1830215	Dadu Khot	F	8	8	7	9	80.00	Yes
6	C1830218	Guruprasad Kalachandra	F	6	7	8	7	69.12	Yes
7	C1830219	Jyoti Vharate	F	8	8	9	9	85.50	Yes
8	C1830230	Mayuri Khavare	F	6	8	7	8	78.00	Yes
9	C1830232	Nandini Patil	F	9	9	8	9	88.00	Yes
10	C1830233	Nikunj Potadar	F	7	8	8	9	85.25	Yes
11	C1830234	Omkar Patil	F	9	10	9	10	90.87	Yes
12	C1830236	Pooja Shreyakar	F	8	9	8	9	84.50	Yes
13	C1830237	Pooja Chougule	F	9	9	8	9	86.37	Yes
14	C1830240	Prabhavati Patil	F	8	9	8	9	84.00	Yes
15	C1830241	Pradnya Kamble	F	8	7	8	9	78.25	Yes
16	C1830242	Pradnya Magadum	F	8	8	7	9	83.62	Yes
17	C1830244	Prathamesh Modi	F	8	7	9	8	81.25	Yes
18	C1830245	Pratik Patil	F	8	6	7	8	72.37	Yes
19	C1830247	Praveen Sadalage	F	8	7	7	9	82.37	Yes

K.L.E.S G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

20	C1830250	Priyanka Heggannnavar	F	8	9	8	9	85.75	Yes
21	C1830255	Robin Sattigeri	F	6	5	7	8	63.62	Yes
22	C1830256	Rohan Kumbhar	F	7	6	7	6	69.00	Yes
23	C1830259	Rutuja G Narawade	F	8	9	8	10	86.00	Yes
24	C1830260	Rutuja K Patil	F	9	8	9	10	84.62	Yes
25	C1830262	Rutuja T Patil	F	9	9	9	9	90.62	Yes
26	C1830263	Saiprem Patil	F	7	8	8	7	70.81	Yes
27	C1830269	Shivani Ammanagi	F	9	8	7	9	85.25	Yes
28	C1830276	Soumya Ghosarwade	F	8	9	8	9	83.62	Yes
[illegible]9	C1830277	Srushti Desai	F	8	7	8	7	79.00	Yes
30	C1830279	Sushma Khot	F	7	8	8	9	80.00	Yes
31	C1830283	Vinashri Gurav	F	9	9	10	9	85.62	Yes
32	C1830285	Vidya Kondekar	F	7	8	7	7	77.37	Yes
33	C1830287	Vinayak Ingale	F	8	8	9	9	84.12	Yes

Growth Rate = 33/33 * 100 =100 %

HOD
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Department of Commerce
K.L.E's G. I. B. College, Nipani.

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DEPARTMENT OF COMMERCE

Academic growth of slow learners after extra tests, assignments and guidance for the year 2019-20

B. Com V & VI Semester

Sr. No.	Register No.	*Name of the student*	% at Entry	V Sem		VI Sem		% at Exit	Growth
				T_1	T_2	T_1	T_2		
1	C1730204	Akshay Babannavar	F	5	4	5	6	F	No
2	C1730207	Amit Sannaik	F	5	4	4	5	57.89	Yes
3	C1730236	Mohammad Bilal Pathan	F	4	5	5	5	F	No
4	C1730237	Mohammad Bagban	F	5	4	5	5	50.68	Yes
5	C1730239	Niranjan Chavan	F	4	5	4	4	F	No
6	C1730240	Niranjan Patil	F	5	4	5	5	F	No
7	C1730255	Pratik Dandale	F	5	5	4	5	F	No
8	C1730267	Sachin Badadavar	49.71	5	6	6	5	F	No
9	C1730274	Satish Shirole	F	5	5	6	5	F	No
10	C1730278	Shraddha Patil	F	4	6	5	6	67.65	Yes
11	C1730279	Shubham Kamagouda	F	5	4	4	5	F	No
12	C1730280	Shubham Puthane	F	6	5	7	7	64.35	Yes
13	C1730282	Snehal Chougule	F	5	4	5	5	F	No
14	C1730300	Vaishnav Pol	59.14	7	8	7	8	61.22	Yes

Growth Rate = 05/14 * 100 = 35.71%

K.L.E'S G.I.BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

HOD

Head
Department of Commerce
K.L.E's G. I. B. College, Nipani.

K. L. E Society's
G. I. Bagewadi Arts, Science & Commerce College, Nipani
Department of Economics

Students List of Slow Learners for the year 2019-20
B.A I & II Semester

Sl. No	Name of the Students	% at Entry PU	% at Exit II Sem I Test	II Test	%	Growth
1	Abhijeet Patil	45.00	3	6		
2	Akshata Hegade	40.00	3	6		
3	Manjunath Kode	43.00	4	6		
4	Naveen Nasalapure	43.00	4	8	Promoted	
5	Shivaji Dhanagar	56.00	4	8		
6	Shivaji Divate	45.00	3	6		
7	Udaya Bilikudare	58.00	4	8		
8	Varsharani Sankapal	53.00	4	8		

Maintenance of Growth =

Students List of Slow Learners for the year 2019-20
B.A III & IV Semester

Sl. No	Name of the Students	% at Entry III Sem	% at Exit IV Sem I Test	II Test	%	Growth
1	Deepali Koot	43.00	3	6	Promoted	

Maintenance of Growth =

Students List of Slow Learners for the year 2019-20
B.A V & VI Semester

Sl. No	Name of the Students	% at Entry V Sem	% at Exit VI Sem I Test	II Test	%	Growth
1	Nameera Inamdar	F	4	8	73.5	100
2	Aarati Khot	45.00	4	8	Ab	-

Maintenance of Growth = 1/1x100=100

H.O.D.
Dept. of Economics
G.I. Bagewadi College, Nipani

K.L.E's G.I. BAGEWADI ARTS, SCI. & COM. COLLEGE NIPANI

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Ref. No. *Date :*

ii. Practice Test

K. L. E. Society's

G. I Bagewadi Arts, Science and Commerce college Nipani.

Department of mathematics

PRACTICE TEST - I

for the year 2019-20

8/10

class :- Bsc I[st] sem

Roll no. :- 137

Date:- 26/10/19

1) state and prove Leibnitz's test for n^{th} derivative of product.

⇒ Statement :- If $y = uv$, where u & v are functions of 'x' having derivatives of n^{th} order then,

$$y_n = {}^nc_0 u_n v + {}^nc_1 u_{n-1} v_1 + {}^nc_2 u_{n-2} v_2 + \cdots + {}^nc_n u v_n,$$

where, suffixes of u & v denotes the number of times they are differentiated & ${}^nc_r$ denoted the number of combination of 'n' different things taken 'r' at a time.

Proof :- We prove this result by using mathematical induction.

i) Let $y = uv$

$$y_1 = u_1 v + u v_1$$

$$y_1 = {}^1c_0 u_1 v + {}^1c_1 u v_1$$

∴ The theorem is true for $n=1$

ii) Let's assume that the theorem is true for $n=m$.

i.e. $y_m = {}^mc_0 u_m v + {}^mc_1 u_{m-1} v_1 + {}^mc_2 u_{m-2} v_2 + \cdots + {}^mc_m u v_m$ —①

iii) Diff. eq^n ① on both side.

$$y_{m+1} = {}^mc_0 [u_{m+1} v + u_m v_1] + {}^mc_1 [u_m v_1 + u_{m-1} v_2] + {}^mc_2 [u_{m-1} v_2 + u_{m-2} v_3] + \cdots + {}^mc_m [u_1 v_m + u v_{m+1}]$$

$$= {}^mc_0 u_{m+1} v + ({}^mc_0 + {}^mc_1) u_m v_1 + ({}^mc_1 + {}^mc_2) u_{m-1} v_2 + ({}^mc_2 + {}^mc_3) u_{m-2} v_3 + \cdots + ({}^mc_{n-1} + {}^mc_m) u_1 v_m + {}^mc_m u v_{m+1}$$

We have $^mC_0 = 1 = {}^{m+1}C_0 = {}^mC_m = {}^{m+1}C_{m+1}$

$\because$ $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^mC_0 + {}^mC_1 = {}^{m+1}C_1 + {}^mC_1 + {}^mC_2 = {}^{m+1}C_2$ & so on.

Substituting this value in the above result,

$$y_{m+1} = {}^{m+1}C_0 u_{m+1} v + {}^{m+1}C_1 u_m v_1 + {}^{m+1}C_2 u_{m-1} v_2 + {}^{m+1}C_3 u_{m-2} v_3 + \cdots + {}^{m+1}C_{m+1} u v_{m+1}$$

$\therefore$ The theorem is true for $n = m+1$

$\therefore$ By mathematical induction the theorem is true for every +ve integer n;

2] State and prove Cauchy's theorem.

$\Rightarrow$ Statement :- If $f(x)$ and $g(x)$ are

① Continuous in $[a,b]$ ② Differentiable in (a,b)

③ $g'(x) \neq 0$ for some $x \in (a,b)$ then $\exists$ atleast one point $c \in (a,b)$ s.t.

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

Proof :- Let's consider the auxilary fu^n $F(x) = f(x) + Ag(x)$ — ①

where 'A' is a constant to be determined s.t. $F(a) = F(b)$

$$\Rightarrow f(a) + A g(a) = f(b) + A g(b)$$

$$-Ag(b) + Ag(a) = f(b) - f(a)$$

$$-A\, g(b) - g(a) = f(b) - f(a)$$

$$-A = \frac{f(b)-f(a)}{g(b)-g(a)} \quad \text{— ②}$$

Since both $f(x)$ & $g(x)$ are continuous in $[a,b]$ & differentiable in (a,b). $F(x)$ is also continuous in $[a,b]$. & differentiable in (a,b).

Also, $F(a) = F(b)$

$\therefore$ $F(x)$ satisfies all the conditions of Roll's theorem

$\exists$ $c \in (a,b)$ s.t. $F'(c) = 0$

from eq^n ①, $F'(x) = f'(x) + A g'(x)$

$$\therefore F'(c) = 0$$

$$f'(c) + A g'(c) = 0$$

$$\Rightarrow -A = \frac{f'(c)}{g'(c)} \quad \text{— ③}$$

from ② & ③,

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)} \quad \text{for some } c \in (a,b)$$

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Department of Mathematics
PRACTICE TEST - II
For the year 2019-20

7/10

Class :- BSc I sem
Date:-
Roll no :- 137

1) If Δ' is reciprocal of determinant Δ of order 4, then prove that $\Delta' = \Delta^3$

$\Rightarrow$ $$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

The reciprocal determinant,
$$\Delta' = \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix}$$

Consider, $\Delta\Delta'$
$$\Delta\Delta' = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} a_{11}A_{11}+a_{12}A_{12}+\cdots+a_{14}A_{14} & a_{12}A_{21}+a_{12}A_{22}+\cdots+a_{14}A_{24} & \cdots & a_{11}A_{41}+\cdots+a_{14}A_{44} \\ a_{21}A_{11}+a_{22}A_{12}+\cdots+a_{24}A_{14} & a_{21}A_{21}+a_{22}A_{22}+\cdots+a_{24}A_{24} & \cdots & a_{21}A_{41}+\cdots+a_{24}A_{44} \\ a_{31}A_{11}+a_{32}A_{12}+\cdots+a_{34}A_{14} & a_{31}A_{21}+a_{32}A_{22}+\cdots+a_{34}A_{34} & \cdots & a_{31}A_{41}+\cdots+a_{34}A_{44} \\ a_{41}A_{11}+a_{42}A_{12}+\cdots+A_{44}A_{14} & a_{41}A_{21}+a_{42}A_{22}+\cdots+a_{44}A_{34} & \cdots & a_{41}A_{41}+\cdots+a_{44}A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} \Delta & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & \Delta \end{vmatrix} = \Delta^4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$\Delta\Delta' = \Delta^4 . 1$

$\Delta' = \Delta^4 . \Delta^{-1} = \Delta^{4-1}$

$\boxed{\Delta' = \Delta^3}$

2) Expand $\sin^5\theta$ interms of sines of multiple of θ.

$\Rightarrow$ We have, $\sin\theta = \frac{1}{2i}\left[z - \frac{1}{z}\right]$

$$\sin^5\theta = \left[\frac{1}{2i}\left(z - \frac{1}{z}\right)\right]^5$$

$$= \left(\frac{1}{2i}\right)^5 \left(z - \frac{1}{z}\right)^5$$

$$\sin^5\theta = \frac{1}{32i}\left(z - \frac{1}{z}\right)^5$$

Binomial theorem,

$$\sin^5\theta = \frac{1}{32i}\left[5c_0 z^5 - 5c_1 z^4 \frac{1}{z} + 5c_2 z^3 \frac{1}{z^2} - 5c_3 z^2 \frac{1}{z^3} + 5c_4 z \cdot \frac{1}{z^4} - 5c_5 \frac{1}{z^5}\right]$$

$5c_0 = 5c_5 = 1$, $5c_1 = 5c_4 = 5$, $5c_2 = 5c_3 = 10$

$$\sin^5\theta = \frac{1}{32i}\left[z^5 - 5z^3 + 10z - 10\frac{1}{z} + 5\frac{1}{z^3} - \frac{1}{z^5}\right]$$

$$= \frac{1}{32i}\left[\left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)\right]$$

$$= \frac{1}{32i}\left[2i\sin5\theta - 5(2i\sin3\theta) + 10(2i\sin\theta)\right]$$

$$\sin^5\theta = \frac{1}{16}\left[\sin5\theta - 5\sin3\theta + 10\sin\theta\right]$$

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Date

KLE·Societys

G. I. Bogewadi Arts, Science & Commerce College, Nipani

Department of Mathematics

For the year 2019-20

Roll No:- 37 Marks obtained:- 1/10

Class - BSc I sem Date Max Marks: 10

Answer the any two of the following

1) State and prove Leibnitz's test for n^{th} derivative of product?

→ Statement :- If $y = uv$ where u & v are function of x having derivatives of n^{th} order then

$y_n = {}^nC_0 u_n v_0 + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots + {}^nC_n u v_n$

where suffixes of uv denot the number of time there differentiated and ${}^nC_r$ denotes the number of combinations of n different things taken r at a time

proof:- We prove that result by using mathematical induction

case i) let $y = uv$

$y_1 = u\frac{dv}{dt} + v\frac{du}{dt}$

$y_1 = u_1 v + u v_1$

$y_1 = {}^1C_0 u_1 v + {}^nC_1 u v_1$ $[\because {}^1C_0 = 1 = {}^1C_1]$

The theorem is true for $n = 1$

case ii) let us assume that the theorem is true for $n = m$

$y_m = {}^mC_0 u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \dots + {}^mC_m u v_m$ — (1)

case iii) differentiate equation (1) on both side we get

$y_{m+1} = {}^mC_0 [u_{m+1} v + u_m v_1] + {}^mC_1 [u_m v_1 + u_{m-1} v_2] + {}^mC_2 [u_{m-1} v_2 + u_{m-2} v_3] + \dots + {}^mC_m [v_m u_1 + u v_{m+1}]$

$y_{m+1} = {}^mC_0 u_{m+1} v + ({}^mC_0 + {}^mC_1) u_m v_1 + ({}^mC_1 + {}^mC_2) u_{m-1} v_2 + ({}^mC_2 + {}^mC_3) v_{m-2} v_3 + \dots + ({}^mC_{m-1} + {}^mC_m) u_1 v_m + {}^mC_m u v_{m+1}$

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∴ we have $^mC_0 = 1 = {}^{m+1}C_0 = {}^mC_m = {}^{m+1}C_{m+1}$

Since $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^mC_0 + {}^mC_1 = {}^{m+1}C_1$ &

$^mC_1 + {}^mC_2 = {}^{m+1}C_2$ & so on

Substituting these value in the above result we get

$$y_{m+1} = {}^{m+1}C_0\, u_{m+1}V + {}^{m+1}C_1\, u_m V_1 + {}^{m+1}C_2\, u_{m-1}V_2 + {}^{m+1}C_3\, u_{m-2}V_3 + \cdots + {}^{m+1}C_{m+1}\, U V_{m+1}$$

∴ The theorem is true for $n = m+1$

∴ By mathematical induction the theorem is true for every positive integer n.

2) Expand $\tan x$ by maclaurin's Theorem up to the power containing x^5

→ Giving maclaurin's series

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \frac{f^{v}(0)x^5}{5!} + \cdots$$

Let $f(x) = \tan x$

$f'(x) = \sec^2 x$

$f''(x) = 2\sec x(\sec x \tan x)$

$= 2\sec^2 x \tan x$

$f'''(x) = 2(\sec^2 x \sec^2 x + \tan x\, 2\sec^2 x \tan x)$

$= 2\sec^4 x + 4\sec^2 x \tan^2 x$

$f^{iv}(x) = 2 \times 4\sec^3 x(\sec x \tan x) + 4[\sec^2 x\, 2\tan x \sec^2 x + \tan^2 x\, 2\sec x(\sec x \tan x)]$

$f^{iv}(x) = 8\sec^4 x \tan x + 8\sec^4 x \tan x + 8\sec^2 x \tan^3 x$

$f^{v}(x) = 8[\sec^4 x \sec^2 x + \tan x\, 4\sec^3 x(\sec x \tan x)]$
$+ 8[\sec^4 x \sec^2 x + \tan x\, 4\sec^3 x(\sec x \tan x)]$
$+ 8[\sec^2 x\, 3\tan^2 x \sec^2 x + \tan^3 x\, 2\sec x(\sec x \tan x)]$

$f^{v}(x) = 8[\sec^6 x + 4\sec^4 x \tan^2 x] + 8[\sec^6 x + 4\sec^4 x \tan^2 x] + 24\sec^4 x \tan^2 x + 16\tan^4 x \sec^2 x]$

$f^{v}(x) = 16[\sec^6 x + 4\sec^4 x \tan^2 x] + 24\sec^4 x \tan^2 x + 16\tan^4 x \sec^2 x$

$f^{v}(0) = 16[1+0] + 24(0) + 16(0)$

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$f^{V}(0) = 16$

$f^{IV}(0) = 0$

$f'''(0) = 2$

$f''(0) = 0$

$f'(0) = 1$

∴ maclaurin's series

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \frac{f'''(0)}{3!}x^3 + f^{IV}(0)\frac{x^4}{4!} + f^{V}(0)\frac{x^5}{5!}$$

$$\tan x = 0 + x + 0 + \frac{2}{3\times2\times1}x^3 + 0 + \frac{16}{54321}x^5$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5$$

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G I Bagewadi Arts Science & Commerce college, Nipani

Department of mathematics

for the year 2019-20

5/10

Roll No:- 37 Marks obtained:-

class:- BSc I Sem Date Max Marks: 10

Answer the any two of the following

1) If Δ' is reciprocal of determinant Δ of order 4 then prove that $\Delta' = \Delta^3$

→ $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$ be a determinant of order 4

$\Delta' = \begin{vmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{vmatrix}$

$\Delta\Delta' = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} \begin{vmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{vmatrix}$

$\Delta\Delta' = \begin{vmatrix} a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}+a_{14}A_{14} & a_{11}A_{21}+\dots+a_{14}A_{24} & a_{11}A_{31}+\dots+a_{14}A_{34} & a_{11}A_{41}+\dots+a_{14}A_{44} \\ a_{21}A_{11}+\dots+a_{24}A_{14} & a_{21}A_{21}+\dots+a_{24}A_{24} & a_{21}A_{31}+\dots+a_{24}A_{34} & a_{21}A_{41}+\dots+a_{24}A_{44} \\ a_{31}A_{13}+\dots+a_{34}A_{14} & a_{31}A_{21}+\dots+a_{34}A_{24} & a_{31}A_{31}+\dots+a_{34}A_{34} & a_{31}A_{41}+\dots+a_{34}A_{44} \\ a_{41}A_{11}+\dots+a_{44}A_{14} & a_{41}A_{21}+\dots+a_{44}A_{24} & a_{41}A_{31}+\dots+a_{44}A_{34} & a_{41}A_{41}+\dots+a_{44}A_{44} \end{vmatrix}$

$\Delta\Delta' = \begin{vmatrix} \Delta & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & \Delta \end{vmatrix}$

$$\Delta\Delta' = \Delta^4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\Delta\Delta' = \Delta^4 [I]$$

$$\Delta\Delta' = \Delta^4$$

dividing both the side Δ

$$\Delta' = \frac{\Delta^4}{\Delta}$$

$$\boxed{\Delta' = \Delta^3}$$

2) Expand $\sin^5\theta$ in terms of sines of multiples of θ

→ We have $\sin\theta = \frac{1}{2i}\left[z - \frac{1}{z}\right]$

$$\Rightarrow \sin^5\theta = \left[\frac{1}{2i}\left[z - \frac{1}{z}\right]\right]^5$$

$$= \left(\frac{1}{2i}\right)^5 \left(z - \frac{1}{z}\right)^5$$

$$\sin^5\theta = \frac{1}{32i}\left[z - \frac{1}{z}\right]^5$$

$$= \frac{1}{32i}\left[{}^5C_0 z^5 - {}^5C_1 z^4 \frac{1}{z} + {}^5C_2 z^3 \frac{1}{z^2} - {}^5C_3 z^2 \frac{1}{z^3} + {}^5C_4 z \frac{1}{z^4} - {}^5C_5 \frac{1}{z^5}\right]$$

$${}^5C_0 = {}^5C_5 = 1$$

$${}^5C_1 = {}^5C_4 = 5$$

$${}^5C_2 = {}^5C_3 = 10$$

$$= \frac{1}{32i}\left[z^5 - 5z^3 + 10z - 10\frac{1}{z} + \frac{5}{z^3} - \frac{1}{z^5}\right]$$

$$= \frac{1}{32i}\left[\left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)\right]$$

$$= \frac{1}{32i}\left[2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)\right]$$

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$$= \frac{2i}{32i}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

$$= \frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

$$\sin^5\theta = \frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

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K.L.E. Society's
G.I. Bagewadi Arts, Science And Commerce College Nipani
Department of mathematics
Practice Test - II
For the year : 2019-20

4/10

Roll No: 140
Class : B.Sc. I[st] Sem   Date:- 26-10-2019

1) Prove that a set $N \times N$ is countable where N is set of naturals.

⟹ proof: $N = \{1, 2, 3 \ldots\}$

$N \times N = \{(a,b)\ a, b \in N\}$

$N \times N = \{(1,1)(1,2)(1,3) \ldots$
$(2,1)(2,2)(2,3) \ldots$
$(3,1)(3,2)(3,2) \ldots\}$

$A_1 = \{(1,1)(1,2)(1,3) \ldots\}$
$A_2 = \{(2,1)(2,2)(2,3) \ldots\}$
$A_3 = \{(3,1)(3,2)(3,3) \ldots\}$

— — — — — —

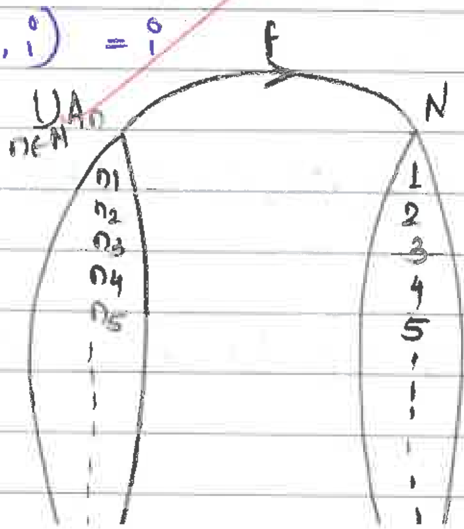
$A_n = \{(n,1)(n,2)(n,3) \ldots\}$

— — — — — —

clearly $N \times N = \bigcup_{n \in N} A_n$

define $f : \bigcup_{n \in N} A_n \to N$ define by

$f(n, i) = i$



⟹ From Venn Diagram it is clear that f is bijective

$\Rightarrow \bigcup_{n \in N} A_n \sim N$

$\Rightarrow N \times N = \bigcup_{n \in N} A_n \sim N$

$\Rightarrow N \times N$ is denumerable

$\Rightarrow N \times N$ is Countable.

2) Expand $\sin^5\theta$ in terms of Sines of multiples of θ.

⟹ Solution:- we have $\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$

$\Rightarrow \sin\theta = \left[\frac{1}{2i}\left(z - \frac{1}{z}\right)\right]^5 = \left(\frac{1}{2i}\right)^5\left(z - \frac{1}{z}\right)^2$

$= \frac{1}{32i}\left(z - \frac{1}{z}\right)^5$

$= \frac{1}{32i}\left[{}^5C_0 z^5 - {}^5C_1 z^4 \frac{1}{z} + {}^5C_2 z^3 \frac{1}{z^2} - {}^5C_3 z^2 \frac{1}{z^3} + {}^5C_4 z \frac{1}{z^4} - {}^5C_5 \frac{1}{z^5}\right]$

$= {}^5C_0 = {}^5C_5 = 1$

${}^5C_1 = {}^5C_4 = 5$

${}^5C_2 = {}^5C_3 = 10$

$= \frac{1}{32i}\left[z^5 - 5z^3 + 10z - 10\frac{1}{z} + 5\frac{1}{z^3} - \frac{1}{z^6}\right]$

$= \frac{1}{32i}\left[\left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)\right]$

$= \frac{1}{32i}\left[2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)\right]$

$\sin^5\theta = \frac{1}{16}\left[\sin 5\theta - 5\sin^3\theta + 10\sin\theta\right]$

CLASS | SUBJECT
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K.L.E. Society's
G. I. Bagewadi Arts, Science And Commerce College Nipani.
Department of Mathematics

Practice Test - I
For the year 2019-20

4/10

Class :- B.Sc Ist sem Date: 26-10-2019
Roll No :- 145

Answer the following questions:

1) State and prove Leibnitz's tests for n^{th} derivative of product.

⟹ Statement: If $y = uv$, where u and v are functions of x having derivatives of n^{th} order then
$y_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + --- {}^nC_n u v_n$
where suffixes are u & v denate the number of times they are differentiated and ${}^nC_r$ denates the number of combinations of n different things taken 'r' at a time.

Proof: We prove this result by using mathematical induction

case i) let $y = uv$
$\Longrightarrow y_1 = u_1 v + v_1 u$
$= {}^1C_0 u_1 v + {}^1C_1 u v_1$ $\{ \because {}^1C_0 = 1 = {}^1C_1$
$\therefore$ The Theorem is true for $n = 1$.

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Case ii) Let us assume that the theorem is true for $n=m$

i.e $y_m = {}^mC_0 u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \dots + {}^mC_m u v_m$ —①

Case iii) : Differentiate eq^n ① on both sides

$y_{m+1} = {}^mC_0 [u_{m+1} v + u_m v_1] + {}^mC_1 [u_m v_1 + u_{m-1} v_2] + {}^mC_2 [u_{m-1} v_2 + u_{m-2} v_3] + \dots + {}^mC_m [u_1 v_m + u v_{m+1}]$

$= {}^mC_0 u_{m+1} v + ({}^mC_0 + {}^mC_1) u_m v_1 + ({}^mC_1 + {}^mC_2) u_{m-1} v_2 + ({}^mC_2 + {}^mC_3) u_{m-2} v_3 + \dots ({}^mC_{m-1} + {}^mC_m) u_1 v_m + {}^mC_m u v_{m+1}$

[we have ${}^mC_0 = 1 = {}^{m+1}C_0 = {}^mC_m = {}^{m+1}C_{m+1}$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^mC_0 + {}^mC_1 = {}^{m+1}C_1$ & ${}^mC_1 + {}^mC_2 = {}^{m+1}C_2$ & so on]

Substituting these value in the above results we get

$y_{m+1} = {}^{m+1}C_0 u_{m+1} v + ({}^{m+1}C_1 u_m v_1) + ({}^{m+1}C_2 u_{m-1} v_2) + {}^{m+1}C_3 u_{m-2} v_3 + \dots {}^{m+1}C_{m+1} u v_{m+1}$

$\therefore$ The Theorem is true for $n = m+1$

$\therefore$ by Mathematical Induction the theorem is true for every positive integer n.//

CLASS	SUBJECT
ROLL NO.	DATE

2) Explain $\tan x$ by Maclaurin's Theorem up to the power containing x^5.

⟹ Sol^n:- let $y = \tan x$

$y = f(x) = \tan x \implies f(0) = 0$

$f'(x) = \sec^2 x \implies f'(0) = 1$

$f''(x) = 2\sec^2 x \tan x \implies f''(0) = 0$

$f'''(x) = 2\sec^2 x \cdot \sec^2 x + 2\tan x \cdot \sec^2 x \tan x$

$\implies f'''(0) = 2$

$f^{iv}(x) = 2(4\sec^3 x \cdot \sec x \tan x) + 2(2\sec^2 x + 2\tan x \sec^2 x + \tan^2 x \cdot 4\sec x \tan x$

$= 0 + 0 + 0 + 0 \implies f^{iv}(0) = 0$

$f^{iv}(x) = 8\sec^2 x \cdot \tan x + 4\sec^2 x + 2\sec^2 x \tan x + 4\sec x \tan^3 x$

$f^{v}(x) = 8[\sec^2 x \cdot \sec^2 x + 2\tan^2 x \sec^2 x] + 8\sec^2 x \cdot \tan x + 2[\sec^4 x + 2\tan^2 x \sec^2 x] + 4[\sec^3 x \cdot 3\tan x \cdot + \tan^4 x \cdot \sec x]$

$f^{v}(0) = 8[1+0] + 8[0] + 2[1] + 4[0]$

$= 8 + 0 + 2 + 0 = 10$

$f^{v}(0) = 10$

∴ We have Maclaurin's Series

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} f^{v}(0) + \ldots$$

→ $\tan x = 0 + x(1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(0) + \frac{x^5}{5!}(10)$

$$= x + \frac{x^3}{3} + \frac{x^4}{6} + \frac{x^5}{12}$$

Kle Society's
G.I. Bagewadi Arts, Science and Commerce College Nipani
Department of Mathematics

Practice test - I for the year 2019-20

Class - BSc I sem Date - 26/10/2019

Roll No - 199

3/10

1) State and prove Leibnitz's test for nth derivative of product.

→ Statement :- If $y = uv$ where u & v are function of x having derivatives of n^{th} order then

$y_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots + {}^nC_n u_0 v_n$

where suffees of u, v denot the number of time there differentiated and ${}^nC_r$ denotes the number of combinations of n different things taken 'r' at a time.

proof - we prove this result by using mathematical induction case i) let $y = uv$

$\implies y_1 = u \frac{dv}{dt} + dv \frac{dy}{dt}$

$y_1 = u_1 v + u v_1$

$y_1 = {}^1C_0 u_1 v + {}^1C_1 u v_1 \quad [\because {}^1C_0 = 1 = {}^1C_1]$

∴ The Theorem is true for $n=1$

Case-II) let us assume that the theorem is true for $n=m$

ie $y_m = {}^mC_0 u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \dots + {}^mC_m u v_m$ —(1)

case-III - differentiate equation (1) in both the side

$y_{m+1} = {}^mC_0 [u_{m+1} v + u_m v_1] + {}^mC_1 [u_m v_1 + v_{m+1} v_2] + {}^mC_m$
$[u_m v_1 + u v_{m+1}] + {}^mC_m [v_m u_1 + u v_{m+1}]$

$y_{m+1} = {}^mC_0 u_{m+1} v + ({}^mC_0 + {}^mC_1) u_m v_1 + ({}^mC_1 + {}^mC_2)$
$u_m + v_2 + ({}^mC_2 + {}^mC_3) u_{m-2} v_3 + \dots + ({}^mC_{m-1} + {}^mC_m)$
$u_2 v_m + {}^mC_m u v_{m+1})$

we have ${}^mC_0 = 1 = {}^{m+1}C_0 = {}^mC_m = {}^{m+1}C_{m+1}$

Since ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r = {}^mC_0 + {}^mC_1 = {}^{m+1}C_1$ &

${}^mC_1 + {}^mC_2 \Rightarrow {}^{m+1}C_2$ & so on

Substituting these value in the above result we get

$y_{m+1} = {}^{m+1}C_0 u_{m+1} v + {}^{m+1}C_1 u_m v_1 + {}^{m+1}C_2 u_{m-1} v_2 + {}^{m+1}C_3 u_{m-2}$
$v_3 + \dots + {}^{m+2}C_{m+1} u v_{m+1}$

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∴ The theorem is true of $n = m+1$

∴ By mathematical induction the theorem

∴ is true for every positive integer 'n'

2) Expand $\tan x$ by Maclaurin's Theorem up to the powers containing x^5

⟹ Given maclaurin's series :

$$F(x) = f(0) + F'(0)x + \frac{F''(0)}{2!}x^2 + \frac{F'''(0)}{3!}x^3 + \frac{F^{IV}(0)}{4!}x^4 + \dots$$

let $f(x) = \tan x$

$F'(x) = \sec^2 x$. $F''(x) = 2\sec x\,(\sec x \cdot \tan x)$

$= 2\sec^2 x \tan x$

$F'''(x) = 2(\sec^2 x \cdot \sec^2 x + \tan x \; 2\sec^2 x \cdot \tan x)$

$= 2\sec^4 x + 4\sec^2 x \cdot \tan^2 x$

$F^{IV}(x) = 2\times 4 \sec^3 x \;(\sec x + \tan x) + 4[\sec^2 x \; 2\tan x \cdot \sec^2 x + \tan^2 x \; 2\sec x\,(\sec x \tan x)]$

$F^{V}(x) = 8([\sec^4 x \cdot \sec^2 x + \tan x \; 4\sec^3 x\,(\sec x \tan x)]$
$+ 8[\sec^4 x \; \sec^2 x + \tan x \; 4\sec^3 x \;(\sec x \cdot \tan x)]$
$+ 8(\sec^2 x \; 3\tan^2 x \; \sec^2 x + \tan^2 x \cdot 2\sec x\,(\sec x \cdot \tan x)$

$F^{V}(x) = 8[\sec^6 x + 4\sec^4 x \; \tan^2 x] + 8[\sec^6 x + 4\sec^4 x \; \tan^2 x] + 24 \sec^4 x \tan^2 x + 16\tan^4 x \sec^2 x]$

$F^{V}(x) = 16[\sec^6 x + 4\sec^4 x \tan x] + 24 \sec^4 x \tan^2 x + 16\tan^4 x \sec^2 x$

$F^{V}(0) = 16[1+0] + 24(0) + 16(0)$

$F^{V}(0) = 16$. $F^{IV}(0) = 0$. $F'''(0) = 2$ $F'(0) = 1$

∴ maclaurin's series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{IV}(0)}{4!}x^4 + \frac{f^{V}(0)}{5!}x^5$$

$$\tan x = 0 + x + 0 + \frac{2}{3\times2\times1}x^3 + 0 + \frac{16}{5\times4\times3\times2\times1}x^5$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots$$

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G. I. Bagewadi Arts, science and commerce college, Nipani

Department of Mathematics.
Practice Test - II
for the year 2019-20 Date - 16/10

Class - BSc - I sem
Roll No - 187

1] If Δ' is reciprocal of determinant Δ of order 4 then prove that $\Delta' = \Delta^3$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$ be a determinant of order 4.

$$\Delta' = \begin{vmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} \cdot \begin{vmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}+a_{14}A_{14} & a_{11}A_{21}+a_{12}A_{22}+\_\_+a_{14}A_{24} & a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}+a_{14}A_{34} & a_{11}A_{41}+\_\_+a_{14}A_{44} \\ a_{21}A_{11}+\_\_+a_{24}A_{14} & a_{21}A_{21}+\_\_+a_{24}A_{24} & a_{21}A_{31}+\_\_+a_{24}A_{34} & a_{21}A_{41}+\_\_ \\ a_{31}A_{11}+\_\_+a_{34}A_{14} & a_{31}A_{21}+\_\_+a_{34}A_{24} & a_{31}A_{31}+\_\_a_{34}A_{34} & a_{31}A_{41}+\_\_ \\ a_{41}A_{11}+\_\_+a_{44}A_{14} & a_{41}A_{21}+\_\_+a_{44}A_{24} & a_{41}A_{31}+\_\_+a_{44}A_{34} & a_{44}A_{34}\_\_A_{44}A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} \Delta & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & \Delta \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$\Delta\Delta' = \Delta^4(1) = \Delta\Delta' = \Delta^4$

dividing both the side $\Delta \Rightarrow \Delta' = \Delta^4/\Delta$

$\Delta' = \Delta^{4-1}$ $\Delta' = \Delta^3$ //

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a) Expand $\sin^5\theta$ in terms of sines of multiples of θ

⟹ we have $\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$

$$\sin^5\theta = \left[\frac{1}{2i}\left(z - \frac{1}{z}\right)\right]^5 = \left(\frac{1}{2i}\right)^5 \left(z - \frac{1}{z}\right)^5$$

$$\sin^5\theta = \frac{1}{32i}\left(z - \frac{1}{z}\right)^5$$

$$= \frac{1}{32i}\left[{}^5C_0 z^5 - {}^5C_1 z^4 \frac{1}{z} + {}^5C_2 z^3 \frac{1}{z^2} - {}^5C_3 z^2 \frac{1}{z^3} + {}^5C_4 z \frac{1}{z^4} - {}^5C_5 \frac{1}{z^5}\right]$$

${}^5C_0 = {}^5C_5 = 1$

${}^5C_1 = {}^5C_4 = 5$

${}^5C_2 = {}^5C_3 = 10$

$$= \frac{1}{32i}\left[z^5 - 5z^3 + 10z - 10\frac{1}{z} + 5\frac{1}{z^3} - \frac{1}{z^5}\right]$$

$$= \frac{1}{32i}\left[\left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)\right]$$

$$= \frac{1}{32i}\left[2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)\right]$$

$$= \frac{2i}{32i}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

$$= \frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

$$\sin^5\theta = \frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

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G. I. Bagewadi Art's science and commerce college, Nipani

Department of Mathematics

Practical test - I for the year 2019-20

Class - BSc - II sem Date -

Roll No - 187

3/10

1) State and prove Fermat's theorem in number theory

→ Statement : If 'p' is a prime and $(a, p) = 1$ then $a^{p-1} - 1$ is divisible by p

i.e. $a^{p-1} \equiv 1 \pmod{p}$

or $a^p \equiv a \pmod{p}$

proof - we have $(x_1+x_2)^p$

$(x_1+x_2)^p = {}^pC_0 x_1^p + {}^pC_1 x_1^{p-1} x_2 + {}^pC_2 x_1^{p-2} x_2^2 + {}^pC_3 x_1^{p-3} x_1^3$

$+ \dots + {}^pC_{p-1} x_1 x_2^{p-1} + {}^pC_p x_2^p$

$(x_1+x_2)^p = x_1^p + p x_1^{p-1} x_2 + \frac{p(p-1)}{2!} x_1^{p-2} x_2^2 + \dots + \dots$

$\frac{p(p-(p-2))}{(p-1)!} x_1 x_2^{p-1} + x^p$

$= x_1^p + x_2^p +$ terms divisible by p

$\therefore (x_1+x_2)^p \equiv (x_1^p + x_2^p) \pmod{p}$

& $(x_1+x_2+x_3+ \dots + x_a)^p = (x_1^p + x_2^p + x_3^p + \dots + x_a^p) \pmod{p}$

Substituting $x_1 = x_2 = x_3 = x_4 = \dots = x_a = 1$ is eqn (1)

we get

$a^p = a \pmod{p}$ —(2)

but $(a, p) = 1$ ∴ we can cacell the common factor 'a' in eqn (2) we get

$a^{p-1} \equiv 1 \pmod{p}$

∴ $a^{p-1} - 1 \equiv 0 \pmod{p}$

⟹ $a^{p-1} - 1$ is divisible by p

2) Find sum and number of divisors of 3600

$2^4 \times 3^2 \times 5^2$

$p_1 = 2$ $p_2 = 3$ $p_3 = 5$

$\alpha_1 = 4$ $\alpha_2 = 2$ $\alpha_3 = 2$

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$$d(N) = (\alpha_1 + 1)\ (\alpha_2 + 1)\ (\alpha_3 + 1)$$
$$= (4+1)\ (2+1)\ (2+1)$$

$$d(N) = 45$$

$$S(N) = \left(\frac{p_1^{\alpha_1+1} - 1}{p_1 - 1}\right) \left(\frac{p_2^{\alpha_2+1} - 1}{p_2 - 1}\right) \left(\frac{p_3^{\alpha_3+1} - 1}{p_3 - 1}\right)$$
$$= \left(\frac{2^{4+1} - 1}{2-1}\right) \left(\frac{3^{2+1} - 1}{3-1}\right) \left(\frac{5^{2+1} - 1}{5-1}\right)$$
$$= \left(\frac{2^5 - 1}{1}\right) \left(\frac{3^3 - 1}{2}\right) \left(\frac{5^3 - 1}{4}\right)$$
$$= \left(\frac{32-1}{1}\right) \left(\frac{27-1}{2}\right) \left(\frac{125-1}{4}\right)$$
$$= (31) \left(\frac{26}{2}\right) \left(\frac{124}{4}\right)$$
$$= (31)\ (13)\ (31)$$
$$S(N) = 12493$$

G. I Bagewadi KLE's society's Arts, science and Commerce College, Nipani

Department of mathematics

Practice test - II for the year 2019-20

Class - BSc - II sem Date -

Roll No - 133.

1) Derive the formula of radius of curvature $\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$ for the curve

⟹ let $y=f(x)$ be given Cartesian curve and let ψ be the angle made by the tangent at P with x-axis. Then by defination of slope

$\tan\psi$ = slope of tangent

$\tan\psi = dy/dx$

ie $dy/dx = \tan\psi$ ie $y_1 = \tan\psi$

Differentiate both the side w.r.t 's' we get

$\frac{d}{ds}\left(\frac{dy}{dx}\right) = \sec^2\psi \frac{d\psi}{ds}$ {As s, ψ, x & y are functions of each others}

ie $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$ {By chaine rule}

$\frac{d^2y}{dx^2} \cdot \frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$

ie $\frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{ds} \frac{ds}{dx}$

$y_2 = (1+\tan^2\psi)\, 1/\rho \sqrt{1+(dy/dx)^2}$

$y_2 = (1+y_1^2)^{3/2} \frac{1}{\rho}$

$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$ //

2) Prove that spirals $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally

⟹ Given curves $r^n = a^n \cos n\theta$ —(1)

$r^n = b^n \sin n\theta$ —(2)

Take log on both sides

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∴ eqn ① becomes

$\log r^n = \log a^n + \log \cos n\theta$

$n \log r = n \log a + \log \cos n\theta$ diff. w.r.t θ we get

$n \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} (-\sin n\theta) n$

$n \frac{1}{r} \frac{dr}{d\theta} = -\frac{\sin n\theta}{\cos n\theta} n$

$\frac{1}{r} \frac{dr}{d\theta} = -\tan n\theta$

$\cot \phi_1 = -\tan n\theta$

ie eqn ② becomes

$\log r^n = \log b^n + \log \sin n\theta$

$n \log r = n \log b + \log \sin n\theta$

$n \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta} (\cos n\theta) n$

$n \frac{1}{r} \frac{dr}{d\theta} = \frac{\cos n\theta}{\sin n\theta} n$

$\frac{1}{r} \frac{dr}{d\theta} = \cot n\theta$

$\cot \phi_2 = \cot n\theta$

Clearly $\cot \phi_1 \cdot \cot \phi_2 = (-\tan n\theta)(\cot n\theta)$

$\cot \phi_1 \cdot \cot \phi_2 = -1$

⟹ curve cut at right angle and hence ~~angl~~ angle between them is 90°

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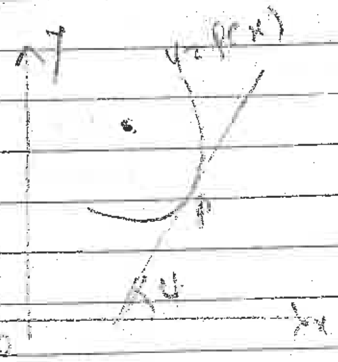
G. I. Bagewadi Arts Science and Commerce College, Nipani
Department of Mathematics
Practice Test - I for the year 2019-20

Class . BSc - III sem. Date
Roll No - 142

1) Prove that Monotonic Increasing bounded above sequence is convergent and converges to its lub.

proof - let $\{a_n\}_{n \in N}$ be monotonic increasing and bounded above.
let l be least upper bound [(ub) [supremum]] of $\{a_n\}$. ∴ By defination $a_n < l$ $\forall n$ — (i)
and for $\epsilon > 0$ ∃ +ve integer such that $a_m > l - \epsilon$ - (ii)
And $\{a_n\}$ be monotonic increasing
∴ $\forall n \geq m$ $a_n \geq a_m$ — (iii)
from (i) (ii) & (iii) we have
$l > a_n \geq a_m > l - \epsilon$ $\forall n \geq m$
i.e $l - \epsilon < a_m \leq a_n < l < l + \epsilon$ $\forall n \geq m$
ie $l - \epsilon < a_m < l + \epsilon$ $\forall n \geq m$
ie e $|a_n - l| < \epsilon$ $\forall n \geq m$
⟹ sequence $\{a_n\}$ is convergent & converges to its lub l
Thus monotonic increasing bounded above sequence is convergent and converges to its supremum.



2) Prove that

$$\frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$

⟹ Since u & v are functions of x & y and x & y are functions of r & θ we have

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial \theta}$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

consider $\frac{\partial(u,v)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix}$

$$\begin{vmatrix} \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r} & \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \theta} \\ \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial r} & \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \partial y/\partial \theta \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,\theta)} \;//$$

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Department of Mathematics

practice Test - II for the year 2019-20

Class - BSc III sem Date

Roll No - 142

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1) Find the volume or solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis

→ $x^{2/3} + y^{2/3} = a^{2/3}$ — ①

$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ — ②

we have $\cos^2\theta + \sin^2\theta = 1$ — ③

from eqn ② & ③

$\left(\frac{x}{a}\right)^{2/3} = \cos^2\theta$ and $\left(\frac{y}{a}\right)^{2/3} = \sin^2\theta$

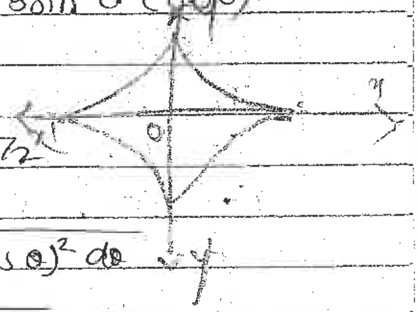
$x = a\cos^3\theta$ and $y = a\sin^3\theta$

Which are parametric equations of eqn ①

$\frac{dx}{d\theta} = -3a\cos^2\theta \cdot (\sin\theta)$ $\frac{dy}{d\theta} = a\,3\sin^2\theta\,(\cos\theta)$

Symmetric about both the axes

is Ist quadrant 'a' varies from 0 to $\pi/2$



Perimetree of astroid

$= 4\int_0^{\pi/2} \sqrt{(-3a\cos^2\theta\sin\theta)^2 + (3a\sin^2\theta\cos\theta)^2}\, d\theta$

$= 4\int_0^{\pi/2} \sqrt{(9a^2\cos^4\theta \cdot \sin^2\theta + 9a^2\sin^4\theta\cos^2\theta)}\, d\theta$

$= 4\int_0^{\pi/2} \sqrt{9a^2\sin^2\theta\cos^2\theta(\cos^2\theta + \sin^2\theta)}\, d\theta$

$4\int_0^{\pi/2} \sqrt{9a^2\sin^2\theta\cos^2\theta\,(1)}\,d\theta = 4\int_0^{\pi/2} 3a\sin\theta\cos\theta\, d\theta$

Divide and multiple by 2

is $\int_0^{\pi/2} 2\times\frac{3}{2}\, 4\sin\theta\cos\theta\, d\theta$

$\frac{4}{2}\int_0^{\pi/2} 3a\,(2\sin\theta \cdot \cos\theta)\, d\theta$

$= 2\int_0^{\pi/2} 3a\sin 2\theta\, d\theta = 6a\int_0^{\pi/2} \sin 2\theta\, d\theta$

$= 6a\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$

$= 6a\left[\frac{-\cos 2\pi/2}{2} + \frac{\cos 0}{2}\right]$

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$= 6a\left[-\frac{\cos 2\pi/2}{2} + \frac{\cos 0}{2}\right]$

$= 6a\left[\frac{-(-1)}{2} + \frac{1}{2}\right] = 6a\left[\frac{1}{2}+\frac{1}{2}\right] = 6a\left[\frac{2}{2}\right]$

$= 6a$ //

2) State and prove langranges theorem for group.

Statement - If 'G' is a finite group and H is a subgroup of G then order of H divides order of G

proof - let $O(G) = n$ let H be a subgroup of G such that $O(H) = m$

Now order of ~~G~~ G is given to be finite Hence the ~~an~~ number of distict right casets of H in G are finite.

let k be the number of destict right cosets of ~~let~~ H in G

Now any two casets have the same number of element and $H = He$ is the ~~cast~~ cosets having in number element ~~right~~ and hence the set of all distinct right cosets of H in G from a partition of G we have number of elements in [G] is equal to km But $O(G) = n$

$\Rightarrow n = mk$

$\Rightarrow m/n$

$\Rightarrow$ i.e $O(H) / O(G)$ //

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PRACTICE TEST - I
For the year 2019-20
Class :- B.SC I Sem Date :- 26/10/2019
Roll No :- 152

I] State and prove leibnitz's test for nth derivative of product.

→ Statement :- If $y = uv$ where u & v are function of x having derivatives of nth order then

$$y_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \cdots + {}^nC_n u v_n$$

where suffixes of uv denot the number of time there differentiated and ${}^nC_r$ denotes the number of combinations of n different things taken 'r' at a time

proof :- we prove this result by using mathematical induction.

Case I :- let $y = uv$

$$\Rightarrow y_1 = u\frac{dv}{dt} + dv\frac{du}{dt}$$

$$y_1 = u_1 v + u v_1$$

$$y_1 = {}^1C_0 u_1 v + {}^1C_1 u v_1 \quad [\because {}^1C_0 = 1 = {}^1C_1]$$

∴ The Theorem is true for $n=1$

Case II :- let us assume that the theorem is true for $n=m$

i.e $y_m = {}^mC_0 u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \cdots + {}^mC_m u v_m$ — ①

Case III :- differentiate equation ① on both the side.

Modern

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$y_{m+1} = {}^mC_0 [u_{m+1}v + u_m v_1] + {}^mC_1 [u_m v_1 + v_{m-1} v_2]$
$+ {}^mC_2 [u_{m-1} v_1 + u_{m-2} v_3] + \dots$
$+ {}^mC_m [v_m u_1 + u v_{m+1}]$

$y_{m+1} = {}^mC_0 u_{m+1} v + ({}^mC_0 + {}^mC_1) u_m v_1 + ({}^mC_1 + {}^mC_2) u_{m-1} v_2 + ({}^mC_2 + {}^mC_3) u_{m-2} v_3 + \dots$
$\dots + ({}^mC_{m-1} + {}^mC_m) u_1 v_m + {}^mC_m u v_{m+1}$

we have ${}^mC_0 = 1 = {}^{m+1}C_0$, ${}^mC_m = {}^{m+1}C_{m+1}$

since ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, ${}^mC_0 + {}^mC_1 = {}^{m+1}C_1$, ${}^mC_1 + {}^mC_2 = {}^{m+1}C_2$ & so on.

Substituting these value in the above result we get.

$y_{m+1} = {}^{m+1}C_0 u_{m+1} v + {}^{m+1}C_1 u_m v_1 + {}^{m+1}C_2 u_{m-1} v_2 + {}^{m+1}C_3 u_{m-2} v_3 + \dots + {}^{m+1}C_{m+1} u v_{m+1}$

∴ The theorem is true for $n = m+1$

∴ By mathematical induction the theorem is true for every positive integer 'n'

2. Expand $\tan x$ by Maclaurin's Theorem up to the power containing x^5.

→ Giving Maclaurin's series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!}x^4 + \frac{f^{v}(0)}{5!}x^5 + \dots$$

let $f(x) = \tan x$

$f'(x) = \sec^2 x$

$f''(x) = 2\sec x (\sec x \tan x)$
$= 2\sec^2 x \tan x$

$f'''(x) = 2(\sec^2 x \sec^2 x + \tan x \, 2\sec^2 x \tan x)$
$= 2\sec^4 x + 4\sec^2 x \tan^2 x$

$f^{iv}(x) = 2 \times 4 \sec^3 x (\sec x \tan x)$
$+ 4[\sec^2 x \, 2\tan x \sec^2 x + \tan^2 x \, 2\sec x (\sec x \tan x)]$

ROLL NO.	DATE	

$f^{iv}(x) = 8\sec^4 x \tan x + 8\sec^4 x \tan x + 8\sec^2 x \tan^3 x$

$f^{v}(x) = 8[\sec^4 x \sec^2 x + \tan x \; 4\sec^3 x(\sec x \tan x)]$
$+ 8[\sec^4 x \sec^2 x + \tan x \; 4\sec^3 x(\sec x \tan x)]$
$+ 8[\sec^2 x \; 3\tan^2 x \sec^2 x + \tan^3 x \; 2\sec x(\sec x \cdot \tan x)]$

$f^{v}(x) = 8[\sec^6 x + 4\sec^4 x \tan^2 x]$
$+ 8[\sec^6 x + 4\sec^4 x \tan^2 x$
$+ 24\sec^4 x \tan^2 x + 16\tan^4 x \sec^2 x]$

$f^{v}(x) = 16[\sec^6 x + 4\sec^4 x \tan x]$
$+ 24\sec^4 x \tan^2 x + 16\tan^4 x \sec^2 x$

$f^{v}(0) = 16[1+0] + 24(0) + 16(0)$

$f^{v}(0) = 16$

$f^{iv}(0) = 0$

$f'''(0) = 2$

$f''(0) = 0$

$f'(0) = 1$

$\therefore$ maclaurin's series

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$
$+ \frac{f^{iv}(0)}{4!}x^4 + \frac{f^{v}(0)}{5!}x^5$

$\tan x = 0 + x + 0 + \frac{2}{3\times2\times1}x^3 + 0 + \frac{16}{5\times4\times3\times2\times1}x^5$

$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5$

ROLL NO. | DATE

K. L. E Society's
G. I. Bagewadi Arts, Science and Commerce College Nippani
Department of mathematics
PRACTICE TEST – II
For the year - 2019-20

Class :- B. Sc - I Sem Date :-

Roll No :- 152

9/10

I] If Δ' is reciprocal of determinant Δ of order 4 then prove that $\Delta' = \Delta^3$

→ $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$ be a determinant of order 4

$$\Delta' = \begin{vmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} \begin{vmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}+a_{14}A_{14} & a_{11}A_{21}+a_{12}A_{22}+\cdots+a_{13}A_{24} & a_{11}A_{31}+\cdots+a_{14}A_{34} & a_{11}A_{41}+\cdots+a_{14}A_{44} \\ a_{21}A_{11}+\cdots+a_{24}A_{14} & a_{21}A_{21}+\cdots+a_{24}A_{24} & a_{21}A_{31}+\cdots+a_{24}A_{34} & a_{21}A_{41}+\cdots+a_{24}A_{44} \\ a_{31}A_{11}+\cdots+a_{34}A_{14} & a_{31}A_{21}+\cdots+a_{34}A_{24} & a_{31}A_{31}+\cdots+a_{34}A_{34} & a_{31}A_{41}+\cdots+a_{34}A_{44} \\ a_{41}A_{11}+\cdots a_{44}A_{14} & a_{41}A_{21}+\cdots+a_{44}A_{24} & a_{41}A_{31}+\cdots+a_{44}A_{34} & a_{41}A_{41}+\cdots a_{44}A_{44} \end{vmatrix}$$

$$\Delta\Delta' = \begin{vmatrix} \Delta & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & \Delta \end{vmatrix}$$

ROLL NO. DATE

$$\Delta A' = \Delta^4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$\Delta \Delta' = \Delta^4 (1)$

$\Delta \Delta' = \Delta^4$

dividing both the side Δ

$\Delta' = \dfrac{\Delta^4}{\Delta}$

$\Delta' = \Delta^{4-1}$

$\Delta' = \Delta^3$

2) Expand $\sin^5\theta$ in terms of sines of multiples of θ.

→ we have $\sin\theta = \dfrac{1}{2i}\left(z - \dfrac{1}{z}\right)$

$\Rightarrow \sin^5\theta = \left[\dfrac{1}{2i}\left(z - \dfrac{1}{z}\right)\right]^5$

$= \left(\dfrac{1}{2i}\right)^5 \left(z - \dfrac{1}{z}\right)^5$

$\sin^5\theta = \dfrac{1}{32i}\left(z - \dfrac{1}{z}\right)^5$

$= \dfrac{1}{32i}\Big[{}^5C_0 z^5 - {}^5C_1 z^4 \dfrac{1}{z} + {}^5C_2 z^3 \dfrac{1}{z^2} - {}^5C_3 z^2 \dfrac{1}{z^3} + {}^5C_4 z \dfrac{1}{z^4} - {}^5C_5 \dfrac{1}{z^5}\Big]$

${}^5C_0 = {}^5C_5 = 1$

${}^5C_1 = {}^5C_4 = 5$

${}^5C_2 = {}^5C_3 = 10$

$= \dfrac{1}{32i}\left[z^5 - 5z^3 + 10z - 10\dfrac{1}{z} + 5\dfrac{1}{z^3} - \dfrac{1}{z^5}\right]$

$= \dfrac{1}{32i}\left[\left(z^5 - \dfrac{1}{z^5}\right) - 5\left(z^3 - \dfrac{1}{z^3}\right) + 10\left(z - \dfrac{1}{z}\right)\right]$

$= \dfrac{1}{32i}\left[2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)\right]$

STUDENT'S NAME		TOTAL MARKS OBTAINED
CLASS	SUBJECT	
ROLL NO.	DATE	

$$= \frac{2i}{32i}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

$$= \frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

$$\sin^5\theta = \frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$$

K.L.E. Society's

G. I. Bagewadi Arts, Science and Commerce College Nipani

Department of Mathematics

Practise test - I

For the year 2019-20

6/10

Class :- BSc II sem

Roll no :- 180

Date :- 15/10/2020

1) State and prove Fermat's theorem in number theory.

⟹ Statement :-

If 'p' is a prime and $(a,p) = 1$ then $a^{p-1} - 1$ is divisible by p.

i.e. $a^{p-1} \equiv 1 \pmod p$ [or $a^p \equiv a \pmod p$]

Proof :- We have,

$$(\alpha_1+\alpha_2)^p = {}^pC_0\,\alpha_1^p + {}^pC_1\,\alpha_1^{p-1}\alpha_2 + {}^pC_2\,\alpha_1^{p-2}\alpha_2^2 + \cdots\cdots + {}^pC_{p-1}\,\alpha_1\alpha_2 + {}^pC_p\,\alpha_2$$

$$(\alpha_1+\alpha_2)^p = \alpha_1^p + p\alpha_1^{p-1} + \frac{p(p-1)}{2!}\alpha_1^{p-2}\alpha_2^2 + \cdots + \frac{p(p-1)\cdots p-(p-2)}{(p-1)!}\alpha_1\alpha_2^{p-1} + \alpha_2^p$$

$$(\alpha_1+\alpha_2)^p = \alpha_1^p + \alpha_2^p + \text{terms divisible by } p.$$

$$\therefore (\alpha_1+\alpha_2)^p \equiv (\alpha_1^p + \alpha_2^p) \pmod p$$

lll^{rly} we can show that,

$$(\alpha_1+\alpha_2+\alpha_3+\cdots\cdots+\alpha_a)^p \equiv (\alpha_1^p+\alpha_2^p+\alpha_3^p+\cdots\cdots+\alpha_a^p) \pmod p \quad -①$$

Substituting $\alpha_1 = \alpha_2 = \cdots\cdots = \alpha_a = 1$ in eq^n ①

we get,

$$a^p \equiv a \pmod p \longrightarrow ②$$

But $(a,p) = 1$

∴ We can cancel the common factor 'a' in eq^n ②

we get,

$$a^{p-1} \equiv 1 \pmod p$$

$$\therefore a^{p-1} - 1 \equiv 0 \pmod p$$

$\Longrightarrow a^{p-1} - 1$ is divisible by p.

2] Find sum and number of divisors of 3600.

$\Longrightarrow$ $3600 = 2^4 \times 3^2 \times 5^2$

Here $p_1 = 2$, $p_2 = 3$, $p_3 = 5$

$\alpha_1 = 4$, $\alpha_2 = 2$, $\alpha_3 = 2$

$$\therefore \quad d(N) = (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$$

$$= (4+1)(2+1)(2+1)$$

$$= 5 \times 3 \times 3$$

$$d(N) = 45$$

$$S(N) = \left(\frac{p_1^{\alpha_1+1} - 1}{p_1 - 1}\right)\left(\frac{p_2^{\alpha_2+1} - 1}{p_2 - 1}\right)\left(\frac{p_3^{\alpha_3+1} - 1}{p_3 - 1}\right)$$

$$= \left(\frac{2^{4+1} - 1}{2-1}\right)\left(\frac{3^{2+1} - 1}{3-1}\right)\left(\frac{5^{2+1} - 1}{5-1}\right)$$

$$= \left(\frac{2^5 - 1}{1}\right)\left(\frac{3^3 - 1}{2}\right)\left(\frac{5^3 - 1}{4}\right)$$

$$= \left(\frac{32-1}{1}\right)\left(\frac{27-1}{2}\right)\left(\frac{125-1}{4}\right)$$

$$= (31)\left(\frac{\overset{13}{\cancel{26}}}{\cancel{2}}\right)\left(\frac{\overset{31}{\cancel{124}}}{\cancel{4}_1}\right)$$

$$= 31 \times 13 \times 31$$

$$S(N) = 12493$$

K.L.E. Society's

G I Bagewadi Karres Arts, Science & Commerce College, Nipani

Department of Mathematics

Practice test - II

For the year 2019-20

7/10

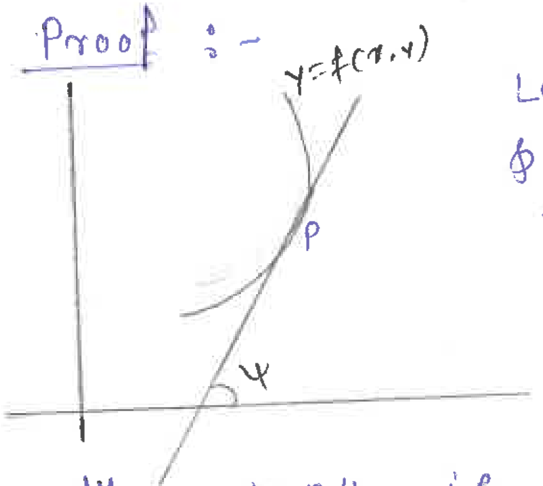
Class :- BSc II sem

Date :- 15/10/2020

Roll no. :- 180

1) Derive the formula of radius of curvature $\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$ for the curve.

=) Proof :-



Let $y=f(x)$ be given cartesion curve & let ψ be the angle made by the tgt. at P with x-axis.

Then by the defn of slope.

tan ψ = slope of tangent

$\tan\psi = \frac{dy}{dx}$

i.e. $\frac{dy}{dx} = \tan\psi$ i.e. $y_1 = \tan\psi$

Diff. both the sides w.r.t. 's' we get

$$\Rightarrow \frac{d}{ds}\left(\frac{dy}{dx}\right) = \sec^2\psi \frac{d\psi}{ds}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)\left(\frac{dx}{ds}\right) = \sec^2\psi \frac{d\psi}{ds}$$

$$\frac{d^2y}{dx^2} \cdot \frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$$

$$\text{i.e. } \frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{ds} \cdot \frac{ds}{dx}$$

$$y_2 = 1+\tan^2\psi \cdot \frac{1}{\rho}\sqrt{1+(dy/dx)^2}$$

$$y_2 = (1+y_1^2) \cdot 1/\rho \sqrt{1+y_1^2}$$

$$y_2 = (1+y_1^2)^{3/2} \cdot \frac{1}{\rho}$$

$$\boxed{\rho = \frac{(1+y_1^2)^{3/2}}{y_2}}$$

2) Obtain the reduction formula for $\int \tan^n x\,dx$ where 'n' is +ve integer and hence find the value of $\int \tan^7 dx$ &
$\int_0^{\pi/4} \tan^4 x\,dx$

$\Rightarrow \int \tan^n x\,dx = \int \tan^{n-2} x \cdot \tan^2 x\,dx$

$= \int \tan^{n-2} x\,(\sec^2 x - 1)\,dx$

$= \int \tan^{n-2} x \cdot \sec^2 x\,dx - \int \tan^{n-2} x\,dx$

$\int \tan^n x\,dx = \dfrac{\tan^{n-2+1} x}{n-2+1} - \int \tan^{n-2} x\,dx$

* $\int \tan^7 x\,dx$

We have, $\int \tan^n x\,dx = \dfrac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$ — (1)

Put $n = 7$

$\Rightarrow \int \tan^7 x\,dx = \dfrac{\tan^6 x}{6} - \int \tan^5 x\,dx$ — (2)

$\therefore \int \tan^5 x\,dx = \dfrac{\tan^4 x}{4} - \int \tan^3 x\,dx$ — (3)

$\therefore \int \tan^3 x\,dx = \dfrac{\tan^2 x}{2} - \int \tan x\,dx$

$\therefore \int \tan^3 x\,dx = \dfrac{\tan^2 x}{2} - \log \sec x$

$\therefore \int \tan^7 x\,dx = \dfrac{\tan^6 x}{6} - \dfrac{\tan^4 x}{4} + \dfrac{\tan^2 x}{2} - \log \sec x$

* $\int_0^{\pi/4} \tan^4 x\,dx$

Let $= \int_0^{\pi/4} \tan^4 x\,dx = \int_0^{\pi/4} \tan^{4-2} x \cdot \tan^2 x\,dx$

$= \int_0^{\pi/4} \tan^{4-2} x\,(\sec^2 x - 1)\,dx$

$= \int_0^{\pi/4} \tan^{4-2} x \sec^2 x\,dx - \int_0^{\pi/4} \tan^2 x\,dx$

$u_4 = \left.\dfrac{\tan^3 x}{3}\right]_0^{\pi/4} - \int_0^{\pi/4} \tan^2 x\,dx$

$u_4 = \dfrac{1}{3}\left[\tan^3(\pi/4) - \tan^3(0)\right] - \int_0^{\pi/4} \tan^2 x\,dx$

$u_4 = \dfrac{1}{3} - u_2$

$u_4 + u_2 = \dfrac{1}{3}$

K.L.E societys

G.J. Bagewadi Arts, Science and commerce college Nipani

Department of mathematics

for the year

Roll NO:- 87

Marks obtained

class:- BSc II sem

Max Marks:- 10

Answer the any two of the following

1. state the prove Fermat's theorem in number theory.

→ Statement → If p is a prime and $(a,p)=1$ then $a^{p-1}-1$ is divisible by p

i.e. $a^{p-1} \equiv 1 \pmod{p}$

$[$ or $a^p \equiv a \pmod{p}]$

Proof :- We have $(x_1+x_2)^p$

$$(x_1+x_2)^p = {}^pC_0 x_1^p + {}^pC_1 x_1^{p-1}x_2 + {}^pC_2 x_1^{p-2}x_2^2 + {}^pC_3 x_1^{p-3}x_1^3 + \cdots + {}^pC_{p-1} x_1 x_2^{p-1} + {}^pC_p x_2^p$$

$$(x_1+x_2)^p = x_1^p + p x_1^{p-1}x_2 + \frac{p(p-1)}{2!} x_1^{p-2}x_2^2 + \cdots + \frac{p[p-(p-2)]}{(p-1)!} x_1 x_2^{p-1} + x_2^p$$

$= x_1^p + x_2^p +$ terms divisible by p.

$\therefore (x_1+x_2)^p \equiv (x_1^p + x_2^p) \pmod{p}$

Similarly we can show that

$(x_1+x_2+x_3+\cdots+x_a)^p \equiv (x_1^p+x_2^p+x_3^p+\cdots x_a^p) \pmod{p}$ ①

We get

$a^p = [a \pmod{p}]$ —②

but $(a,p)=1 \therefore$ We can cancell the common factor a $\therefore$ eqn ② We get

$a^{p-1} \equiv 1 \pmod{p}$

$\therefore a^{p-1}-1 \equiv 0 \pmod{p}$

$\Rightarrow a^{p-1}-1$ is divisible by p

$\therefore p$ is a prime & $(a,p)=1$

2 find sum and number of divisors of 3600.

→ $2^4 \times 3^2 \times 5^2$

$P_1 = 2$ $P_2 = 3$ $P_3 = 5$

$\alpha_1 = 4$ $\alpha_2 = 2$ $\alpha_3 = 2$

$$d(N) = (\alpha_1+1)(\alpha_2+1)(\alpha_3+1)$$

$$= (4+1)(2+1)(2+1)$$

$$d(N) = 45$$

$$S(N) = \left(\frac{P_1^{\alpha_1+1}-1}{P_1-1}\right)\left(\frac{P_2^{\alpha_2+1}-1}{P_2-1}\right)\left(\frac{P_3^{\alpha_3+1}-1}{P_3-1}\right)$$

$$= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{2+1}-1}{3-1}\right)\left(\frac{5^{2+1}-1}{5-1}\right)$$

$$= \left(\frac{2^5-1}{1}\right)\left(\frac{3^3-1}{2}\right)\left(\frac{5^3-1}{4}\right)$$

$$= \left(\frac{32-1}{1}\right)\left(\frac{27-1}{2}\right)\left(\frac{125-1}{4}\right)$$

$$= (31)\left(\frac{26}{2}\right)\left(\frac{124}{4}\right)$$

$$= (31)(13)(31)$$

$$S(N) = 12493$$

K.L.E Societys

G. I. Bagewadi Arts Science & commerce college Nipani

Department of Mathematics

for the year

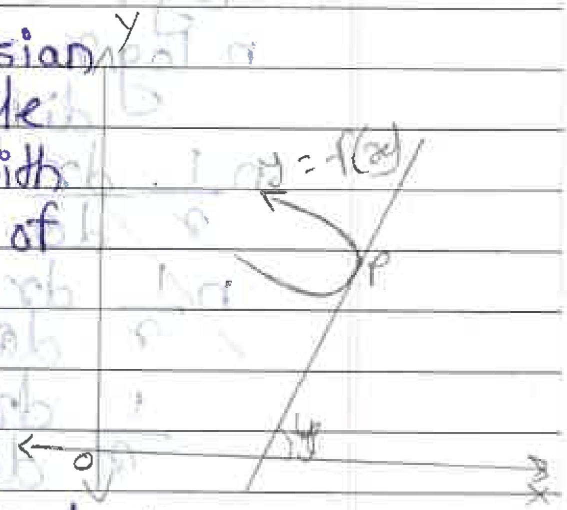
Roll NO:- 87 Marks obtained:- 10/10

class:- Bsc II sem Max Marks:- 10

Answer the any two of the following

1 Derive the formula of radius of curvature $\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$ for the curve.

→ Let $y=f(x)$ be given Cartesian curve and let ψ be the angle made by the tangent at P with x-axis. Then by defination of slope.



$\tan\psi$ = slope of tangent.

$\tan\psi = \frac{dy}{dx}$

i.e $\frac{dy}{dx} = \tan\psi$ i.e. $y_1 = \tan\psi$

Differentiate both the side w.r.t. 's' we get

$\frac{d}{ds}\left(\frac{dy}{dx}\right) = \sec^2\psi \frac{d\psi}{ds}$ {As s, y, x, y are fun^s of each others}

i.e. $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$ {By chaine rule}

$\frac{d^2y}{dx^2}\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$

i.e. $\frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{ds}\frac{ds}{dx}$

$y_2 = (1+\tan^2\psi)\frac{1}{\rho}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$y_2 = (1+y_1^2)\frac{1}{\rho}\sqrt{1+y_1^2}$

Date

$$y_2 = (1+y_1^2)^{3/2} \frac{1}{\rho}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

2. Prove that spirals $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.

→ Given Curves : $r^n = a^n \cos n\theta$ — ①

$r^n = b^n \sin n\theta$ — ②

Take log on both side.

∴ eq^n ① becomes.

$$\therefore \log r^n = \log a^n + \log \cos n\theta$$

$$n \log r = n \cdot \log a + \log \cos n\theta$$

diff. w.r.t. θ we get,

$$n \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} (-\sin n\theta) n$$

$$n \frac{1}{r} \frac{dr}{d\theta} = \frac{-\sin n\theta}{\cos n\theta} n$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\tan n\theta.$$

$$\cot \phi_1 = -\tan n\theta.$$

i.e. eq^n ② becomes

$$\log r^n = \log b^n + \log \sin n\theta.$$

$$n. \log r = n. \log b + \log \sin n\theta.$$

$$n. \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta} (\cos n\theta) n.$$

$$n \frac{1}{r} \frac{dr}{d\theta} = \frac{\cos n\theta}{\sin n\theta} n$$

$$\frac{1}{r} \frac{dr}{d\theta} = \cot n\theta$$

$$\cot \phi_2 = \cot n\theta.$$

Clearly $\cot \phi_1 \cot \phi_2 = (-\tan n\theta)(\cot n\theta)$

$\cot \phi_1 \cot \phi_2 = -1$

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G. I. Bagewadi Arts, Science and Commerce College

Nipani

Practice Test - I

For the year = 2019-20

PAGE:
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Roll No: 160

Date: 15/10/2020

Class: B.sc IInd Sem

1) State And Prove Fermat's Theorem in number Theory.

⟹ Statement:

If p is a prime and $(a, p) = 1$.

then $a^{p-1} - 1$ is divisible by p

i.e. $a^{p-1} \equiv 1 \pmod{p}$ [or $a^p \equiv a \pmod{p}$]

Proof: We have,

$$(x_1 + x_2)^p = {}^pC_0 x_1^p + {}^pC_1 x_1^{p-1} x_2 + {}^pC_2 x_1^{p-2} x_2^2 + \cdots + {}^pC_{p-1} x_1 x_2^{p-1} + {}^pC_p x_2^p$$

$$= x_1^p + p x_1^{p-1} x_2 + \frac{p(p-1)}{2!} x_1^{p-2} x_2^2 + \cdots + \frac{p(p-1)\cdots[p-(p-2)]}{(p-1)!} x_1 x_2^{p-1} + x_2^p$$

$= x_1^p + x_2^p$ + terms divisible by p.

$\therefore (x_1 + x_2)^p = (x_1^p + x_2^p) \pmod{p}$

Similarly we can show that

$(x_1 + x_2 + x_3 \cdots x_a)^p \equiv (x_1^p + x_2^p + x_3^p + \cdots + x_a^p) \pmod{p}$ — (1)

Substituting $x_1 = x_2 = \cdots x_a = 1$ in eqn (1)

we get

$a^p \equiv a \pmod{p}$ — (2)

But $(a, p) = 1$ therefore we can cancel the common factor a in eqn (2)

we get

$a^{p-1} \equiv 1 \pmod{p}$

$\therefore a^{p-1} - 1 \equiv 0 \pmod{p}$

⟹ $a^{p-1} - 1$ is divisible by p.

Sukhi

Corollary :- If p is prime
$\Rightarrow n^p \equiv n \pmod p$

2) Write the condition that equation $ax^2+by^2+cz^2+2gzx+2fyz+2hxy+2ux+2vy+2wz+d=0$ represents sphere.

$\Rightarrow$ Proof :- Let us take general eq^n of second degree in x, y, z is $ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$ — ①

If it passes through origin it has vertex as origin then cone passing through origin then
$\therefore d=0$

If $P(x_1, y_1, z_1)$ be any pt on through cone then eq^n the P lies on generator of the cone.
And hence eq^n of generator passing through P and vertex origine is

$$\frac{x-0}{x_1-0} = \frac{y-0}{y_1-0} = \frac{z-0}{z_1-0} = r$$

$\Rightarrow x = x_1 r, \quad y = r y_1, \quad z = r z_1$

$\therefore$ Any pt on the generator $Q(rx_1, ry_1, rz_1)$ which lies on ① for all values of r
$\therefore$ eq^n ① become

$\Rightarrow ar^2x_1^2 + br^2y_1^2 + cr^2z_1^2 + 2r^2fy_1z_1 + 2r^2gz_1x_1 + 2r^2hx_1^2y_1 + 2urx_1 + 2vry_1 + 2wrz_1 = 0$

i.e $r^2(ax_1^2+by_1^2+cz_1^2+2fy_1z_1+2gz_1x_1+2hx_1y_1) + 2r(ux_1+vy_1+wz_1) = 0 \ \forall r$

This is true only when

$ax_1^2+by_1^2+cz_1^2+2fy_1z_1+2gz_1x_1+2hx_1y_1=0$ — ③
$ux_1+vy_1+wz_1=0$ — ④

from eq^n ③ & ④
(If $r^2p + rq = 0 \ \forall r$ then $p=0, q=0$)
$u=0, v=0$ & $w=0$ from ④

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Then the eqn ① becomes

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

∴ Hence eqn of cone with vertex at origin is

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

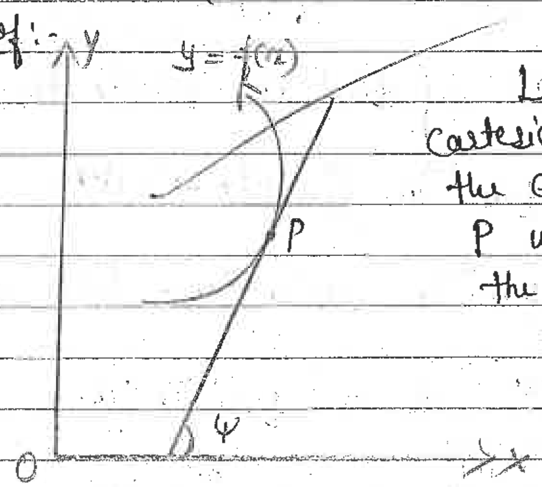
which is homogeneous eqn of second degree in x, y & z.

K.L.E. Society's
G.T. Bagewadi Arts, Science and Commerce College Nipani
Department of Mathematics
Practice Test - II
For the year _ _ _ _ _
Roll No: 160
class : B.sc IInd sem
Date: 15/10/2020

5/10

1) Derive the formula of radius of curvature $\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$ for the curve

⟹ proof:



Let $y = f(x)$ be given cartesion curve & let ψ be the angle made by the tgt at P with x-axis then by the dyenation of slope.

tan ψ = slope of the tgs

$\tan\psi = \frac{dy}{dx}$

ie $\frac{dy}{dx} = \tan\psi$ ie $y_1 = \tan\psi$

Diff w.r.t 'S' we get,

$\frac{d}{ds}\left(\frac{dy}{dx}\right) = \sec^2\psi$

ie $\frac{d}{ds}\left(\frac{dy}{dx}\right)\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$ { by chain rule

$\frac{d^2y}{dx^2}\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$

ie $\frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{ds}\frac{ds}{dx}$

$y_2 = 1+\tan^2\psi \frac{1}{\rho}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ $\left\{ \because \frac{ds}{dx} = \sqrt{1+y_1^2}\right.$

$y_2 = (1+y_1^2)\frac{1}{\rho}\sqrt{1+y_1^2}$

$$y_2 = (1+y_1^2)^{3/2} \frac{1}{\rho}$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

2) Obtain the reduction formula for $\int \tan^n x \, dx$ where n is positive integer and hence find the value of $\int \tan^7 x \, dx$ & $\int_0^{\pi/4} \tan^4 x$

⟹ Let $\int \tan^n x \, dx = \int \tan^{n-2} x \tan^2 x \, dx$

$$= \int \tan^{n-2} x (\sec^2 x - 1) \, dx$$

$$= \int \tan^{n-2} x \sec^2 x \, dx - \tan^{n-2} x \, dx$$

$$= \frac{\tan^{n-2+1} x}{n-2+1} - \int \tan^{n-2} x \, dx.$$

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx.$$

i) $\int_0^{\pi/4} \tan^4 dx$

$$\int_0^{\pi/4} \tan^4 x \, dx = \frac{\tan^{4-1} x}{4-1} - \int_0^{\pi/4} \tan^{4-2} x \, dx$$

$$= \frac{\tan^3 x}{3} - \int_0^{\pi/4} \tan^2 x \, dx \quad \text{— (1)}$$

$n = 2$

$$\int_0^{\pi/4} \tan^2 x \, dx = \frac{\tan x}{1} - \int_0^{\pi/4} \tan^0 x \, dx$$

$$= \tan x - x \quad \text{— (2)}$$

$$\int_0^{\pi/4} \tan^4 x \, dx = \frac{\tan^3 x}{3} - \tan x + x$$

$$= \frac{\tan^3 \pi/4}{3} - \tan \pi/4 + \pi/4 - \frac{\tan^3 0}{3} + \tan 0 - 0$$

$$= \frac{1}{3} - 1 + \frac{\pi}{4} \quad = -\frac{2}{3} + \frac{\pi}{4}$$

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ii) $\int \tan^7 x \, dx$

we have $\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$ — ①

put $n = 7$

$\int \tan^7 x \, dx = \frac{\tan^6 x}{6} - \int \tan^5 x \, dx$ — ②

put $n = 5$

$\int \tan^5 x \, dx = \frac{\tan^4 x}{4} - \int \tan^3 x \, dx$ — ③

$n = 3$

$\int \tan^3 x \, dx = \frac{\tan^2 x}{2} - \int \tan x \, dx$ — ④

$\therefore \int \tan^7 x \, dx = \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} - \log \sec x$ //

K.L.E. Society's

G. I. Bagewadi Arts, Science And Commerce,

College, Nipani

Department of Mathematics

Practice Test - I

For the year :-

Class :- B.Sc II sem   Date :-

Roll no :- 152

7/10

Answer the following questions.

1) State and prove Fermat's thm in number theory.

⇒ Statement :- If 'p' is a prime number and 'n' is a positive integer and relatively prime to each other, then prove that, $n^{p-1} \equiv 1 \pmod{p}$

proof :- We shall first prove that,

$\forall a, b \in m$ then,

$$(a+b)^p \equiv (a^p + b^p) \pmod{p}$$

where 'p' is prime no.

Now, expanding $(a+b)^p$ by using binomial thm, we get.

$$(a+b)^p = a^p + {}^pC_1 a^{p-1} b + {}^pC_2 a^{p-2} b^2 + \cdots + {}^pC_{p-2} a^2 b^{p-2} + {}^pC_{p-1} a b^{p-1} + b^p$$

$$\Rightarrow (a+b)^p - (a^p + b^p) = {}^pC_1 a^{p-1} b + {}^pC_2 a^{p-2} b^2 + \cdots + {}^pC_{p-2} a^2 b^{p-2} + {}^pC_{p-1} a b^{p-1}$$

$$\Rightarrow (a+b)^p - (a^p + b^p) = \sum_{r=1}^{p-1} {}^pC_r\, a^{p-r} b^r \qquad \text{——} \; (1)$$

Now, $${}^pC_r = \frac{p!}{(p-r)!\, r!} \quad \text{where } 1 \le r \le p-1$$

But $p!$ is divisible by 'p' and 'p' is co-prime
{∵ p is co-prime to 1, 2, 3 - - - r ($r < p$ and p is prime) and also p is co-prime to their product $= r!$ }

Also for the same reasons, p is co-prime to $(p-r)!$.

$$\therefore {}^pC_r = \frac{p!}{(p-r)!\, r!} \text{ is divisible by } p.$$

∴ ∃ an integer 'k_r' such that, ${}^pC_r = p(k_r)$

put ${}^pC_r = p(k_r)$ in eqⁿ (1), we have

$$(a+b)^p - (a^p + b^p) = p \sum_{r=1}^{p-1} k_r\, a^{p-r} b^r$$ which

is divisible by p.

$$\therefore p \,/\, (a+b)^p - (a^p + b^p)$$

or $(a+b)^p \equiv (a^p + b^p) \pmod p$

IIIrly $(a+b+c)^p \equiv (a^p + b^p + c^p) \pmod p$

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Generally, we can write in this way, we have

$$(a_1 + a_2 + a_3 + \text{------} + a_n)^p \equiv (a_1^p + a_2^p + \text{---} + a_n^p) \pmod p$$

put $a_1 = a_2 = a_3 = \text{---} = a_n = 1$

$\therefore (1+1+ \cdots + 1)^p \equiv (1^p + 1^p + \cdots + 1^p) \pmod p$

$\Rightarrow n^p \equiv n \pmod p$

$\Rightarrow n^{+1} \cdot n^{-1} \cdot n^p \equiv n \pmod p \qquad \because n^{+1} \cdot n^{-1} = 1$

$\Rightarrow n(n^{p-1}) \equiv n \pmod p$

$\Rightarrow n^{p-1} \equiv 1 \pmod p$

Hence proof.

2) Find sum and number of divisors of 3600

$\Rightarrow$ Given

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$\therefore 3600 = 2^4 \times 3^2 \times 5^2$

Here $P_1 = 2$, $P_2 = 3$ and $P_3 = 5$

$\alpha_1 = 4$, $\alpha_2 = 2$ and $\alpha_3 = 2$

$\therefore$ Number of divisors are :

$d(N) = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$

$= (1+4)(1+2)(1+2)$

$= 5 \times 3 \times 3$

$= 45$

Sum of the divisors are,

$$\sigma(N) = \left(\frac{P_1^{\alpha_1+1} - 1}{P_1 - 1}\right)\left(\frac{P_2^{\alpha_2+1} - 1}{P_2 - 1}\right)\left(\frac{P_3^{\alpha_3+1} - 1}{P_3 - 1}\right)$$

$$= \left(\frac{2^{4+1} - 1}{2 - 1}\right)\left(\frac{3^{2+1} - 1}{3 - 1}\right)\left(\frac{5^{2+1} - 1}{5 - 1}\right)$$

$$= \left(\frac{2^5 - 1}{1}\right)\left(\frac{3^3 - 1}{2}\right)\left(\frac{5^3 - 1}{4}\right)$$

$$= (32 - 1)\left(\frac{27 - 1}{2}\right)\left(\frac{125 - 1}{4}\right)$$

$$= 31 \times \frac{26}{2} \times \frac{124}{4}$$

$$= 31 \times 13 \times 41$$

$$= 16,523$$

K.L.E. Society's

G.I. Bagewadi Arts, Science And Commerce College, Nipani

Department of Mathematics

Practice Test - II

For the year :-

Class :- B.Sc II sem. Date :-

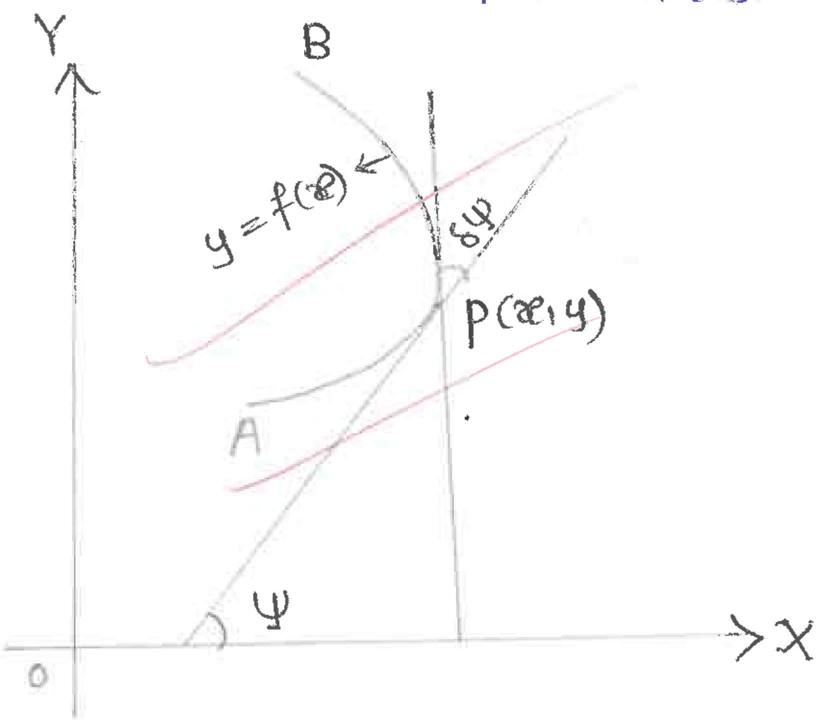
Roll No :-

8/10

Answer the following questions.

1) Derive the formula of radius of curvature $\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$ for the curve

⇒ consider the eqⁿ of curve $y = f(x)$. Let ψ be the angle made by the tangent at $p(x, y)$ with the direction of x- axis.



We know that, by the defn of slope,

Slope of the tangent = $\tan\psi$

$$\Rightarrow \frac{dy}{dx} = \tan\psi$$

Diff. w.r. to 's' on both sides,

$$\Rightarrow \frac{d}{ds}\left(\frac{dy}{dx}\right) = \sec^2\psi \cdot \frac{d\psi}{ds}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right)\cdot\frac{dx}{ds} = \sec^2\psi\cdot\frac{d\psi}{ds}$$ ∵ chain rule.

$$\Rightarrow \frac{d^2y}{dx^2}\cdot\frac{dx}{ds} = \sec^2\psi\cdot\frac{d\psi}{ds}$$

i.e. $$\frac{d^2y}{dx^2} = \sec^2\psi\cdot\frac{d\psi}{ds}\cdot\frac{ds}{dx}$$

i.e. $$y_2 = (1+\tan^2\psi)\cdot\frac{1}{\rho}\cdot\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow y_2 = (1+y_1^2)\cdot\frac{1}{\rho}\sqrt{1+y_1^2}$$

$$\Rightarrow y_2 = \frac{(1+y_1^2)^{3/2}}{\rho}$$

$$\Rightarrow \rho = \frac{(1+y_1^2)^{3/2}}{y_2}.$$

2) prove that spirals $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.

$\Rightarrow$ Given curves are,

$r^n = b^n \sin n\theta$

$\Rightarrow \log r^n = \log b^n + \log \sin n\theta$

$\Rightarrow n \log r = n \log b + \log \sin n\theta$

Diff. w.r. to 'θ', we get

$\Rightarrow n \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta} \cdot \cos n\theta \cdot n$

$\Rightarrow \frac{1}{r} \cdot \frac{dr}{d\theta} = \frac{\cos n\theta}{\sin n\theta}$

$\Rightarrow r \cdot \frac{d\theta}{dr} = \tan n\theta$

$\Rightarrow \tan \phi_1 = \tan n\theta$ —— (1)

And $r^n = a^n \cos n\theta$

$\Rightarrow \log r^n = \log a^n + \log \cos n\theta$

$\Rightarrow n \log r = n \log a + \log \cos n\theta$

Diff. w.r. to 'θ', we get

$\Rightarrow n \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} (-\sin n\theta \cdot n)$

$\Rightarrow \frac{1}{r} \cdot \frac{dr}{d\theta} = \frac{-\sin n\theta}{\cos n\theta}$

$\Rightarrow r \cdot \frac{d\theta}{dr} = -\frac{\cos n\theta}{\sin n\theta}$

$\Rightarrow \tan \phi_2 = -\cot n\theta$ —— (2)

Now, the condition of orthogonality is,

$$\tan\phi_1 \cdot \tan\phi_2 = -1$$

So that, from ① and ②,

$$\tan\phi_1 \cdot \tan\phi_2 = -\tan n\theta \times \cot n\theta$$

$$\tan\phi_1 \cdot \tan\phi_2 = -1$$

Hence, the given curves are intersect orthogonally.

ROLL NO. DATE

K.L.E Society's
G.I. Bagewadi Art's, Science and commerce College, Nippani
Department of Mathematics
PRACTICE TEST - I
For the year - 2019-20

Class :- B.Sc II Sem Date :-
Roll No :- 60

8/10

1] State and prove Fermat's theorem in number theory.

→ Statement :- If p is a prime and $(a, p) = 1$ then $a^{p-1} - 1$ is divisible by p

i.e $a^{p-1} \equiv 1 \pmod p$

[or $a^p \equiv a \pmod p$]

proof :- we have $(x_1 + x_2)^p$

$$(x_1+x_2)^p = {}^pC_0 x_1^p + {}^pC_1 x_1^{p-1} x_2 + {}^pC_2 x_1^{p-2} x_2^2 + {}^pC_3 x_1^{p-3} x_2^3 + \dots + {}^pC_{p-1} x_1 x_2^{p-1} + {}^pC_p x_2^p$$

$$(x_1+x_2)^p = x_1^p + p x_1^{p-1} x_2 + \frac{p(p-1)}{2!} x_1^{p-2} x_2^2 + \dots$$

$$\dots + \frac{p(p-(p-2))}{(p-1)!} x_1 x_2^{p-1} + x_2^p$$

$\therefore x_1^p + x_2^p +$ terms divisible by p

$\therefore (x_1+x_2)^p \equiv (x_1^p + x_2^p) \pmod p$

similarly we can show that

$(x_1 + x_2 + x_3 + \dots + x_n)^p \equiv (x_1^p + x_2^p + x_3^p + \dots + x_n^p) \pmod p$ ①

Substituting $x_1 = x_2 = x_3 = x_4 = \dots = x_n = 1$ in eqn ①

we get

$a^p \equiv a \pmod p$ —②

but $(a, p) = 1$ $\therefore$ we can cancel the common factor 'a' in eqn ② we get

$a^{p-1} \equiv 1 \pmod p$

$\therefore a^{p-1} - 1 \equiv 0 \pmod p$

$\Rightarrow a^{p-1} - 1$ is divisible by p

STUDENT'S NAME		TOTAL MARKS OBTAINED
CLASS	SUBJECT	
ROLL NO.	DATE	

2) Find sum and number of divisors of 3600

$2^4 \times 3^2 \times 5^2$

$P_1 = 2 \quad P_2 = 3 \quad P_3 = 5$

$\alpha_1 = 4 \quad \alpha_2 = 2 \quad \alpha_3 = 2$

$d(N) = (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$

$= (4+1)(2+1)(2+1)$

$d(N) = 45$

$$S(N) = \left(\frac{P_1^{\alpha_1+1} - 1}{P_1 - 1}\right)\left(\frac{P_2^{\alpha_2+1} - 1}{P_2 - 1}\right)\left(\frac{P_3^{\alpha_3+1} - 1}{P_3 - 1}\right)$$

$$= \left(\frac{2^{4+1} - 1}{2 - 1}\right)\left(\frac{3^{2+1} - 1}{3 - 1}\right)\left(\frac{5^{2+1} - 1}{5 - 1}\right)$$

$$= \left(\frac{2^5 - 1}{1}\right)\left(\frac{3^3 - 1}{2}\right)\left(\frac{5^3 - 1}{4}\right)$$

$$= \left(\frac{32 - 1}{1}\right)\left(\frac{27 - 1}{2}\right)\left(\frac{125 - 1}{4}\right)$$

$$= (31)\left(\frac{26}{2}\right)\left(\frac{124}{4}\right)$$

$$= (31)(13)(31)$$

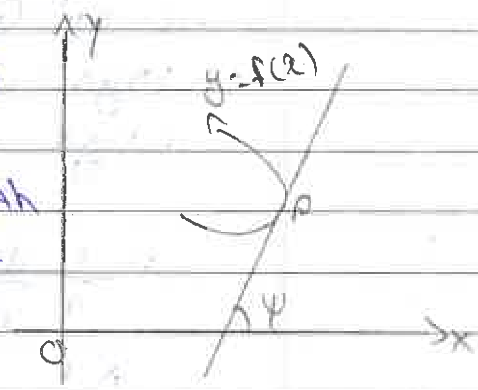
$S(N) = 12493$

K. L. E Society's
G. I. Bagewadi Arts, Science and Commerce College, Nippani.
Department of mathematics
PRACTICE TEST - II
For the year - 19-20

Class :- B.Sc II Sem Date :-
Roll No :- 60

1] Derive the formula of radius of curvature $\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$ for the curve

→ let $y = f(x)$ be given Cartesian curve and let ψ be the angle made by the tangent at P with x-axis. then by defination of slope



$\tan\psi$ = Slope of tangent

$\tan\psi = \frac{dy}{dx}$

i.e $\frac{dy}{dx} = \tan\psi$ i.e $y_1 = \tan\psi$

Differentiate both the side w.r.t 'S' we get.

$\frac{d}{ds}\left(\frac{dy}{dx}\right) = \sec^2\psi \frac{d\psi}{ds}$ {As S, ψ, x & y are functions of each others}

i.e $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$ {By chaine rule}

$\frac{d^2y}{dx^2}\frac{dx}{ds} = \sec^2\psi \frac{d\psi}{ds}$

i.e $\frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{ds}\frac{ds}{dx}$

$y_2 = (1+\tan^2\psi)\frac{1}{\rho}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$y_2 = (1+y_1^2)\frac{1}{\rho}\sqrt{1+y_1^2}$

Modern

ROLL NO. DATE

$$y_2 = (1+y_1^2)^{3/2} \frac{1}{\rho}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

2] Prove that spirals $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.

→ Given curves $r^n = a^n \cos n\theta$ — ①

$r^n = b^n \sin n\theta$ — ②

Take log on both side

i.e eq^n ① becomes

$$\log r^n = \log a^n + \log \cos n\theta$$

$$n \log r = n \log a + \log \cos n\theta$$

diff w.r.t θ we get

$$n \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} (-\sin n\theta) n$$

$$n \frac{1}{r} \frac{dr}{d\theta} = -\frac{\sin n\theta}{\cos n\theta} n$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\tan n\theta$$

$$\cot \phi_1 = -\tan n\theta$$

i.e eq^n ② becomes

$$\log r^n = \log b^n + \log \sin n\theta$$

$$n \log r = n \log b + \log \sin n\theta$$

$$n \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta} (\cos n\theta) n$$

$$n \frac{1}{r} \frac{dr}{d\theta} = \frac{\cos n\theta}{\sin n\theta} n$$

$$\frac{1}{r} \frac{dr}{d\theta} = \cot n\theta$$

$$\cot \phi_2 = \cot n\theta$$

STUDENT'S NAME		TOTAL MARKS OBTAINED
CLASS	SUBJECT	
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Clearly $\cot\phi_1 \cot\phi_2 = (-\tan n\theta)(\cot n\theta)$

$\cot\phi_1 \cot\phi_2 = -1$

$\Rightarrow$ Curve cut at right angle and hence angle between them is 90°

K. L. E Society's

G. I. Bagewadi Arts, Science and Commerce college, Nipani.

Department of Mathematics

Practice test - I

For the year 19-20

Class :- BSc III[rd] sem

Roll no :- 201

Date :- 26/10/19

4/10

1) Prove that monotonic increasing bounded above sequence is convergent and converges to its lub.

Proof :- Let $\{a_n\}_{n \in N}$ be monotonic increasing and bounded above.

Let 'l' be lub (supremum) of $\{a_n\}$.

$\therefore$ By def^n, $a_n < l \ \forall n$ —(i)

and for $\varepsilon > 0$, $\exists$ +ve integer 'm' such that

$a_m > l - \varepsilon$ — (ii)

And $\{a_n\}$ be monotonic increasing.

$\therefore \forall \ n \geq m$, $a_n \geq a_m$ —(iii)

from (i) & (ii) & (iii) we have,

$l > a_n \geq a_m > l - \varepsilon$, $\forall n \geq m$

i.e. $l - \varepsilon < a_m \leq a_n < l < l + \varepsilon \quad \forall n \geq m$

i.e. $l - \varepsilon < a_n < l + \varepsilon \quad \forall n \geq m$

i.e. $|a_n - l| < \varepsilon \quad \forall n \geq m$

$\Rightarrow$ sequence $\{a_n\}$ is convergent & converges to its lub 'l'.

Thus monotonic increasing bounded above Sequence is convergent & converges to it supremum.

2) Prove that.

$$\frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$

<u>Proof</u> :- Since u & v are functions of r & θ and r & θ are functions of x & y, we have,

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}, \quad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial y}$$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial v}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}, \quad \frac{\partial v}{\partial y} = \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial v}{\partial \theta}\cdot\frac{\partial \theta}{\partial y}$$

Consider.

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} & \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial y} \\ \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial v}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} & \frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial v}{\partial \theta}\cdot\frac{\partial \theta}{\partial y} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix} \begin{vmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)}$$

i.e. $$\frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$

K. L. E. Society's

G. I. Bagewadi Arts, Science & Commerce College,
Nipani.

Department of Mathematics

Practice test - II

For the year 2019-20

3/10

class :- BSc IIIrd sem   Date :- 26/10/19

Roll no :- 201

1) Find the volume of solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ above the x-axis.

$\Rightarrow$ Let equation of curve is,

$x^{2/3} + y^{2/3} = a^{2/3}$. The parametric equation of the curve are, $x = a\cos^3 t$, $y = a\sin^3 t$, Here 't' varies from 0 to $\pi/2$

Required volume $\rightarrow$ = 2 x volume generated by the curve in the first quadrant.

$$= 2 \times \int_0^{\pi/2} \pi y^2 \frac{dx}{dt} . dt$$

$$= 2\pi \int_0^{\pi/2} a^2$$

2) State and prove Lagrange's Theorem for groups.

⇒ Statement :- The order of a subgroup of a finite group 4 is a divisor of the order of the group G, i.e. H is a subgroup of finite group G then $o(H)/o(G)$

Proof :- Let the order of finite group $G = o(G) = n$, and Let H be a subgroup of G. i.e. $H = o(H) = m$.

Consider the right cosets

$$Ha = \{h_1 a, h_2 a, h_3 a \cdots h_m a\}$$

where. $H = \{h_1, h_2, h_3 \cdots h_m\}$ and $a \in G$

Now all the elements of Ha are distinct, otherwise suppose.

$$h_i a = h_j a$$

$$h_i a a^{-1} = h_j a a^{-1} \quad \because \text{ operate } a^{-1} \text{ on both sides}$$

$$h_i e = h_j e$$

$$h_i = h_j$$

which is not true,

since all the elements in H are distinct.

since the group G is finite then number of such right cosets will be finite.

Let the number of distinct right cosets be k, then the union of k distinct right cosets of H in G is equal to G.

$$G = Ha_1 \cup Ha_2 \cup Ha_3 \cdots \cdots \cup Ha_k$$

Then each coset is having 'm' elements.

But G is having 'n' elements as order of

$$G = o(G) = n$$

$$\therefore n = mk$$

$$\Rightarrow m/n \quad \text{where } m \in Z$$

$$\Rightarrow o(H) / o(G)$$

Thus, the order of the subgroup H divides the order of the group G.

PAGE :
DATE : / /

K.L.E. Society's
G.I. Bagewadi Arts, Science and Commerce College Nipani
Department of mathematics.
practice Test - I

Roll No: 227 Date: 26/10/2019

class : B.sc III rd Sem

1/10

1) Prove that monotonic increasing bounded above seq. is convergent and converges to its lub.

⟹ Proof:- Let $\{a_n\}_{n\in N}$ be monotonic increasing and bounded above.

let l be least upper bound of $\{a_n\}$

∴ By def^n $a_n < l \ \forall n$ — ①

& for $\varepsilon > 0$ ∃ +ve integer m. s.t $a_m > l - \varepsilon$ — ②

and $\{a_n\}$ be monotonic increasing

∴ $\forall \ n \geq m, \ a_n \geq a_m$ — ③

from ① ② & ③ we have,

$l > a_n \geq a_m > l - \varepsilon \ \forall \ n \geq m$

i.e $l - \varepsilon < a_m \leq a_n < l < l + \varepsilon \ \forall \ n \geq m$

i.e $l - \varepsilon < a_n < l + \varepsilon \ \forall \ n \geq m$

i.e $|a_n - l| < \varepsilon \ \forall \ n \geq m$

⟶ seq $\{a_n\}$ is convergent and converges to least upper bound 'l'

Thus monotonic increasing bounded above sequence is convergent and converges to its supremum.

Sulekha

2) prove that $\frac{\partial(u\ v)}{\partial(r,\theta)} \frac{\partial(r\ \theta)}{\partial(x\ y)} = \frac{\partial(u\ v)}{\partial(x\ y)}$

⟹ Solⁿ:- Since u & v are functions of x & y and x & y are functions.

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta}$$

Consider $\frac{\partial(u\ v)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix}$

$$\begin{vmatrix} \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} \\ \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u\ v)}{\partial(r\ \theta)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u\ v)}{\partial(r\ \theta)} = \frac{\partial(u\ v)}{\partial(x\ y)} \frac{\partial(x\ y)}{\partial(r,\theta)}$$

$$\therefore \frac{\partial(u\ v)}{\partial(r\ \theta)} \frac{\partial(r\ \theta)}{\partial(x\ y)} = \frac{\partial(u\ v)}{\partial(x\ y)}$$

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DATE : / /

K. L. E. Society's
G. I. Bagewadi Arts, Science And Commerce
college Nipani
Department of Mathematics
Practice Test - II

4/10

Roll No:- 227
Date: 26/10/2019
class :- B.sc III rd sem.

1) Find the volume of solid generated by resolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.

⇒ Solution: Given $x^{2/3} + y^{2/3} = a^{2/3}$ — ①

$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ — ②

$\cos^2\theta + \sin^2\theta = 1$ — ③

from ② & ③ $\left(\frac{x}{a}\right)^{2/3} = \cos^2\theta$, $\left(\frac{y}{b}\right)^{2/3} = \sin^2\theta$

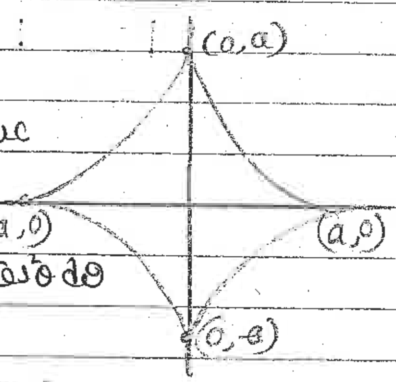
raise the power 3/2

$x = a\cos^3\theta$ & $y = a\sin^3\theta$

which are parametric eqn of ①

$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$, $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

symmetric about both the axis :



perimetre of astroids = 4 × length of arc present 1st quadrent 0 to π/2

$= 4\int_0^{\pi/2} \sqrt{9a^2\cos^4\theta\sin^2\theta + 9a^2\sin^4\theta\cos^2\theta}\, d\theta$

$= 4\int_0^{\pi/2} \sqrt{9a^2\cos^2\theta\sin^2\theta(\sin^2\theta + \cos^2\theta)}\, d\theta$

$= 4\int_0^{\pi/2} \sqrt{9a^2\cos^2\theta\sin^2\theta\,(1)}\, d\theta$

$= 4 \times 3a \int_0^{\pi/2} \cos\theta\sin\theta\, d\theta$

$$= 12a \int_0^{\pi/2} \frac{\cos 2\theta}{2}$$

$$= 12a \left[\frac{\sin 2\theta}{2} \right]_0^{\pi/2}$$

$$= \overset{6}{\cancel{12}}a \left[\frac{\sin 2 \times \pi/2}{\cancel{2}} - \frac{\sin 0}{2} \right]$$

$$= 6a$$

~~2) Solve $y = 2px + y^2 pq$~~

$\Rightarrow$ ~~Soln: $y = 2px + y^2 pq$~~ — (1)

~~$2px = y - y^2 pq$~~

~~$2x = \frac{y}{p} - \frac{y^2}{q}$~~ — (2)

~~diff w.r.t y~~

~~$$2\frac{1}{p} = \left[\frac{1}{p}(1) + y\left(\frac{-1}{p^2}\right)\frac{dp}{dy}\right] - \left[\frac{1}{q}(2y) + y^2\left(\frac{-1}{q^2}\right)\frac{dq}{dy}\right]$$~~

~~$$\frac{2}{p} = \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} - \left(\frac{2y}{q} - \frac{y^2}{q^2}\frac{dq}{dy}\right)$$~~

~~$$\therefore \frac{1}{p} + \frac{y}{p^2}\frac{dp}{dy}$$~~

PAGE :
DATE : / /

2) State and prove Laugrage's Theorem for group.

⇒ Statement: If G is a finite group and H is subgroup of G then order of H divides order of G

Proof:- Let $O(G) = n$ Let H be a subgroup of G Such that $O(H) = m$
Now order of G is given to be finite
Hence the number of distinct right Cosets of H in G are finite
Let 'K' be the number of distinct right cosets of H in G
Now any 2 cosets have the sums number of element and H C He
is the cosets having 'm' number of elements and hence the set of all distinct right co-sets of H in G forms a partition of G we have number of elements in (G) is equal to Km But

$O(G) = n$

$n = mK$

$m | n$

ie $O(H) / O(G)$

K.L.E Societys

G.I. Bagewadi Arts Science & Commerce College Nipani

Department of Mathematics

for the year

Roll No:- 224 Marks obtained 1/10

class:- BSc III sem Max marks :- 10

Answer the any two of the following

1) Prove that monotonic convergent and converges of it's Lub.

proof :- let $\{a_n\}_{n \in N}$ be monotonic increasing and bounded above

let 'l' be least upper bound (Lub) [supremum] of $\{a_n\}$

$\therefore$ By defination $a_n \le l \quad \forall n$ — (i)

and for $\varepsilon > 0$ $\exists$ +ve integer such that

$a_m > l - \varepsilon$ — (ii)

And $\{a_n\}$ be monotonic increasing

$\therefore \forall \; n \geq m \quad a_n \geq a_m$ — (iii)

$l > a_n \geq a_m > l - \varepsilon \quad \forall \; n \geq m$ | from (i) & (ii), (iii)

ie $l - \varepsilon < a_m \le a_n < l < l + \varepsilon \quad \forall \; n \geq m$

ie $l - \varepsilon < a_n < l + \varepsilon \quad \forall \; n \geq m$

ie $|x_n - l| < \varepsilon \quad \forall \; n \geq m$

=> sequence $\{a_n\}$ is convergent & converges to its Lub l.

Thus monotonic increasing bounded above sequence is convergent and converges to its supremum

GoodLuck Page No. Date

2) Prove that $\frac{\partial(u\,v)}{\partial(r\,\theta)} \times \frac{\partial(r\,\theta)}{\partial(x\,y)} = \frac{\partial(u\,v)}{\partial(x\,y)}$

→ Since u & v are functions of x & y and x & y are functions of r & θ we have

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta}$$

consider

$$\frac{\partial(u\,v)}{\partial(r\,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} \\ \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u\,v)}{\partial(r\,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u\,v)}{\partial(r\,\theta)} = \frac{\partial(u\,v)}{\partial(x\,y)} \cdot \frac{\partial(x\,y)}{\partial(r\,\theta)}$$

$$\frac{\partial(u\,v)}{\partial(r\,\theta)} \cdot \frac{\partial(r\,\theta)}{\partial(x\,y)} = \frac{\partial(u\,v)}{\partial(x\,y)}$$

#

K.L.E Societys

G J Bagewadi Arts Science & commerce college Nipani

Department of Mathematics

for the year

Roll No:- 224 Marks obtained:- 8/10

Class:- BSc III sem Max marks:- 10

Answer the any two of the following:

1 Find the volume of solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ above the x-axis.

→ $x^{2/3} + y^{2/3} = a^{2/3}$ —①

$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ —②

We have $\cos^2\theta + \sin^2\theta = 1$ —③

from eq^n ② & ③.

$\left(\frac{x}{a}\right)^{2/3} = \cos^2\theta$ and $\left(\frac{y}{a}\right)^{2/3} = \sin^2\theta$

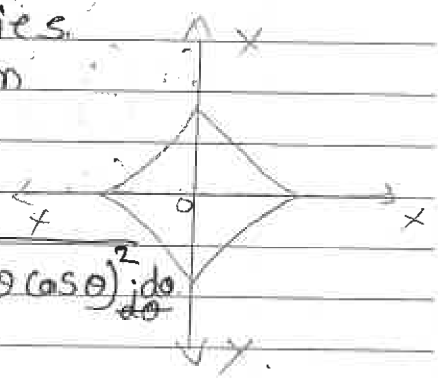
$x = a\cos^3\theta$ and $y = a.\sin^3\theta$

Which are parametric eq^n of eq^n ①.

$\frac{dx}{d\theta} = a\,3\cos^2\theta(-\sin\theta)$ $\frac{dy}{d\theta} = a\,3\sin^2\theta(\cos\theta)$

Symmetric about both the axies.

in Ist qudrant 'a' veries from 0 to $\pi/2$.



perimeter of asteroid.

$$4\int_0^{\pi/2} \sqrt{(-3a\cos^2\theta\sin\theta)^2 + (3a\sin^2\theta\cos\theta)^2}\,d\theta$$

$$4\int_0^{\pi/2} \sqrt{(9a^2\cos^4 a\sin^2\theta + 9a^2\sin^4\theta\cos^2\theta)}\,d\theta.$$

$$4\int_0^{\pi/2} \sqrt{9a^2\sin^2\theta\cos^2\theta(\cos^2\theta + \sin^2\theta)}\,d\theta$$

$$4\int_0^{\pi/2} \sqrt{9a^2 \sin^2\theta \cos^2\theta (1)} \cdot d\theta$$

$$4\int_0^{\pi/2} 3a \sin\theta \cdot \cos\theta \cdot d\theta$$

divide & multiplied by 2.

$$4\int_0^{\pi/2} 2 \times \frac{3}{2} a \sin\theta \cdot \cos\theta \cdot d\theta$$

$$\frac{4}{2}\int_0^{\pi/2} 3a (2\sin\theta \cdot \cos\theta) d\theta$$

$$2\int_0^{\pi/2} 3a \cdot \sin 2\theta \cdot d\theta$$

$$6a\int_0^{\pi/2} \sin 2\theta \cdot d\theta$$

$$6a\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$6a\left[\frac{-\cos 2\pi/2}{2} + \frac{\cos 0}{2}\right]$$

$$6a\left[\frac{-\cos\pi}{2} + \frac{\cos 0}{2}\right]$$

$$6a\left(\frac{-(-1)}{2} + \frac{1}{2}\right)$$

$$6a\left(\frac{1}{2} + \frac{1}{2}\right)$$

$$6a\left(\frac{2}{2}\right)$$

$$6a$$

K.L.E. Society's

G.I. Bagewadi Arts, Science And Commerce College, Nipani

Department of Mathematics

Practice Test - I

For the year :

Class :- B.Sc III sem   Date :-

Roll No :- 175

6/10

Answer the following questions.

1) prove that monotonic increasing bounded above seq. is convergent and converges to it's lub.

⇒ Let $\{a_n\}_{n\in N}$ be monotonic increasing and bounded above.

Let 'l' be lub (supremum) of $\{a_n\}$

$\therefore$ By defn, $a_n < l \quad \forall n$ ——— (i)

and for $\varepsilon > 0$, $\exists$ +ve integer 'm' s.t.

$$a_m > l - \varepsilon \quad \text{——— (ii)}$$

And $\{a_n\}$ be monotonic increasing,

$\therefore \forall n \geq m$, $a_n \geq a_m$ ——— (iii)

Now from (i), (ii) and (iii), We have

$$l > a_n \geq a_m > l - \varepsilon$$

i.e. $l - \varepsilon < a_m \leq a_n < l < l + \varepsilon \quad \forall\ n \geq m$

i.e. $l - \varepsilon < a_n < l + \varepsilon \quad \forall\ n \geq m$

i.e. $|a_n - l| < \varepsilon \quad \forall\ n \geq m$

$\Rightarrow$ seq $\{a_n\}$ is convergent and converges to it's lub 'l'.

Thus, Monotonic increasing bounded above seq. is convergent and converges to it's lub.

2) prove that $[(p \longrightarrow r) \vee (q \longrightarrow r)] \leftrightarrow [(p \wedge q) \longrightarrow r]$ is a tautology.

p	q	r	$p \to r$	$q \to r$	$a \vee b$	$p \wedge q$	$(p \wedge q) \to r$	$c \leftrightarrow d$
			a	b	c		d	
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	T
T	F	T	T	T	T	F	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	T	T	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

Here, the last column contains only true values, Hence it is a tautology.

K.L.E. Society's

G. I. Bagewadi Arts, Science And Commerce College, Nipani.

Department of Mathematics.

Practice test - II

9/10

For the year :-

Class :- B.Sc III sem Date :-

Roll No :- 175

Answer the following questions.

1) Find the volumn of solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ above the x-axis.

→ Given eqn of asteroid,

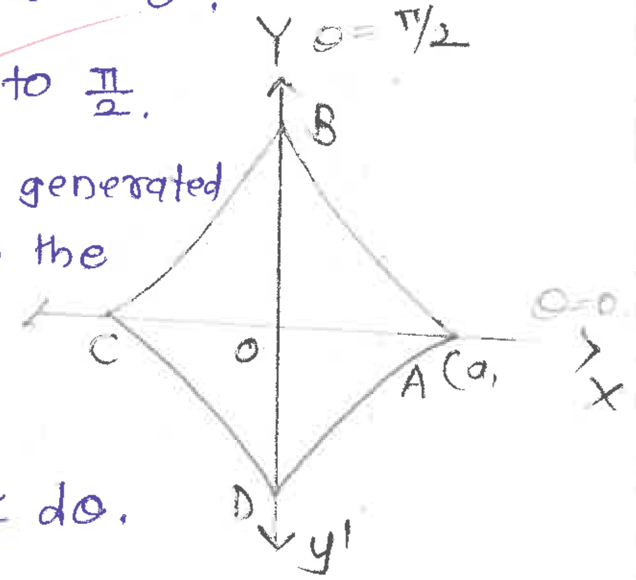
$$x^{2/3} + y^{2/3} = a^{2/3}$$

the parametric eqns of the asteroid is,

$$x = a\cos^3\theta \quad \text{and} \quad y = a\sin^3\theta .$$

Here 'θ' varies from 0 to $\frac{\pi}{2}$.

Req. volume = 2 · volume generated by arc in the first quadrant.



$$= 2 \times \int_0^{\pi/2} \pi y^2 \frac{dx}{d\theta}\, d\theta .$$

$$= 2\pi \int_0^{\pi/2} (a\sin^3\theta)^2 \, 3a\cos^2\theta\,(-\sin\theta)\, d\theta$$

$$= 2\pi \int_0^{\pi/2} -a^2 \sin^6\theta \; 3a\cos^2\theta \sin\theta \, d\theta$$

$$= -2\pi \times 3a^3 \int_0^{\pi/2} \sin^7\theta \cos^2\theta \, d\theta.$$

$$= -6a^3\pi \left[\frac{6\times4\times2\times1}{9\times7\times5\times3\times1} \right]$$

$$= -6\pi a^3 \times \frac{16}{9\times35}$$

$$= -\frac{32\pi a^3}{105}.$$

i.e. $V = \dfrac{32\pi a^3}{105}$.

---

2) State and prove Lagrange's thm for groups.

⇒ Statement :→ If 'G' is a finite group and 'H' is subgroup of 'G' then order of H divides order of G.

Proof :→ let $O(G) = n$. and 'H' be a subgroup of 'G' s.t $O(H) = m$.

Now, order of 'G' is given to be finite. Hence, the no. of distinct right coset of H in G are finite.

let 'k' be the no. of distinct right cosets of in G.

Now, any two cosets have the same no of elements and $H = He$ is a coset having 'm' no of elements and since, the set of all distinct right cosets of 'H' in G form a partition of 'G'

We have, the no. of elts in G is equal to $k \cdot n$.

i.e. $O(G) = n$

$\Rightarrow n = mk$

i.e. $m \mid n$

$O(H) \mid O(G)$.

ROLL NO. | DATE

K. L. E Society's
G. I. Bagewadi Arts . Science and commerce college Nippani
Department of Mathematics
PRACTICE TEST - I
For the year - 2019-20

Class :- B.Sc. III Sem   Date :- 26/10/2019
Roll No :- 83

5/10

1] Prove that monotonic increasing bounded above sequence is convergent and converges to its lub.

proof :- let $\{a_n\}_{n\in N}$ be monotonic increasing and bounded above
let 'l' be least upper bound (lub) [supremum] of $\{a_n\}$
∴ By defination $a_n < l \ \forall n$ — (i)
and for $\varepsilon > 0$ ∃ +ve integer such that
$a_m > l - \varepsilon$ — (ii)
And $\{a_n\}$ be monotonic increasing
∴ $\forall n \geq m$, $a_n \geq a_m$ — (iii)
from (i) (ii) & (iii) we have
$l > a_n \geq a_m > l - \varepsilon \quad \forall . n \geq m$
i.e $l - \varepsilon < a_m \leq a_n < l < l + \varepsilon \quad \forall n \geq m$
i.e $l - \varepsilon < a_n < l + \varepsilon \quad \forall n \geq m$
i.e $|a_n - l| < \varepsilon \quad \forall n \geq m$
⇒ sequence $\{a_n\}$ is convergent & converges to its lub 'l'
Thus monotonic increasing bounded above sequence is convergent and converges to it's supremum.

Modern

2] Prove that

$$\frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$

→ Since u & v are functions of x & y and x & y are functions of r & θ we have

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta}$$

Consider $\frac{\partial(u,v)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix}$

$$\begin{vmatrix} \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} \\ \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,\theta)}$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$

CLASS | SUBJECT
ROLL NO. | DATE

K. L. E Society's
G. I. Bagewadi Arts, Science and commerce college Nippani
Department of Mathematics
PRACTICE TEST - II
For the year -

Class :- B.Sc - III Sem Date :- 26/10/2019
Roll No :- 83

7/10

1) Find the volume of solid generated by revolving the astenoid $x^{2/3} + y^{2/3} = a^{2/3}$ above the x-axis.

→ $x^{2/3} + y^{2/3} = a^{2/3}$ —— ①

$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ —— ②

we have $\cos^2\theta + \sin^2\theta = 1$ —— ③

From eqn ② & ③

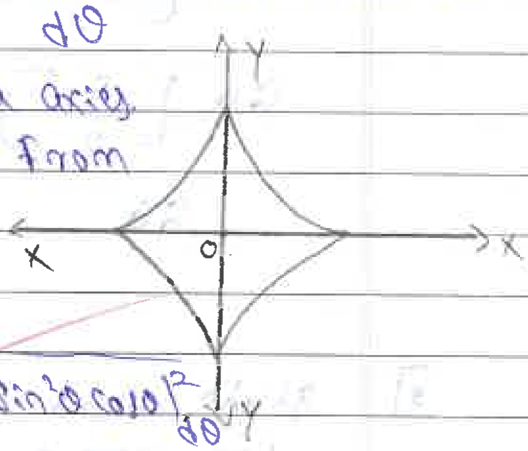
$\left(\frac{x}{a}\right)^{2/3} = \cos^2\theta$ and $\left(\frac{y}{a}\right)^{2/3} = \sin^2\theta$

$x = a\cos^3\theta$ and $y = a\sin^3\theta$

which are parametric equations of eqn ①

$\frac{dx}{d\theta} = a\,3\cos^2\theta\,(-\sin\theta)$ $\frac{dy}{d\theta} = a\,3\sin^2\theta\,(\cos\theta)$

Symmetric about both the axies.
in Ist quadrant θ veries from 0 to π/2



perimeter of astenoid.

$4\int_0^{\pi/2}\sqrt{(-3a\cos^2\theta\sin\theta)^2 + (3a\sin^2\theta\cos\theta)^2}\,d\theta$

$4\int_0^{\pi/2}\sqrt{(9a^2\cos^4\theta\sin^2\theta + 9a^2\sin^4\theta\cos^2\theta)}\,d\theta$

$4\int_0^{\pi/2}\sqrt{9a^2\sin^2\theta\cos^2\theta\,(\cos^2\theta + \sin^2\theta)}\,d\theta$

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$4\int_0^{\pi/2} \sqrt{9a^2 \sin^2\theta \cos^2\theta}\,(1)\,d\theta$

$4\int_0^{\pi/2} 3a \sin\theta \cos\theta \, d\theta$

divide and multiplie by 2

$4\int_0^{\pi/2} 2\times\frac{3}{2} a \sin\theta \cos\theta \, d\theta$

$\frac{4}{2}\int_0^{\pi/2} 3a(2\sin\theta\cos\theta)\, d\theta$

$2\int_0^{\pi/2} 3a \sin 2\theta \, d\theta$

$6a\int_0^{\pi/2} \sin 2\theta \, d\theta$

$6a\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$

$6a\left[-\frac{\cos 2\pi/2}{2} + \frac{\cos 0}{2}\right]$

$6a\left[-\frac{\cos\pi}{2} + \frac{\cos 0}{2}\right]$

$6a\left[-\frac{(-1)}{2} + \frac{1}{2}\right]$

$6a\left[\frac{1}{2}+\frac{1}{2}\right]$

$6a\left[\frac{2}{2}\right]$

$6a$

2] State and prove Lagrange's Theorem for group.

→ statement :- If G is a finite group and H is a subgroup of G then order of H divides order of G.

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Proof :- let $O(G) = n$ let H be a subgroup of G such that $O(H) = m$

Now order of G is given to be finite

Hence the number of distict right cosets of H in G are finite.

let 'k' be the number of distict right cosets of H in G

Now any two cosets have the same number of element and $H = He$ is the cosets having 'm' number elements and hence the set of all distict right cosets of H in G forms a partition of G we have number of elements in (G) is equal to km

But $O(G) = n$

$\Rightarrow n = mk$

$\Rightarrow m | n$

i.e $O(H) | O(G)$.

K. L. E SOCIETYS

G. I. Bagewadi Arts Science and commerce College
Nipani

DEPARTMENT OF MATHEMATICS

For the year 2019-20

Roll No. :- 87 Class :- BSc IV sem

Date :- 15/10/2020

4/10

* Answer the following question:-

1) Prove that series $\Sigma 1/n^p$ is convergent if $p>1$ & divergent if $p\le 1$

→ Proof:-

Given series $\Sigma 1/n^p = 1 + 1/2^p + 1/3^p + \cdots + 1/n^p + \cdots$

i) let $p > 1$

Now $1/2^p = 1$

$$\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^p} + \frac{1}{2^p} \qquad \because \frac{1}{3^p} < \frac{1}{2^p} \quad (p > 1 > 0)$$

$$\text{i.e } \frac{1}{(2^1)^p} + \frac{1}{(2^2-1)^p} < \frac{2}{2^p} = \frac{1}{2^{p-1}}$$

$$\& \quad \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p} < \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p}$$

$$\text{i.e } \frac{1}{(2^2)^p} + \frac{1}{(2^2+1)^p} + \frac{1}{(2^2+2)} + \frac{1}{(2^2+3)^p} < \frac{4}{4^p} = \frac{1}{4^{p-1}} = \frac{1}{(2^{p-1})^2}$$

$$\text{Simillarly } \frac{1}{8^p} + \cdots + \frac{1}{15^p} < \frac{1}{(2^{p-1})^3}$$

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Adding all these we get

$$1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \cdots + \frac{1}{7^p}\right) + \cdots < 1 + \frac{1}{2^{p-1}} + \frac{1}{(2^{p-1})^2} + \cdots$$

$$\text{i.e } \sum_{n=1}^{\infty} \frac{1}{n^p} < \sum_{n=1}^{\infty} \frac{1}{(2^{p-1})^{n-1}}$$

$$\& \quad \sum_{n=1}^{\infty} \frac{1}{(2^{p-1})^{n-1}} = \sum_{n=1}^{\infty} r^{n-1} \text{ is geometric series with}$$

$$r = \frac{1}{2^{n-1}} < 1 \quad \text{as } p > 1$$

& hence it is convergent

$\therefore$ from (1) by I comp. test $\sum 1/n^p$ is also convergent

ii) If $p = 1$ then $\sum_{n=1}^{\infty} 1/n^p = \sum_{n=1}^{\infty} 1/n$ is divergent by cauchy's critieria for convergence.

iii) If $p < 1$ then $n^p < n$ $\quad (3^{1/2} < 3)$

$$\frac{1}{n^p} > \frac{1}{n}$$

$$\Rightarrow \sum 1/n^p > \sum 1/n$$

& $\sum 1/n$ is divergent by II comparision test

$\sum 1/n^p$ is also divergent

Thus $\sum_{n=1}^{\infty} 1/n^p$ is convergent if $p > 1$
& divergent if $p \le 1$.

2) Find $\frac{d}{dt}[\vec{A}(t)\ \vec{B}(t)\ \vec{C}(t)]$

$\Longrightarrow$ we know that $[\vec{A}\ \vec{B}\ \vec{C}] = \vec{A}\,(\vec{B} \times \vec{C})$
differentiating with respetive t.

i) $\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}] = \frac{d}{dt}[\vec{A} \cdot (\vec{B} \times \vec{C})]$

$$= \vec{A}\,\frac{d}{dt}(\vec{B} \times \vec{C}) + \frac{d\vec{A}}{dt}\,(\vec{B} \times \vec{C})$$

$$= \vec{A}\left\{\vec{B} \times \frac{d\vec{C}}{dt} + \frac{d\vec{B}}{dt} \times \vec{C}\right\} + \frac{d\vec{A}}{dt} \cdot (\vec{B} \times \vec{C})$$

$$= \vec{A}\left(\vec{B} \times \frac{d\vec{C}}{dt}\right) + \vec{A}\left(\frac{d\vec{B}}{dt} + \vec{C}\right) + \frac{d\vec{A}}{dt}(\vec{B} \times \vec{C})$$

$$= \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right]$$

$$\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}] = \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right]$$

3) State & Prove Lelibnitz's test for convergence of alternating series

Statement :- Let $\sum (-1)^{n-1} u_n$ be alternating series & it is converge -nt if i) $u_n \geq u_{n+1}\ \forall n$

ii) $\lim_{n\to\infty} u_n = 0$.

→ Proof :-

Given Series $\sum (-1)^{n-1} u_n$.

$= u_1 - u_2 + u_3 + u_4 + \text{------}$

let S_n be n^{th} Partial sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} u_n$ Then $S_{2n} = u_1 - u_2 + u_3 - u_4 + \text{---}\ u_{2n-1} - u_n$.

$= u_1 - (u_2 - u_3) + (u_4 + u_5)$ $(1/c + 1/h_2)\,\rho u$

$= u_1 - [((u_2 - u_3) + (u_4 - u_5) + (u_6 - u_7) + \text{------} + (u_{2n-2} - u_n + u_{2n})$

$= u_3 - ((u_2 - u_3)(u_4 - u_5) + (u_6 - u_7) + \text{---} + (u_{2n-2} - u_{2n-1}) + u_n)$

$= u_1 - (+ve) + (+ve) + (+ve) + \text{----} + (+ve)$

$\because u_n \geq u_{n+1}\ \forall n$

$= u_1 - (+ve\ \text{no.}) < u_1$

$S_{2n} < u_1\ \forall n$

$\Rightarrow$ Seq $\{S_n\}$ is bounded → ①

Next we have to prove sequence

$\{S_{2n}\} = \{S_2, S_4, S_6 \text{---}\}$

consen $S_{2n+2} - S_n = u_{2n+1} - u_{2n+2} \geq 0$.

$\because u_{n+1} \geq u_{n+1}\ \forall n$

i.e $S_{2n+2} - S_{2n} \geq 0$

$\Rightarrow S_{2n+2} \geq S_{2n} \ \forall n$.

$\Rightarrow$ Seq $\{S_{2n}\}$ is monotonic increasing — ②

Thus from ① & ②

Seq. $\{S_{2n}\}$ is bounded & monotonic increasing & hence it is convergent

Let it convergence to l i.e $\lim_{n\to\infty} S_{2n} = l$

Simillarly seq. $S_{2n-1} = \{S_1, S_3, S_5 - - - -\}$ is convergent

we have to prove that $\{S_{2n-1}\}$ also convergce to l

i.e $\lim_{n\to\infty} S_{2n-1} = \lim_{n\to\infty} [U_1 - U_2 + U_3 - U_4 + - - - - + U_{2n-1}]$

$= \lim_{n\to\infty} [(U_1 - U_2 + U_3 - U_4 + - - - - - + U_{2n-1} - U_{2n}) + U_{2n}]$

$= \lim_{n\to\infty} (S_{2n} + U_{2n})$

$= \lim_{n\to\infty} S_{2n} + \lim_{n\to\infty} U_{2n}$

$= l + 0 \quad \because \lim_{n\to\infty} U_n = 0$

$\Rightarrow$ Seq $\{S_{2n-1}\}$ also converge to l

Thus sequence $\{S_{2n}\}$ & $\{S_{2n-1}\}$ convergent & converge to l & hence sequence $\{S_n\}$ is convergent.

Then by defination series $\sum(-1)^{n-1} U_n$ is convergent

Hence alternating series $\sum(-1)^{n-1} U_n$ is convergent if i) $U_n \geq U_{n+1} \ \forall n$

ii) $\lim_{n\to\infty} U_n = 0$.

K.L.E. SOCIETY'S

G. I. Bagewadi Arts Science and Commerce College
Nipani

DEPARTMENT OF MATHEMATICS

For the year 2019-20

Roll No.:- 87 | Class:- B.Sc IV Sem

Date:- 15/10/2020

4/10

* Answer the following questions.

1) Find the fourier expansion for the function $f(x) = |x|$ in $(-1\ 1)$

⟹ Let us consider $f(x) = |x|$ in $(-l\ l)$

→ $2l = 2l \Rightarrow l = l$.

∴ Required fourier series for the function $f(x) = |x|$ in $(-l\ l)$ is given by.

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \rightarrow (1)$$

Now $f(x) = |x| = \begin{cases} x & \text{if } x \in (0\ l) \\ -x & \text{if } x \in (-l, 0) \end{cases}$

$$f(-x) = |x| = x = f(x)$$

∴ The function is even function

or $\phi(-x) = x$ $\phi(x) = -x$ as intervals $(-l\ l)$

$$\phi(-x) = -x = \phi(x)$$

⟹ $f(x)$ is an even function

∴ $b_n = 0$.

Now we Proceed to find a_0 & a_n as follows

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$= \frac{1}{l}\int_{-l}^{l} |x|\,dx = \frac{1}{l}\left\{\int_{-l}^{0} -x\,dx + \int_{0}^{l} x\,dx\right\}$$

$$= \frac{1}{l}\left\{\left(-\frac{x^2}{2}\right]_{-l}^{0} + \left(\frac{x^2}{2}\right]_{0}^{l}\right\}$$

$$= \frac{1}{l}\left[-\left(0 - \frac{l^2}{2}\right) + \frac{l^2}{2} + 0\right]$$

$$= \frac{1}{l}\left(\frac{l^2}{2} + \frac{l^2}{2}\right)$$

$$= \frac{1}{l}\left[\frac{2l^2}{2}\right] \Rightarrow l$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{l}\int_{-l}^{l} |x| \cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{l}\int_{0}^{l} x\cos\left(\frac{n\pi x}{l}\right)dx$$

2) Define kernel of homomorphism and prove that ker f is normal to subgroup.

→ Defination :- Let (G, o) & $(H, *)$ be groups

Let $\phi : (G, o) \rightarrow (H, *)$ be a group homomorphism then kernel of ϕ is the subset of the domain of ϕ defined as

$\ker(\phi) = \{x \in G : \phi(x) = e_H\}$.

Proof :—

Let $f : G \rightarrow G'$ be a homomorphism & e, e' be the identity elts of G & G' respectively then

$\ker f = \{x \in G \mid f(x) = e'\}$

Now as $e \in G$ $f(e) = e'$

$\therefore e \in \ker f$ $\ker f \neq \phi$

Next let $x, y \in \ker f$ then

$f(x) = e'$ $f(y) = e'$

consider $f(xy^{-1}) = f(x)\, f(y^{-1})$

$= e' [f(y)]^{-1}$

$= e' (e')^{-1}$

$= e'$

$f(xy^{-1}) = e'$

$\Rightarrow xy^{-1} \in \ker f$

Thus $\forall x, y \in \ker f$ $xy^{-1} \in \ker f$

$\therefore \ker f$ is subgroup G

Now let $h \in \ker f$

$\Rightarrow f(h) = e'$ & $a \in G$

then $f(aa^{-1}) = f(a) . f(ha^{-1})$

$= f(a)\, f(h)\, f(a^{-1})$

$= f(a) . e' [f(a)]$

$= e' f(a) (f(a))^{-1}$

$= e' e'$

$f(aha^{-1}) = e'$

$aha^{-1} \in \ker f$

$\rightarrow \ker f$ is normal subgroup of G

$\forall$ $h \in \ker f$ & $\forall a \in G$

3) Find particular integral for $\frac{1}{f(D)} e^{ax}$.

$\rightarrow$ Let $f(D) y = f(x) \rightarrow (1)$

i.e $f(D)\, y = e^{ax}$

$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots + a_{n-1} D + a_n) y = e^{ax} \rightarrow (2)$

$\frac{d}{dx} = D$ $\frac{d^2}{dx^2} = D^2$ & so on.

$D(e^{ax}) = a e^{ax}$ $D^2 (e^{ax}) = a^2 e^{ax}$

$\dots D^n(e^{ax}) = a^n e^{ax}$.

$\therefore$ Equation (2) becomes

$[D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots + a_{n-1} D + a_n]$

$f(D)\, y = f(a)\, y$

or $f(D) e^{ax} = f(a) e^{ax}$

operating $\frac{1}{f(D)}$ on both the side we get

$$\frac{1}{f(D)} f(D) e^{ax} = \frac{1}{f(D)} f(a) e^{ax}$$

$$e^{ax} = \frac{1}{f(D)} f(a) e^{ax}.$$

$$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}.$$ Provided $f(a) \neq 0$. —③

Suppose $f(a) = 0$ then $(D-a)$ is factor of $f(D)$

$\therefore f(D) = (D-a)\phi(D)$ where $\phi(a) \neq 0$

$$\frac{1}{f(D)} e^{ax} = \frac{1}{(D-a)\phi(D)} e^{ax}$$

$$= \frac{1}{(D-a)} \left[\frac{1}{\phi(D)} e^{ax} \right]$$

$$= \frac{1}{(D-a)} \left[\frac{1}{\phi(a)} e^{ax} \right]$$ by eqn③ $\phi(a) \neq 0$

$$= \frac{x e^{ax}}{\phi(a)} = \frac{x e^{ax}}{f'(a)}$$ $f'(a) \neq 0$

$\therefore$ P.I of $\frac{1}{f(D)} e^{ax} = \frac{e^{ax}}{f(a)}$ if $f(a) \neq 0$

$$= \frac{x e^{ax}}{f'(a)}$$ if $f'(a) \neq 0$ $f(a) = 0$

$$= \frac{x^2 e^{ax}}{f''(a)}$$ if $f''(a) \neq 0$ $f'(a) = 0$ ∮

So ~~on~~ ———

K.L.E. SOCIETY'S

G.I. Bagewadi Arts. Science and commerce college Nippani

DEPARTMENT OF MATHEMATICS

PRATICE TEST - II

for the year:

Class : B.Sc IV sem Date:

Roll No 214

6/10

Answer the following question

i) Find perticular integral for $\frac{1}{f(D)} e^{ax}$

⇒ let $f(D) y = \phi(x)$

ie $f(D) y = e^{ax} \longrightarrow ①$

$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_{n-1} D + a_n) y = e^{ax} \longrightarrow ②$

$\frac{d}{dx} = D, \quad \frac{d^2}{dx^2} = D^2$ & so on

$D(e^{ax}) = a e^{ax}, \quad D^2(e^{ax}) = a^2 e^{ax} \cdots$

$D^n(e^{ax}) = a^n e^{ax}$

eq^n ② becomes

$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_{n-1} D + a_n) y$

$= (a^n + a_1 a^{n-1} + a_2 a^{n-2} + \cdots + a + a_n) y$

$\Rightarrow f(D) y = f(a) y$

or $f(D) e^{ax} = f(a) e^{ax}$

operating $\frac{1}{f(D)}$ on both the side we get

$\frac{1}{f(D)} f(D) e^{ax} = \frac{1}{f(D)} f(a) e^{ax}$

$e^{ax} = \frac{1}{f(D)} f(a) e^{ax}$

or $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax} \quad \cdot f(a) \neq 0$

1) Define the term ~~homo~~ ~~kern~~ homomorphism. and prove that kerf is normal subgroup G.

Proof: let $f: G \to G'$ be homomorphism and e, e' be the identity elts of G and G' respectively then $\ker f = \{x \in G \mid f(x) = e'\}$

Now as $e \in G$ $f(e) = e'$.

$e \in \ker f$, $\ker f \neq 0$

Next let $x, y \in \ker f$ then $f(x) = e'$ $f(y) = e'$

Consider $f(xy^{-1}) = f(x)f(y^{-1}) = e'(f(y))^{-1}$

$= e'(e')^{-1}$

$f(xy^{-1}) = e'$

$xy^{-1} \in \ker f$

Thus $\forall x, y \in \ker f$ $xy^{-1} \in \ker f$

$\therefore \ker f$ is subgroup of G

Now let $h \in \ker f$

$\Rightarrow f(h) = e'$ & $a \in G$

then $f(aha^{-1}) = f(a) f(ha^{-1})$

$= f(a) f(h) f(a^{-1})$ f is

$= f(a) e' [f(a)]^{-1}$ homomorphism

$= e'e'$

$f(aha^{-1}) = e'$

$\Rightarrow aha^{-1} \in \ker f$

$\Rightarrow \ker f$ is normal subgroup of G

$\forall h \in \ker f$ & $\forall a \in G$

K.L.E SOCIETY'S

G. I. Bagewadi Arts, Science, and commerce college, Nipani

DEPARTMENT OF MATHEMATICS

For the year

Practice Test - I

Roll No: 214

Date:

Class: B.Sc. IV sem

4/10

Ans Answer the following question:-

1) Prove that series $\sum \frac{1}{n^p}$ is convergent if $p>1$ divergent if $p \leq 1$

Proof:

Given series $\sum \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$

i) let $p>1$

Now $1/2^p = 1$

$$\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^p} + \frac{1}{2^p} \qquad \because \frac{1}{3^p} < \frac{1}{2^p} \quad (p>1>0)$$

ie $\frac{1}{(2^1)^p} + \frac{1}{(2^2-1)^p} < \frac{2}{2^p} = \frac{1}{2^{p-1}}$

$$\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p} < \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p}$$

ie $\frac{1}{(2)^p} + \frac{1}{(2^2+1)^p} + \frac{1}{(2^2+2)^p} + \frac{1}{(2^2+3)^p} < \frac{4}{4^p} = \frac{1}{4^{p-1}} = \frac{1}{(2^{p-1})^2}$

Similarly $\frac{1}{8^p} + \cdots + \frac{1}{15^p} < \frac{1}{(2^{p-1})^3}$

— — — — — — — — —

adding all these we get

$$1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \cdots + \frac{1}{7^p}\right) + \cdots < 1 + \frac{1}{2^{p-1}} + \frac{1}{(2^{p-1})^2} + \cdots$$

ie $\sum_{n=1}^{\infty} \frac{1}{n^p} < \sum_{n=1}^{\infty} \frac{1}{(2^{p-1})^{n-1}}$

& $\sum_{n=1}^{\infty} \frac{1}{(2^{p-1})^{n-1}} = \sum_{n=1}^{\infty} r^{n-1}$ is geometric series with

$r = \frac{1}{2^{p-1}} < 1$ as $p>1$

and hence it is convergent

$\therefore$ from ① by comp. test $\sum \frac{1}{n^p}$ is also convergent

ii) If $p=1$ then $\sum_{n=1}^{\infty} \frac{1}{n^p} = \sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. cauchy's criteria for convergence

iii) If $p<1$ then $n^p<n$ $(3^{1/2}<3)$

$$\frac{1}{n^p} > \frac{1}{n}$$

$$\Rightarrow \sum \frac{1}{n^p} > \sum \frac{1}{n}$$

and $\sum \frac{1}{n}$ is divergent by II comparision

$\sum \frac{1}{n^p}$ is also divergent

Thus $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p>1$ & divergent if $p\le 1$

) Find $\frac{d}{dt}[\vec{A(t)}\ \vec{B(t)}\ \vec{c(t)}]$

$\Rightarrow$ W.K.T. $[\vec{A}\ \vec{B}\ \vec{C}] = \vec{A}\cdot(\vec{B}\times\vec{C})$ diff w.r. to 't'

(i) $\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}] = \frac{d}{dt}[\vec{A}\cdot(\vec{B}\times\vec{C})$

$$= \vec{A}\frac{d}{dt}(\vec{B}\times\vec{C}) + \frac{d\vec{A}}{dt}(\vec{B}\times\vec{C})$$

$$= \vec{A}\left\{\vec{B}\times\frac{d\vec{C}}{dt} + \frac{d\vec{B}}{dt}\times\vec{C}\right\} + \frac{d\vec{A}}{dt}(\vec{B}\times\vec{C})$$

$$= \vec{A}\left(\vec{B}\times\frac{d\vec{C}}{dt}\right) + \vec{A}\left(\frac{d\vec{B}}{dt}\times\vec{C}\right) + \frac{d\vec{A}}{dt}(\vec{B}\times\vec{C})$$

$$= \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right]$$

$$\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}] = \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right]$$

K.L.E Society's

G.I. Bagewadi Arts, Science and Commerce College Nipani

Department of Mathematics

PRACTICE TEST - I/II

~~For the year~~

Class ~~-~~ B.sc IV Sem Date - 15/10/2020

Roll. no - 100

Roll. no - 20
Reg. no - S1717626

SONA ROOPA Page No. 19
Date

CLASS - IV Sem

4/5

I] Answer the following questions.

1) Prove that Series $\sum \frac{1}{n^p}$ is convergent if $p>1$ and divergent if $p\leq 1$.

→ Statement →
Let $\sum_{n=1}^{\infty} u_n$ is Series of +ve terms S.t
$$\lim_{n\to\infty} n\left[\frac{u_n}{u_{n+1}} - 1\right] = l$$ then u_n is

i) Series convergent if $p>1$.
ii) Divergent if $p<1$
iii) Test fails if $p=1$.

Proof :-
Let $\sum u_n$ is series of +ve terms.
$\therefore u_n > 0 \ \forall n$.
comparing this series with P - series.
$$\sum v_n = \sum \frac{1}{n^p}$$

which is convergent if $p>1$ & is divergent if $p<1$

Now
$$\frac{v_n}{v_{n+1}} = \frac{\frac{1}{n^p}}{1/(n+1)^p} = \frac{(n+1)^p}{n^p}$$
$$= \left(1+\frac{1}{n}\right)^p$$
$$= {}^pC_0 + {}^pC_1 \, 1/n + {}^pC_2 \, 1/n^2 + \dots {}^pC_p \, 1/n^p$$
$$= 1 + p \cdot \frac{1}{n} + \frac{p(p-1)}{2!}\frac{1}{n^2} + \dots \frac{1}{n^p} \quad (1)$$

case i) If $P > 1$ &
choose l, s.t $P \geq l > 1$

$\implies \sum v_n = \sum \frac{1}{n^P}$ is convergent $(P>1)$

$\therefore$ By 4th comparision test series $\sum v_n$ is convergent.

$$\therefore \frac{u_n}{u_{n+1}} \geq \frac{v_n}{v_{n+1}}$$

$$\implies \frac{u_n}{u_{n+1}} \geq 1 + P.\frac{1}{n} + \frac{P(P-1)}{2!}\frac{1}{n^2} + \text{-------} + \frac{1}{n^P}$$

$$\implies \left(\frac{u_n}{u_{n+1}} - 1\right) \geq P.\frac{1}{n} + \frac{P(P-1)}{2!}\frac{1}{n^2} + \text{------} + \frac{1}{n^P}$$

$$\implies \left(\frac{u_n}{u_{n+1}} - 1\right) \geq \frac{1}{n}\left[P + \frac{P(P-1)}{2!}\frac{1}{n} + \frac{P(P-1)(P-2)(P-3)}{3!}\frac{1}{n^2} + \text{-------} + \frac{1}{n^{P-1}}\right]$$

$\implies$ Both side multiply 'n' we get

$$\implies n\left(\frac{u_n}{u_{n+1}} - P\right) \geq \left[P + \frac{P(P-1)}{2!}\frac{1}{n} + \frac{P(P-1)(P-2)(P-3)}{3!}\frac{1}{n^2} + \text{------} + \frac{1}{n^{P-1}}\right]$$

$$\implies \lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - P\right) \geq \lim_{n\to\infty} P + \frac{P(P-1)}{2!}\frac{1}{n} + \text{-----} + \frac{1}{n^{P-1}}$$

$$\implies l \geq P + 0 + 0 \text{ ----- } + 0$$

$l \geq P$ which is true.

$\therefore \sum u_n$ is convergent if $P > 1$.

case ii) :-

If $P < 1$.
choose l s.t $P \leq l < 1$.

$\Rightarrow \sum v_n = \sum \frac{1}{n^p}$ is divergent.

$\therefore$ By IVth comparision test $\sum u_n$ is divergent if $\frac{u_n}{u_{n+1}} \leq \frac{v_n}{v_{n+1}}$

$$\Rightarrow \frac{u_n}{u_{n+1}} \leq 1 + p.\frac{1}{n} + \frac{p(p-1)}{2!}\frac{1}{n^2} + \cdots + \frac{1}{n^p}$$

$$\Rightarrow \left(\frac{u_n}{u_{n+1}} - 1\right) \leq p.\frac{1}{n} + \frac{p(p-1)}{2!}\frac{1}{n^2} + \cdots + \frac{1}{n^p}$$

$$\Rightarrow \left(\frac{u_n}{u_{n+1}} - 1\right) \leq \frac{1}{n}\left(p + \frac{p(p-1)}{2!}\frac{1}{n} + \cdots + \frac{1}{n^{p-1}}\right)$$

multiplying 'n' we get

$$\Rightarrow n\left(\frac{u_n}{u_{n+1}} - 1\right) \leq p + \frac{p(p-1)}{2!}\frac{1}{n} + \cdots \frac{1}{n^{p-1}}$$

$$\Rightarrow \lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) \leq p + 0 + 0 \cdots + 0$$

$\Rightarrow l \leq p$. which is true.

$\Rightarrow \sum u_n$ is divergent if $p < 1$

Case iii) :-

if $p = 1$ test fails

for the series $\sum \frac{1}{n}$ & $\sum \frac{1}{n(\log n)^2}$

$$\therefore \lim_{n\to\infty} n\left[\frac{u_n}{u_{n+1}} - 1\right] = \lim_{n\to\infty} n\left[\frac{1/n}{1/n+1}\right]$$

$$= \lim_{n\to\infty} n\left[\frac{n+1}{n}\right]$$

$$= \lim_{n\to\infty}\left(1 - \lim_{n\to\infty}\left(\frac{u_n}{u_{n+1}}\right)\right)$$

$$= 1$$

$$\Rightarrow \lim_{n\to\infty} n\left[\frac{u_n}{u_{n+1}} - 1\right] = \lim_{n\to\infty} n\left[\frac{1}{n(\log n)^2}(n+1)(\log(n+1))^2\right]$$

$$= \lim_{n\to\infty} n\left[\frac{n+1}{n} \cdot \frac{\log(n+1)^2}{\log n^2}\right]$$

$= 1$

∴ But $\sum 1/n$ is divergent & $\sum \frac{1}{n(\log n)^2}$ is convergent.

2) Find $\frac{d}{dt}[\vec{A}(t)\ \vec{B}(t)\ \vec{c}(t)]$.

⟶ If $\vec{A}(t)$, $\vec{B}(t)$ and $\vec{c}(t)$ are differentiable vector factors of a scalar variable t then

$$\frac{d}{dt}[\vec{A}\vec{B}\vec{C}](t) = \left[\frac{d}{dt}\vec{A}\cdot\vec{B}\cdot\vec{C}\right](t) + \left[\vec{A}\ \frac{d}{dt}\vec{B}\ \vec{C}\right](t) + \left[\vec{A}\cdot\vec{B}\cdot\frac{d}{dt}\vec{C}\right](t)$$

Proof:- WKT $[\vec{A}\cdot\vec{B}\cdot\vec{C}](t) = \vec{A}(t)(\vec{B}\times\vec{C})(t)$

Diff w.r.t 't'

$$\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}](t) = \frac{d}{dt}[\vec{A}(t)\cdot(\vec{B}\times\vec{C})(t)]$$

$$= \vec{A}(t)\frac{d}{dt}(\vec{B}\times\vec{C})(t) + \frac{d}{dt}\vec{A}(t)\cdot(\vec{B}\times\vec{C})(t)$$

$$= \vec{A}(t)\left\{\vec{B}(t)\times\frac{d\vec{C}}{dt}(t) + \frac{d\vec{B}(t)}{dt}\times\vec{C}(t)\right\} + \frac{d\vec{A}(t)}{dt}\cdot(\vec{B}\times\vec{C})(t)$$

$$\Longrightarrow \vec{A}(t)\left(\vec{B}(t)*\frac{d\vec{C}(t)}{dt}\right) + \vec{A}(t)\cdot\left(\frac{d}{dt}\vec{B}(t)*\vec{C}\right) + \frac{d}{dt}\vec{A}(t)\cdot(\vec{B}\times\vec{C})(t)$$

$$\Longrightarrow \left[\vec{A}(t)\ \vec{B}(t)\ \frac{d\vec{C}(t)}{dt}\right] + \left[\vec{A}(t)\ \frac{d\vec{B}(t)}{dt}\cdot\vec{C}(t)\right] + \left[\frac{d}{dt}\vec{A}(t)\cdot\vec{B}(t)\ \vec{C}(t)\right]$$

$$\because [\vec{A}\cdot\vec{B}\,\vec{C}](t) = \vec{A}(t)\cdot(\vec{B}\times\vec{C})(t)$$

$$\Longrightarrow \frac{d}{dt}[\vec{A}(t)\,\vec{B}(t)\,\vec{C}(t)] = \left[\frac{d\vec{A}}{dt}\cdot\vec{B}\,\vec{C}\right](t) + \left[\vec{A}\,\frac{d\vec{B}}{dt}\,\vec{C}\right](t) + \left[\vec{A}\cdot\vec{B}\,\frac{d\vec{C}}{dt}\right] //.$$

II] Answer the following questions.

1) Define Kernel of homomorphism and Prove that Ker f is normal Subgroup of G.

→ Defⁿ:- The Kernal is the set of those elements of G whose image is the identity on the other side.

Thus if $f: G \to G'$, f is Homomorphism then $\{K = g \in G \,/\, f(g) = e'\}$ where e' is Identity element in G'.

Proof :-

Let $f: G \to G'$ be a Homomorphism, & e, e' be the Identity elements of G & G' respectively, then

$$\operatorname{Ker} f = \{x \in G \,/\, f(x) = e'\}$$

now $e \in G$

$f(e) = e'$

$\Longrightarrow e \in \operatorname{Ker} f$, $\operatorname{Ker} f \neq \phi$.

next let $x, y \in \operatorname{Ker} f$

then

$$f(x) = e'$$

$$f(y) = e'$$

consider, $f(xy^{-1}) = f(x)\, f(y^{-1})$ [f is Homomorphism]

$= e'\, [f(y)]^{-1}$ $\left(\because f(y^{-1}) = [f(y)]^{-1}\right)$

$= e'\, (e')^{-1}$

Sona Roopa

$\therefore f(xy^{-1}) = e'$

$\Rightarrow xy^{-1} \in \text{Ker} f.$

Thus, for all $x, y \in \text{Ker} f$, $x \cdot y^{-1} \in \text{Ker} f$.

$\therefore$ Ker f is subgroup of G.

now, let $h \in \text{Ker} f$.

$\Rightarrow f(h) = e$ & $a \in G$

then $f(aha^{-1}) = f(a) \cdot f(ha^{-1})$

$= f(a) \cdot f(h) \cdot f(a^{-1})$

$\therefore f$ is Homomorphism

$\Rightarrow f(a) \cdot e' [f(a)]^{-1}$

2) Find particular integral for $\frac{1}{f(D)} e^{ax}$

$\Rightarrow$ let $f(D) y = \phi(x)$

$$f(D) y = e^{ax} \quad \text{——— (1)}$$

$$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_{n-1} D + a_n) y = e^{ax} \text{ — (2)}$$

$\because \frac{d}{dx} = D$, $\frac{d^2}{dx^2} = D^2$ & so on,

$\therefore D(e^{ax}) = ae^{ax}$, $D^2(e^{ax}) = a^2 e^{ax}$ ------

$\therefore$ eq^n (2) becomes

$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots a_{n-1} D + a_n) y = (a^n + a_1 a^{n-1} + a_2 a^{n-2} + \cdots a_{n-1} a + a_n) y$

$$\Rightarrow f(D) y = f(a) y$$

or $f(D) e^{ax} = f(a) e^{ax}$

operating $\frac{1}{f(D)}$ on both the sides we get

$$\therefore \frac{1}{f(D)} \cdot f(D) e^{ax} = \frac{1}{f(D)} f(a) e^{ax}$$

$$e^{ax} = \frac{1}{f(D)} f(a) e^{ax}$$

or

$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ Provided

$f(a) \neq 0$. —— (3)

Sona Roopa

Suppose $f(a)=0$ then $(D-a)$ is a factor of $f(D)$

$\therefore f(D) = (D-a)\phi(D)$ where $\phi(a)\neq 0$

$$\therefore \frac{1}{f(D)} e^{ax} = \frac{1}{(D-a)\phi(D)} e^{ax}$$

$$= \frac{1}{(D-a)}\left[\frac{1}{\phi(a)} e^{ax}\right] \quad \left\{ \because \text{By eq}^n \text{ ③} \; \phi(a)\neq 0 \right.$$

$$= \frac{x e^{ax}}{\phi(a)} = \frac{x e^{ax}}{f'(a)} \text{ if } f'(a)\neq 0$$

$\therefore$ Particular integral of

$$\frac{1}{f(D)} e^{ax} = \frac{e^{ax}}{f(a)} \text{ if } f(a)\neq 0$$

$$= \frac{x e^{ax}}{f'(a)} \text{ if } f'(a)\neq 0$$

$$= \frac{x^2 e^{ax}}{f''(a)} \text{ if } f''(a)\neq 0.$$

B.Sc Vth Sem.

I] Answer the following questions.

1) Prove that $\frac{\pi^3}{24} < \int_0^{\pi} \frac{x^2\,dx}{5+3\cos x} < \frac{\pi^3}{6}$

Let $f(x) = \frac{1}{5+3\cos x}$ & $g(x) = x^2$

clearly f & g are continuous in $[0, \pi]$ and hence both are R-integrable & $g(x) = x^2 \geq 0 \;\forall x \in [0, \pi]$ & $f(0) = 1/8$,

$f(\pi) = \frac{1}{5-3} = \frac{1}{2}$

$\therefore m = \frac{1}{8}$ & $M = \frac{1}{2}$

By the Ist mean value theorem and μ b/w $m = \frac{1}{8}$ & $M = \frac{1}{2}$ i.e

$\frac{1}{8} \leq \mu \leq \frac{1}{2}$

S.T $\int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx = \mu \int_0^{\pi} x^2\,dx$

$= \mu \left.\frac{x^3}{3}\right]_0^{\pi}$

$= \frac{\mu}{3}[\pi^3 - 0] = \frac{\mu\pi^3}{3}$

$\therefore \mu = \frac{3}{\pi^3} \int_0^{\pi} \frac{x^3}{5+3\cos x}\,dx$ But we have

$m = \frac{1}{8} \leq \mu \leq M = \frac{1}{2}$

$\frac{1}{8} \leq \frac{3}{\pi^3} \int_0^{\pi} \frac{x^3}{5+3\cos x}\,dx \leq \frac{1}{2}$

multiply throughout by $\pi^3/3$

$$\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \frac{\pi^3}{6}$$

2) Explain Newton - Rapson method of finding real root of $f(x)=0$.

→ let this method is generally used to improve the result obtain by the previous method in minimum number of steps.

let $f(x)=0$ —— ① be given eq^n and the x_0 be the initial approximation to the root of eq^n ① and let $x_1 = x_0+h$ be the correct root $\Rightarrow f(x_1) = f(x_0+h)=0$

Then by Taylor's Series we can express $f(x_1)$ as follows.

$$\Rightarrow f(x_1) = f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \frac{h^3}{3!}f'''(x_0) + \cdots = 0$$

neglecting second and higher order derivatives in the above eq^n then

$$f(x_1) = f(x_0+h) = f(x_0) + hf'(x_0) = 0.$$

$$h = -\frac{f(x_0)}{f'(x_0)}$$

where $f'(x_0) \neq 0 \Rightarrow x_0 = x_0+h = x_0 - \frac{f(x_0)}{f'(x_0)}$

Let us suppose that x_1 is not our required root then we find the next approximation root of the eq^n ① as

$$x_2 = x_1+h = x_0+2h$$

If x_2 is exact root of eq^n ①

$$\Rightarrow f(x_2) = 0 = f(x_1+h)$$

Then again by Taylors Series expansion

$$f(x_2) = f(x_1+h) = f(x_1) + hf'(x_1) + \frac{h^2}{2!} f''(x_1) + \frac{h^3}{3!} f'''(x_2) + \cdots\cdots = 0.$$

neglecting second & higher order derivative in the above eq^n we get

$$f(x_2) = f(x_1+h) = f(x_1) + hf'(x_1) = 0$$

$$\Rightarrow h = -\frac{f(x_1)}{f'(x_1)} \quad \text{where } f'(x_1) \neq 0.$$

∴ The better approximate value x_2 becomes

$$x_2 = x_1 + h = x_1 - \frac{f(x_1)}{f'(x_1)}$$

1ly Starting with x_2, we get the next approximation x_3 as follows,

$$x_3 = x_2 + h = x_3 - \frac{f(x_2)}{f'(x_2)} \quad \& \; f'(x_2) \neq 0.$$

On continuing this process in generally we get $(n+1)^{th}$ approxima^n root as follows,

$$x_{n+1} = x_n + h = x_n - \frac{f(x_n)}{f'(x_n)} \quad \& \; f'(x_n) \neq 0 \quad \forall\, n \geq 0 \quad \text{---(a)}$$

eq^n (a) is called Newton-Raphson method and this formula is called Newton-Raphson formula.

II] Answer the following questions,

1) Prove that $\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\,dx = \pi \sin^{-1} a$

by differendial under integral sign.

→ $F(a) = \int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\,dx$ —— ①

Then by the Lebnidz's Rule

$F'(a) = \int_0^{\pi} \frac{\partial}{\partial a} \frac{\log(1+a\cos x)}{\cos x}\,dx$

$= \int_0^{\pi} \frac{1}{\cos x} \frac{1}{(1+a\cos x)} \cos x\,dx$

$= \int_0^{\pi} \frac{1}{(1+a\cos x)}\,dx$

If $1^2 \geq a^2$, $a=1$, $b=a$

$F'(a) = \frac{1}{\sqrt{1-a^2}} \left[\cos^{-1}\left(\frac{a+\cos x}{1+a\cos x}\right)\right]_0^{\pi}$

$= \frac{1}{\sqrt{1-a^2}} \left[\cos^{-1}\left\{\left(\frac{a-1}{1+a(-1)}\right) - \left(\frac{a+1}{1+a}\right)\right\}\right]$

$= \frac{1}{\sqrt{1-a^2}} \left\{\cos^{-1}(-1) - \cos^{-1}(1)\right\}$

$= \frac{1}{\sqrt{1-a^2}} [\pi - 0]$

$F'(a) = \frac{\pi}{\sqrt{1-a^2}}$ —— ②

Integrate eq^n ④ w.r.t Parameter 'a' we get

$$\int F'(a)\, da = \int \frac{\pi}{\sqrt{1-a^2}}\, da$$

$$F(a) = \pi \sin^{-1} a + C \quad \text{—— ⑤}$$

Put $a = 0$

eq^n ① becomes $F(a) = 0$

eq^n ③ becomes $0 = \pi \sin^{-1}(0) + C$

$$= \pi(0) + C$$

$$C = 0,$$

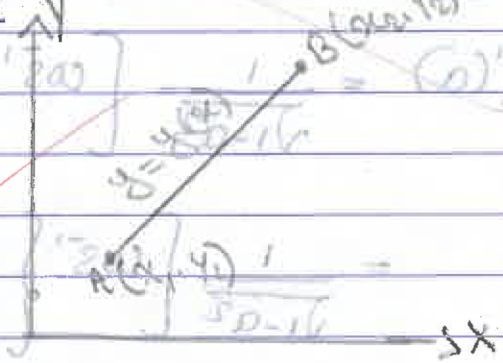
eq^n ③ becomes

$$F(a) = \pi \sin^{-1} a$$

$$\Rightarrow \int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x}\, dx = \pi \sin^{-1} a \,//.$$

2) Prove that geodesics of Plane is a straight line.

→ **Proof :**



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two Points on xy - Plane,

and $y = y(x)$ be curve joining these two Points.

The arc length of curve joining ~~these~~ A & B is given by

$$S = \int_{x_1}^{x_2} ds = \int_{x_1}^{x_2} \sqrt{1+(y')^2}\, dx$$

now Geodesics on xy-Plane is the curve $y = y(x)$ for which S is mini-mum.

$f = \sqrt{1+(y')^2}$ which is independent of x & y.

$\therefore$ corresponding Euler's eq^n is

$$y'' \frac{\partial f}{\partial y'} = 0,$$

$$\Longrightarrow y'' = 0.$$

Integrating $y' = m$

$\Longrightarrow dy = m\,dx$

again Integrate $y = mx + c$

where m & c are constant.

The constant m & c can be determined by using condition that the line passi-ng through A & B.

Thus Geodesics on a plane is Straight line.

B.Sc VI Sem

I] Answer the following questions.

1) State and Prove necessary condition for the function $f(z)$ be an analytic.

→ **Statement** :-

A necessary condition for function $w = f(z) = u(x,y) + iv(x,y)$ be analytic in a domain D, is that u & v satisfy the eq^n $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

which are called cauchy's Riemann eq^n.

Proof :- let $w = f(z) = u(x,y) + iv(x,y)$ be a analytic in a domain D.

By Def^n $f'(z) = \frac{dw}{dz}$ exists $\forall z$ in the domain D.

i.e $f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$

Exists along any Path we choose for $\Delta z \to 0$.

$$f'(z) = \lim_{(\Delta x, \Delta y) \to (0,0)} \left[u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y)\right] - \left[u(x,y) + iv(x,y)\right] \quad \text{—— (1)}$$

$\Delta x + i\Delta y$

As the derivative exist, this limit in unique irrespective of Paths in which $\Delta z \to 0$,

i) let $\Delta z \to 0$ along the x-axis;

along x-axis $y=0$.

$\therefore$ From ①

$$f'(z) = \lim_{\Delta x \to 0} \frac{[u(x+\Delta x, 0) + iv(x+\Delta x, 0) - u(x,0) - iv(x,0)]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} + i \lim_{\Delta x \to 0} \frac{v(x+\Delta x, 0) - v(x,0)}{\Delta x}$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \text{ —— ②}$$

ii) let $\Delta z \to 0$ along the y-axis

Along y-axis $x=0$.

$\therefore \Delta z = i\Delta y$ & $\Delta x = 0$.

$\therefore$ from ①

$$f'(z) = \lim_{\Delta y \to 0} \frac{[u(0, y+\Delta y) + iv(0, y+\Delta y) - u(0,y) - iv(0,y)]}{i\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(0, y+\Delta y) - u(0,y)}{i\Delta y} + \lim_{\Delta y \to 0} \frac{v(0, y+\Delta y) - v(0,y)}{i\Delta y}$$

$$\therefore f'(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$= \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \text{ —— ③}$$

From ② & ③

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ \& } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\Rightarrow \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Thus if $f(z) = u+iv$ is analytic then C-R eqn $u_x = u_y$ & $u_y = -v_x$ are satisfied,

2) State and prove cauchy's Residue theorem,

→ **Statement :-** let $f(z)$ be analytic within and an closed contour c, except at finite no. of Poles $z_1, z_2 ---- z_n$ inside c then $\int_C f(z)dz = 2\pi i (R_1 + R_2 --- R_n)$

$= 2\pi i$ (sum of residues at these poles inside c).

where $R_1, R_2, ---- R_n$ are residues at Poles $z_1, z_2, z_3 ---- z_n$ respectively.

Proof :- By hypothesis $z_1, z_2 ---- z_n$ Poles of $f(z)$ inside c.

∴ function $f(z)$ is not analytic at these points inside C, Hence construct small circles $\gamma_1, \gamma_2, ------ \gamma_n$ around those points then $f(z)$ is analy-dic in the region bounded by closed curves $c, \gamma_1, \gamma_2, \gamma_3 ------$

By cauchys theorem for multi connected region we have

$$\int_c f(z)dz = \gamma_1 \int f(z)dz + \gamma_2 \int f(z)dz + \cdots \cdots \gamma_n \int f(z)dz \quad \text{--- (1)}$$

By Def^n of residue of $f(z)$ we have

$$R_1 = \frac{1}{2\pi i} \gamma_1 \int f(z)dz.$$

where γ_1 is circle around the pole z_1 and R_1 is residue.

$$\therefore \gamma_1 \oint f(z)dz = 2\pi i R_1$$

$$\text{my} \quad \gamma_2 \int f(z)dz = 2\pi i R_2$$

$$\vdots$$

$$\gamma_n \int f(z)dz = 2\pi i R_n$$

Then (1) becomes

$$\int f(z)dz = 2\pi i R_1 + 2\pi i R_2 + \cdots + 2\pi i R_n$$

$$= 2\pi i (R_1 + R_2 + \cdots + R_n)$$

$$= 2\pi i \text{(Sum of residues at these poles inside } c)$$

Thus if $f(z)$ be analytic within & on closed contour c except at finite no. of poles $z_1, z_2 \cdots z_n$ inside c, then

$$\int_c f(z)dz \; dz = 2\pi i (R_1 + R_2 \cdots + R_n)$$
$$= 2\pi i \text{(Sum of residues at these poles inside } c\text{.)}$$

where $R_1, R_2 \cdots R_n$ are residues at poles $z_1, z_2 \cdots z_n$ respectively.

II] Answer the following questions,

1) Find the necessary condition of integrability of the equation $Pdx + Qdy + Rdz = 0$.

→ The total differential eq^n is

$$Pdx + Qdy + Rdz = 0 \quad \text{--- (1)}$$

Let the solution of total differential eq^n be $u = a$ --- (2)

But the total differential is given by

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \quad \text{--- (3)}$$

Since (1) & (3) are same eq^n hence their coefficients are proportional

i.e. $\mu P = \frac{\partial u}{\partial x}$, $\mu Q = \frac{\partial u}{\partial y}$, $\mu R = \frac{\partial u}{\partial z}$.

Of the above three equations Diff. Ist P. w.r.t. y and IInd P. w.r.t. x we get eq^n (4).

Diff. IInd P. w.r.t z and third w.r.t y we get eq^n (5) and diff. third w.r.t. x and Ist w.r.t z we get eq^n (6) as follows.

$$P\frac{\partial \mu}{\partial y} + \mu\frac{\partial P}{\partial y} = \frac{\partial^2 u}{\partial x.\partial y} = Q\frac{\partial \mu}{\partial x} + \mu\frac{\partial Q}{\partial x} \quad \text{--- (4)}$$

$$Q\frac{\partial \mu}{\partial z} + \mu\frac{\partial Q}{\partial z} = \frac{\partial^2 u}{\partial y.\partial z} = R\frac{\partial \mu}{\partial y} + \mu\frac{\partial R}{\partial y} \quad \text{--- (5)}$$

$$R\frac{\partial \mu}{\partial x} + \mu\frac{\partial R}{\partial x} = \frac{\partial^2 u}{\partial z.\partial x} = P\frac{\partial \mu}{\partial z} + \mu\frac{\partial P}{\partial z} \quad \text{--- (6)}$$

Rearranging the terms eq^n ④ & ⑤ & ⑥ as follows.

$$\mu\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = Q\frac{\partial \mu}{\partial x} - P\frac{\partial \mu}{\partial y} \quad \text{—— (7)}$$

$$\mu\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = R\frac{\partial \mu}{\partial y} - Q\frac{\partial \mu}{\partial z} \quad \text{—— (8)}$$

$$\mu\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) = P\frac{\partial \mu}{\partial z} - R\frac{\partial \mu}{\partial x} \quad \text{—— (9)}$$

multiplying eq^n ⑦ by R, eq^n ⑧ by P and eq^n ⑨ by Q & adding we get

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

This is required condition of integrability which can also be written in the determinant as follows.

$$\begin{vmatrix} P & Q & R \\ P & Q & R \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix} = 0.$$

2) Find $L[e^t(\sin 3t \cos 4t)]$

→ let $f(t) = \sin 3t \cos 4t$

$$L[f(t)] = L\left[\frac{1}{2}\{\sin 7t + \sin t\}\right]$$

$$= \frac{1}{2}\{L[\sin 7t] + L[\sin t]\}$$

$$= \frac{1}{2}\left[\frac{7}{s^2+49} + \frac{1}{s^2+1}\right]$$

$$= \frac{1}{2}\left[\frac{7(s^2+1) + (s^2+49)}{(s^2+49)(s^2+1)}\right]$$

$$= \frac{1}{2}\left[\frac{7s^2+7+s^2+49}{(s^2+49)(s^2+1)}\right]$$

$$= \frac{1}{2}\left[\frac{8s^2+56}{(s^2+49)(s^2+1)}\right]$$

$$= \frac{1}{\cancel{2}_1} \cancel{8}^4 \left[\frac{s^2+7}{(s^2+49)(s^2+1)}\right]$$

$$L[\sin 3t \cos 4t] = \frac{4s^2+28}{(s^2+49)(s^2+1)}$$

$$\therefore L[e^t(\sin 3t \cos 4t)] = \frac{2(s-1)^2+28}{[(s-1)^2+49][(s-1)^2+1]}$$

K.L.E Society's

G.I. Bagewadi Art's, Science and Commurce College Nipani

Department of Mathematics

PRACTICE TEST - I

For the year : 2019-20

4/10

Class :- B.Sc IV Sem Date :- 15/10/2020

Roll No. :- 205

1) Proof :- Now given series $\sum \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$

$= 1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}\right) + \ldots$

Case i: $p > 1$

Let, $1 = 1$ —— ①

& $\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^p} + \frac{1}{2^p} = \frac{2}{2^p} = \frac{1}{2^{p-1}}$

i.e $\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^{p-1}}$ —— ②

also $\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p} < \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p} = \left(\frac{1}{2^{p-1}}\right)^2$ —— ③

III^{rly} $\frac{1}{8^p} + \frac{1}{9^p} + \frac{1}{10^p} + \ldots + \frac{1}{15^p} < \left(\frac{1}{2^{p-1}}\right)^3$ —— ④ & so on

Now adding ①, ②, ③, ④ we get

$1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}\right) + \ldots < 1 + \frac{1}{2^{p-1}} + \left(\frac{1}{2^{p-1}}\right)^2 + \left(\frac{1}{2^{p-1}}\right)^3 + \left(\frac{1}{2^{p-1}}\right)^4 + \ldots$

i.e $\sum_{n=1}^{\infty} \frac{1}{n^p} < \left(\frac{1}{2^{p-1}}\right)^{n-1}$ —— ⓐ

& series $\sum_{n=1}^{\infty} \left(\frac{1}{2^{p-1}}\right)^{n-1}$ is geometric series with ratio $x = \frac{1}{2^{p-1}} < 1$ as $p>1$

& $\sum_{n=1}^{\infty} \left(\frac{1}{2^{p-1}}\right)^{n-1}$ is convergent.

∴ from ⓐ & I comp. test $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is also convergent

∴ Series $\sum \frac{1}{n^p}$ is convergent if $p>1$

<u>Case ii</u> : If $p=1$ then $\sum \frac{1}{n^p} = \sum \frac{1}{n}$ is divergent by Cauchy's G.P. of convergent.

<u>Case iii</u> : If $p<1$

$\implies n^p < n$

$\implies \frac{1}{n^p} > \frac{1}{n}$

$\therefore \sum \frac{1}{n^p} > \sum \frac{1}{n}$

& $\sum \frac{1}{n}$ is divergent & by II. Comp. test series $\sum \frac{1}{n^p}$ is also divergent.

Thus $\sum \frac{1}{n^p}$ is convergent if $p>1$ & divergent if $p \le 1$.

2) Let; $\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}]$

$$= \frac{d}{dt}\{\vec{A}\cdot(\vec{B}\times\vec{C})\}$$

$$= \vec{A}\cdot\frac{d}{dt}(\vec{B}\times\vec{C}) + (\vec{B}\times\vec{C})\frac{d\vec{A}}{dt}$$

$$= \vec{A}\cdot\left\{\vec{B}\times\frac{d\vec{C}}{dt} + \frac{d\vec{B}}{dt}\times\vec{C}\right\} + \frac{d\vec{A}}{dt}(\vec{B}\times\vec{C})$$

$$= \vec{A}\cdot\left(\vec{B}\times\frac{d\vec{C}}{dt}\right) + \vec{A}\cdot\left(\frac{d\vec{B}}{dt}\times\vec{C}\right) + \left[\frac{d\vec{A}}{dt}\cdot\vec{B}\cdot\vec{C}\right]$$

$$= \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right]$$

$$= \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right]$$

i.e $\frac{d}{dt}[\vec{A}\ \vec{B}\ \vec{C}] = \left[\frac{d\vec{A}}{dt}\ \vec{B}\ \vec{C}\right] + \left[\vec{A}\ \frac{d\vec{B}}{dt}\ \vec{C}\right] + \left[\vec{A}\ \vec{B}\ \frac{d\vec{C}}{dt}\right]$

KLE Society's

G. I. Bagewadi Arts, Commerce and Science College Nippani

Department of Mathematics

PRACTICE TEST - I

For the year :

7/10

Class : BSC IV Sem

Roll. NO : 206

Date : 15/10/2019

I) Proof :- Now, given series $\sum \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$

$= 1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}\right) + \cdots$

Case i) : $p > 1$

Let, $1 = 1 \longrightarrow ①$

& $\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^p} + \frac{1}{2^p} = \frac{2}{2^p} = \frac{1}{2^{p-1}}$

i.e $\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^{p-1}} \longrightarrow ②$

also, $\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p} < \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p} + \frac{1}{4^p} = \left(\frac{1}{2^{p-1}}\right)^2 \rightarrow ③$

IIIly $\frac{1}{8^p} + \frac{1}{9^p} + \frac{1}{10^p} + \cdots + \frac{1}{15^p} < \left(\frac{1}{2^{p-1}}\right)^3 \rightarrow ④$ & so on

Now adding ①, ②, ③, ④ we get -

$1 + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}\right) + \cdots < 1 + \frac{1}{2^{p-1}}$

$+ \left(\frac{1}{2^{p-1}}\right)^2 + \left(\frac{1}{2^{p-1}}\right)^3 + \left(\frac{1}{2^{p-1}}\right)^4 + \cdots$

i.e $\sum_{n=1}^{\infty} \frac{1}{n^p} < \left(\frac{1}{2^{p-1}}\right)^{n-1} \longrightarrow$ (a)

& Series $\sum_{n=1}^{\infty} \left(\frac{1}{2^{p-1}}\right)^{n-1}$ is geometric series wi[th]

ratio $r = \frac{1}{2^{p-1}} < 1$ as $p > 1$

& $\sum_{n=1}^{\infty} \left(\frac{1}{2^{p-1}}\right)^{n-1}$ is convergent.

$\therefore$ from (a) & I. Comp. Test $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is also conv[ergent]

$\therefore$ Series $\sum \frac{1}{n^p}$ is convergent if $p > 1$

Case ii) :- If $p = 1$ then $\sum \frac{1}{n^p} = \sum \frac{1}{n}$ is diverge[nt]

by cauchy's G.P of convergent.

Case iii) :- If $p < 1$

$\Rightarrow n^p < n$

$\Rightarrow \frac{1}{n^p} > \frac{1}{n}$

$\therefore \sum \frac{1}{n^p} > \sum \frac{1}{n}$

& $\sum \frac{1}{n}$ is divergent & by II. Comp. test series

$\sum \frac{1}{n^p}$ is also convergent.

Let, $\frac{d}{dt}[\vec{A}\,\vec{B}\,\vec{C}]$

$$= \frac{d}{dt}[\vec{A}\cdot(\vec{B}\times\vec{C})]$$

$$= \vec{A}\cdot\frac{d}{dt}(\vec{B}\times\vec{C}) + (\vec{B}\times\vec{C})\cdot\frac{d\vec{A}}{dt}$$

$$= \vec{A}\cdot\left[\vec{B}\times\frac{d\vec{C}}{dt} + \frac{d\vec{B}}{dt}\times\vec{C}\right] + \frac{d\vec{A}}{dt}(\vec{B}\times\vec{C})$$

$$= \vec{A}\cdot\left(\vec{B}\times\frac{d\vec{C}}{dt}\right) + \vec{A}\left(\frac{d\vec{B}}{dt}\times\vec{C}\right) + \left[\frac{d\vec{A}}{dt}\,\vec{B}\,\vec{C}\right]$$

$$= \left[\vec{A}\cdot\vec{B}\,\frac{d\vec{C}}{dt}\right] + \left[\vec{A}\cdot\frac{d\vec{B}}{dt}\cdot\vec{C}\right] + \left[\frac{d\vec{A}}{dt}\,\vec{B}\,\vec{C}\right]$$

$$= \left[\frac{d\vec{A}}{dt}\,\vec{B}\,\vec{C}\right] + \left[\vec{A}\cdot\frac{d\vec{B}}{dt}\,\vec{C}\right] + \left[\vec{A}\cdot\vec{B}\cdot\frac{d\vec{C}}{dt}\right]$$

i.e $\frac{d}{dt}[\vec{A}\,\vec{B}\,\vec{C}] = \left[\frac{d\vec{A}}{dt}\,\vec{B}\,\vec{C}\right] + \left[\vec{A}\cdot\frac{d\vec{B}}{dt}\cdot\vec{C}\right] + \left[\vec{A}\,\vec{B}\,\frac{d\vec{C}}{dt}\right]$

KLE Society's

G. I. Bagewadi Art's. Science and Commerce college. Nippani

Department of Mathematics.

PRACTICE TEST - II

For the year : 2019-20

6/10

Class : BSC IV Sem

Date : 15/10/2020

Roll. NO : 206

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1) Given, $f(x) = |x| = \begin{cases} -x & \text{if } x \in (-1,0) \\ x & \text{if } x \in (0,1) \end{cases}$

Let, $f(x) = |x|$ in $(-1,1)$ & length $l = 1$

The given function is even function $\therefore b_n = 0$

$\therefore a_0 = \frac{1}{l}\int_{-1}^{1} f(x)\,dx$

$= \frac{2}{1}\int_0^1 x.\,dx$

$= 2\left[\frac{x^2}{2}\right]_0^1$

$= [1-0]$

$a_0 = 1$

$a_n = \frac{1}{l}\int_{-1}^{1} f(x)\cos\frac{n\pi x}{l}\,dx$

$= \frac{2}{1}\int_0^1 f(x)\cos\frac{n\pi x}{l}\,dx.$

$= 2\int_0^1 x.\cos n\pi x\,dx.$

$= 2\left[x.\frac{\sin n\pi x}{n\pi} + \frac{1}{n^2\pi^2}\cos n\pi x\right]_0^1$

$$= \frac{2}{n\pi}\left[x \cdot \sin n\pi x + \frac{1}{n\pi}\cos n\pi x\right]_0^1$$

$$= \frac{2}{n\pi}\left[\left(\sin n\pi + \frac{1}{n\pi}\cos n\pi\right) - \left(\frac{1}{n\pi}\right)\right]$$

$$= \frac{2}{n\pi}\left[0 + \left(\frac{(-1)^n}{n\pi} - \frac{1}{n\pi}\right)\right]$$

$$a_n = \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]$$

∴ Required fourier series is given by –

$$f(x) = |x| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\frac{n\pi x}{l}$$

$$|x| = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]\cos n\pi x$$

---

3) Let, $f(D) = D^n + a_n D^{n-1} + a_2 D^{n-2} + \cdots + a_n$

where, $D = \frac{d}{dx}$

Now, $D(e^{ax}) = a \cdot e^{ax}$

$D^2(e^{ax}) = a^2 \cdot e^{ax}$

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$D^n(e^{ax}) = a^n \cdot e^{ax}$

$\Rightarrow f(D) \cdot e^{ax} = f(a) e^{ax}$

$\Rightarrow \frac{1}{f(D)}\left[f(D) \cdot e^{ax}\right] = \frac{1}{f(D)}\left[f(a) \cdot e^{ax}\right]$

$\Rightarrow e^{ax} = f(a) \frac{1}{f(D)} \cdot e^{ax}$

$\Rightarrow \frac{1}{f(a)} e^{ax} = \frac{1}{f(D)} \cdot e^{ax}$

i.e $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} \cdot e^{ax}$ if $f(a) \neq 0$.

Thus P.I for $f(D)y = e^{ax}$ is $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$

if $f(a) \neq 0$.

If $f(a)=0$ then P.I is as follows

if $f(a)=0$ then $(D-a)$ is a factor of $f(D)$.

$\therefore f(D) = (D-a)\phi(D)$

$$\therefore \frac{1}{f(D)} \cdot e^{ax} = \frac{1}{(D-a)\phi(D)} \cdot e^{ax}$$

$$= \frac{1}{\phi(a)} \left[\frac{1}{(D-a)} \cdot e^{ax} \right] \text{ if } \phi(a) \neq 0.$$

$$= \frac{1}{\phi(a)} x \cdot e^{ax}$$

$$= x \cdot \frac{1}{\phi(a)} \cdot e^{ax} \text{ but } \phi(a) = f'(a)$$

$$= x \cdot \frac{1}{f'(D)} \cdot e^{ax} \text{ if } f'(a) \neq 0$$

$$\frac{1}{f(D)} e^{ax} = x^2 \cdot \frac{1}{f''(D)} e^{ax} \text{ if } f'(a) = 0.$$

K.L.E Society's

G.I. Bagewadi Art's Science and Commerce College Nipani

Department of Mathematics

PRACTICE TEST - II

For the year:

4/10

Class : B.Sc. IV Sem   Date: 15/10/2020

Roll. No. : 205

1) Given, $f(x) = |x| = \begin{cases} -x, & \text{if } x \in (-1, 0) \\ x, & \text{if } x \in (0, 1) \end{cases}$

Let $f(x) = |x|$ in $(-1, 1)$ & length $l = 1$

The given function is even function $\therefore b_n = 0$.

$\therefore a_0 = \frac{1}{l} \int_{-1}^{1} f(x)\, dx$

$= \frac{2}{1} \int_{0}^{1} x\, dx$

$= 2 \left[\frac{x^2}{2}\right]_0^1$

$= [1 - 0]$

$a_0 = 1$

$a_n = \frac{1}{l} \int_{-1}^{1} f(x) \cos \frac{n\pi x}{l}\, dx$

$= \frac{2}{1} \int_{0}^{1} f(x) \cos \frac{n\pi x}{1}\, dx$

$$a_n = 2\int_0^1 x \cos n\pi x \, dx$$

$$= 2\left[x \frac{\sin n\pi x}{n\pi} + \frac{1}{n^2\pi^2}\cos n\pi x\right]_0^1$$

$$= \frac{2}{n\pi}\left[x \sin n\pi x + \frac{1}{n\pi}\cos n\pi x\right]_0^1$$

$$= \frac{2}{n\pi}\left[\left(\sin n\pi + \frac{1}{n\pi}\cos n\pi\right) - \left(\frac{1}{n\pi}\right)\right]$$

$$= \frac{2}{n\pi}\left[0 + \frac{(-1)^n}{n\pi} - \frac{1}{n\pi}\right]$$

$$a_n = \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]$$

∴ Required fourier series is given by

$$f(x) = |x| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$$|x| = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right] \cos n\pi x$$

3) Let, $f(D) = D^n + a_n D^{n-1} + a_2 D^{n-2} + \ldots + a_n$ where $D = \frac{d}{dx}$

Now, $D(e^{ax}) = a e^{ax}$

$D^2(e^{ax}) = a^2 e^{ax}$

- - - - - - - - - -

$D^n(e^{ax}) = a^n e^{ax}$

$\Rightarrow f(D)\, e^{ax} = f(a)\, e^{ax}$

$\Rightarrow \frac{1}{f(D)}\left[f(D)\, e^{ax}\right] = \frac{1}{f(D)}\left[f(a)\, e^{ax}\right]$

$\Rightarrow e^{ax} = f(a) \frac{1}{f(D)} e^{ax}$

$\Rightarrow \frac{1}{f(a)} e^{ax} = \frac{1}{f(D)} e^{ax}$

i.e $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ if $f(a) \neq 0$.

Thus PI for $f(D)y = e^{ax}$ is $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$ if $f(a) \neq 0$

If $f(a)=0$ then PI is as follows

if $f(a)=0$ then $(D-a)$ is a factor of $f(D)$

$$\therefore f(D) = (D-a)\phi(D)$$

$$\therefore \frac{1}{f(D)}e^{ax} = \frac{1}{(D-a)\phi(D)}e^{ax}$$

$$= \frac{1}{\phi(a)}\left[\frac{1}{(D-a)}e^{ax}\right] \text{ if } \phi(a) \neq 0$$

$$= \frac{1}{\phi(a)} x e^{ax}$$

$$= x \frac{1}{\phi(a)} e^{ax} \quad \text{but } \phi(a) = f'(a)$$

$$= x \frac{1}{f'(D)} e^{ax} \quad \text{if } f'(a) \neq 0.$$

$$\frac{1}{f(D)}e^{ax} = x^2 \frac{1}{f''(D)} e^{ax} \quad \text{if } f'(a) = 0.$$

K.L.E Society's

G.I. Bagewadi College Nippani

Depagment of Mathematics

Pagdice test-1
fog the yeag 2019-20

Roll no :-152

Magks obtained ——

Class :- B.Sc IV sem

Date :-

6/10

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1) State Leibnitz's test for Convergence of alternating Series

Statement: The alternating series
$\sum_{n=1}^{\infty} (-1)^{n-1} u_n = u_1 - u_2 + u_3 - u_4 + u_5 - \cdots \ (u_n > 0, \forall n)$
Converges if
i) $u_n \geq u_{n+1}, \forall n$ and

ii) $\lim_{n \to \infty} u_n = 0$

Proof: Let S_n denote the n^{th} partial sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} u_n$

$\therefore S_{2n} = u_1 - u_2 + u_3 - u_4 + u_5 - \cdots - u_{2n-1} - u_{2n}$
$= u_1 - (u_2 - u_3)(u_4 - u_5) - \cdots - (u_{2n-2} - u_{2n-1}) - u_{2n}$
$= u_1 - \{(u_2 - u_3) + (u_4 - u_5) + \cdots + (u_{2n-2} - u_{2n-1}) + u_{2n}\}$

$\leq u_1 \ (\because u_n \geq u_{n+1} \ \& \ u_n > 0, \forall n)$
$\therefore S_{2n} \leq u_1, \forall n$
$\rightarrow$ The Sequence (S_{2n}) is bounded above

also $S_{2n+2} = S_{2n} + u_{2n+1} - u_{2n+2}$
$\Rightarrow S_{2n+2} - S_{2n} = u_{2n+1} - u_{2n+2} \geq 0 \ (\because u_n \geq u_{n+1}, \forall n)$
$\therefore S_{2n+2} - S_{2n} \geq 0 \Rightarrow S_{2n+2} \geq S_{2n}, \forall n$
$\rightarrow$ The Sequence $\{S_{2n}\}$ is monotonically increasing

Since every monotonically increasing sequence which is bounded above Converges, therefore the Seq (S_{2n}) Converges.

Let the Sequence $<S_{2n}>$ Converges to S i.e $\lim_{n \to \infty} S_{2n} = S$.
Now,
$\lim_{n \to \infty} S_{2n+1} = \lim_{n \to \infty} (S_{2n} + u_{2n+1}) = \lim_{n \to \infty} S_{2n} + \lim_{n \to \infty} u_{2n+1} = S + 0 = S$
$(\because \lim_{n \to \infty} u_n = 0)$
$\therefore \lim_{n \to \infty} S_{2n+1} = S$

$\therefore$ The Seq $<S_{2n}>$ & $<S_{2n+1}>$ Converges to the Same number S

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→ $\forall \epsilon > 0$ there exist positive integers m_1 & m_2 such that

$|S_{2n} - S| \le \epsilon/2, \forall 2n > m_1$ & $|S_{2n+1} - S| \le \epsilon/2, \forall 2n+1 > m_2$

Let $m = \max(m_1, m_2)$

$\therefore |S_{2n} - S| \le \frac{\epsilon}{2} \; \forall 2n > m$ & $|S_{2n+1} - S| \le \epsilon/2 \; \forall 2n+1 > m$

$\Rightarrow |S_n - S| \le \frac{\epsilon}{2} < \epsilon, \forall n > m$

→ The sequence $<S_n>$ is convergent to S

$\therefore$ The given series is convergent

2) Prove that $\sum \frac{1}{n^p}$ is convergent if $p > 1$ & divergent if $p \le 1$

i) Case ① If $p > 1$.

Proof: we have

$\frac{1}{2^p} + \frac{1}{3^p} < \frac{1}{2^p} + \frac{1}{2^p} \quad \because \frac{1}{3^p} < \frac{1}{2^p}$

$(1/2^p + 1/3^p) < 2/2^p = 1/2^{p-1}$

$1/4^p + 1/5^p + 1/6^p + 1/7^p < 1/4^p + 1/4^p + 1/4^p + 1/4^p$

$= 4/4^p = 1/4^{p-1} = (1/2^{p-1})^2$

III^ly by taking the next 8 terms we get

$\frac{1}{8^p} + \frac{1}{9^p} + \frac{1}{10^p} + \frac{1}{11^p} + \frac{1}{12^p} + \frac{1}{13^p} + \frac{1}{14^p} + \frac{1}{15^p} < \frac{1}{8^p} +$

$\frac{1}{8^p} + \frac{1}{8^p} + \frac{1}{8^p} + \frac{1}{8^p} + \frac{1}{8^p} + \frac{1}{8^p} + \frac{1}{8^p}$

$= \frac{8}{8^p} = \frac{1}{8^{p-1}} = (1/2^{p-1})^3$

$\therefore \frac{1}{1^p} + \left(\frac{1}{2^p} + \frac{1}{3^p}\right) + \left(\frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p}\right) + \cdots$

$< 1 +$

$\frac{1}{2^{p-1}} + \left(\frac{1}{2^{p-1}}\right)^2 + \left(\frac{1}{2^{p-1}}\right)^3 + \cdots$

Pratibha

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The series on the R.H.S is geometric series where ~~ration r~~ ratio $\frac{1}{2^{p-1}} < 1$

$\therefore$ From the geometric series test
The series on the R.H.S is convergent
$\therefore$ By I^{st} Comparision test

The given series is also convergent

$\therefore \sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$

Case ii) If $p \leq 1$
where $p < 1$, $n^p < n$
$\therefore \frac{1}{n^p} > \frac{1}{n} \quad \forall n$

But $\sum \frac{1}{n}$ is divergent ~~according~~ to

$\therefore$ By second Comparision test $\sum \frac{1}{n^p}$ is also divergent

Hence $\sum \frac{1}{n^p}$ is convergent if $p > 1$ & divergent if $p \leq 1$

K.L.E Society's

G I Bagewadi Arts, Commerce, Science College

Nippani

Department of Mathematics

Practice test-II

Roll no: - 152 For the year: - 2019-20

Class : - B.Sc IV sem Marks obtained: -

Date: -

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1) Define kernel of homomorphism and prove that ker f is normal subgroup of G

If G and G' are two group and f (G to G') be a homomorphism from the group e' be the identity element of G' the subset (K of G) K(G) is defined by $K = \{a \in G / f(a) = e'\}$ is called kernel of Homomorphism

Prove that K is normal subgroup of G

We shall prove this thm by two steps

1) K is subgroup of G

2) K is normal in G

3) By definat^n of ker f, K is subgroup of G,

$\therefore f(e) = e'$ where e and e' are identities of G and G'

Hence $e \in K$ thus $K \neq \phi$

Now let $a, b \in K$

then $f(a) = e'$ and $f(b) = e'$

$f(a \cdot b^{-1}) = f(a) \cdot f(b^{-1})$

$= e' (e')^{-1}$

$= e' e' = e'$

Hence $ab^{-1} \in K$ and K is subgroup of G

2) Obtain the fourier series of $f(x) = |x|$ in $(-\pi, \pi)$

Sol^n: $f(x) = |x|$ in $(-\pi, \pi)$

$f(x) = |x|$ in $(-\pi, \pi)$ which is even function

$f(x) = |x| = x$ where ~~x is~~

The fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\left[\pi^2 - 0\right] \quad 2\pi$$

find a_n

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cdot \cos nx\,dx$$

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$$= \frac{2}{\pi}\int_0^{\pi} x \cos nx \, dx$$

$$= \frac{2}{\pi}\left[x \frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left[0 + \frac{\cos n\pi}{n^2}\right] - \left(0 + \frac{1}{n^2}\right)\right]$$

$$= \frac{2}{\pi n^2}\left(\cos n\pi - 1\right)$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$\therefore |x| = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left\{(-1)^n - 1\right\}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$|x| = \frac{\pi}{2} + \frac{2}{\pi}\left\{1(-2)\cos x + 0 + \frac{1}{3^2}(-2)\cos 3x + \frac{1}{5^2}(-2)\cos 5x\right\}$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\left\{\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x\right\}$$

Put $x = \pi$

$$= \pi = \frac{\pi}{2} - \frac{4}{\pi}\left(-1 - \frac{1}{3^2} - \frac{1}{5^2} \cdots\right)$$

$$= \pi - \frac{\pi}{2} = \frac{4}{\pi}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5} + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

K.L.E. SOCIETY'S

G. I. Bagewadi Arts, Science and Commerce college Nipani.

DEPARTMENT OF MATHEMATICS

For the year 2019-20

Roll No. :- [illegible] 01 Class :- B.Sc V sem.

Date :- 26/10/2019

5/10

* Answer the following question:-

State and prove Langrange's interpolation formula for unequal intervals.

⇒ Statement:-

Let $y = f(x)$ be function which takes the values $y_0 = f(x_0)$, $y_1 = f(x_1)$ ------ $y_n = f(x_n)$.

Corresponding to the $x = x_0, x_1, x_2, x_3 ---- x_n$.

(unequally spaced value of x) then.

$$f(x) = \frac{(x-x_1)(x-x_2)---(x-x_n)}{(x_0-x_1)(x_0-x_2)---(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)---(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)---(x_1-x_n)} f(x_1) +$$

$$------- + \frac{(x-x_0)(x-x_1)(x-x_2)-----(x-x_n)}{(x_n-x_0)(x_n-x_1)(x_n-x_2)-----(x_n-x_{n-1})} f(x_n)$$

is called langranges interpolation formula for unequal intervals.

Proof:-

let $y = f(x)$ be a polynomial function of degree n in x which takes the value $f(x_0), f(x_1), f(x_2) ---- f(x_n)$ for $(n+1)$ unequally spaced values of x. Now the polynomial can be written as follows.

$$y = f(x) = a_0(x-x_1)(x-x_2)(x-x_3)---(x-x_n) + a_1(x-x_0)(x-x_2)---(x-x_n) + ------- + a_n(x-x_0)(x-x_1)(x-x_2)-----(x-x_{n-1}) \rightarrow (1)$$

Now we need to find the value of $a_0, a_1, a_2, --- a_n$ as follows.

For that $x = x_0$ in equation (1) we get

$$\Rightarrow y = f(x_0) = a_0(x_0-x_1)(x_0-x_2)(x_0-x_3)----(x_0+x_n)+0+0--$$

$$\Rightarrow y = f(x_0) = a_0(x_0-x_1)(x_0-x_2)(x_0-x_3)------(x_0-x_n)$$

$$a_0 = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)---(x_0-x_n)} \rightarrow (2)$$

Now put $x = x_1$ in equation (1) we get.

$$\Rightarrow y = f(x_1) = a_1(x_1-x_0)(x_1-x_2)(x_1-x_3)----(x_1-x_n) + 0 + 0 ----$$

$$a_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} \longrightarrow (3)$$

On continuing this process for $x = x_n$ in equation (1) we get

$$f(x_n) = a_n (x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1}) + 0 + \cdots$$

$$a_n = \frac{f(x_n)}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \longrightarrow (4)$$

Substituting the values of $a_0, a_1, a_2 \cdots a_n$ from equation (2), (3), (4) in equation (1) we get.

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1) + \cdots + \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} f(x_n)$$

2) Explain Newton-Rapson method for finding real root of $f(x) = 0$.

$\longrightarrow$ This method is generally used to improve the result obtained by previous method in minimum number of steps.

Let $f(x) = 0 \longrightarrow (1)$ be given equation & let x_0 be the initial approximation to the root of equation (1) & let $x_1 = (x_0 + h)$ be the correct root.

$$\Rightarrow f(x_1) = f(x_0 + h) = 0$$

Then by Taylor series we can express $f(x_1)$ as follows.

$$\Rightarrow f(x_1) = f(x_0 + h) = f(x_0) + h f'(x) + \frac{h^2}{2!} f''(x) + \cdots = 0$$

Neglating 2nd and higher order derivatives in above equation.

$\Rightarrow f(x_1) = f(x_0+h) = f(x_0) + h f'(x_0) = 0.$

$$h = \frac{-f(x_0)}{f'(x_0)}. \quad \text{where } f'(x_0) \neq 0$$

Let us suppose that x_1 is not our desired root then we find the next approximation root of the given equation as $x_1\, x_2 = x_1 + h = x_0 + 2h$

If x_2 is exact root of equation ①.

$\rightarrow f(x_2) = 0 = f(x_1+h).$

Then again by taylor's series expansion

$$f(x_2) = f(x_1+h) = f(x_1) + h f'(x_1) + \frac{h^2}{2!} f''(x_1) + \cdots$$

Neglecting 2nd & higher order derivatives in above equation.

$\Rightarrow f(x_2) = f(x_1+h) = f(x_1) + h f'(x_1) + 0 = 0.$

$$h = \frac{-f(x_1)}{f'(x_1)} \quad \text{where } f'(x_1) \neq 0.$$

∴ The better approximate value x_2 becomes

$$x_2 = x_1 + h = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \& \; f'(x_1) \neq 0$$

Similarly starting with x_2 we get the next approximation x_3 as follows.

$$x_3 = x_2 + h = x_2 - \frac{f(x_2)}{f'(x_2)} \quad \& \; f'(x_2) \neq 0.$$

On continuing this process in generally we get the $(n+1)^{th}$ approximation root as following

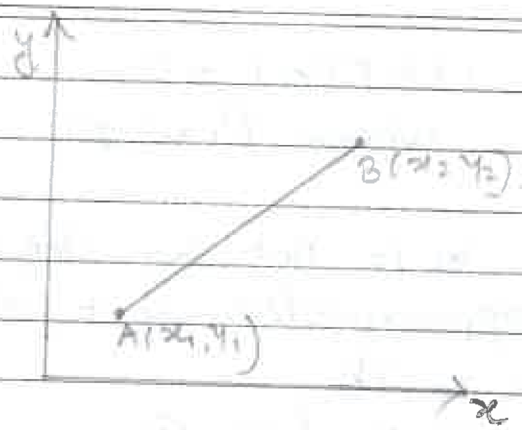
$$x_{n+1} = x_n + h = x_n - \frac{f(x_n)}{f'(x_n)} \longrightarrow (A) \quad \& \; f'(x_n) \neq 0.$$

$\forall n \geq 0.$

Equation (A) is called Newton-Raphson formula.

8) Prove that Geodesics of plane is a straight line.

→ Proof:-



Let $A(x_1, y_1)$ & $B(x_2 y_2)$ be any two points on xy-plane. And $Y = y(x)$ be curve going those two points. The arc length of curve going A & B given by

$$S = \int_{x_1}^{x_2} ds = \int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx$$

Now Geodesis on xy-plane is the curve $Y = Y(x)$ for which S is minimum.

$\Rightarrow$ $F = \sqrt{1+(y')^2}$ which is independent of x & y corresponding Euler's equation is

$$y'' \frac{\partial f}{\partial y'} = 0$$

$$y'' = 0$$

Integrate we get

$$y' = m$$
$$y = mx + C$$

where m & C are constant.

The constant m & c can be determined by using condition that the line passing through A & B.

Thus Geodesics on a plane is straight line.

K.L.E. SOCIETY'S

G. I. Bagewadi Arts, Science and Commerce College Nipani

DEPARTMENT OF MATHEMATICS

For the year.

Roll No. :- 01 Class :- BSC V sem.

Date :- 26/10/2019

* Answer the following questions.

1) State and prove fundamental theorem of integral calculus.

⇒ Statement :-

If $f \in R[a\ b]$ & F is primitive of f then

$$\int_a^b f(x)dx = F(b) - F(a).$$

Proof :-

Now $f \in R[a\ b]$ & hence $\int_a^b f(x)dx$ exists

given that $F(x)$ is primitive of $f(x)$ $\forall x \in R[a\ b)$

$\Rightarrow F'(x) = f(x) \quad \forall x \in [a\ b] \longrightarrow (1).$

$\Rightarrow F(x)$ is differentiable $\forall x \in [a\ b]$

$\Rightarrow F(x)$ is differentiable in each such interval

$[x_{r-1}, x_r] \quad \forall r = 1, 2, 3 \ldots n.$

$\Longrightarrow F(x)$ is continuous on I_r & differentiable on I_r & hence by Langranges mean value theorem $f(\xi_r) \in [x_{r-1}\ x_r]$ such that

$$f'(\xi_r) = \frac{F(x_r) - F(x_{r-1})}{x_r - x_{r-1}}$$

$$\Rightarrow f'(\xi_r)(x_r - x_{r-1}) = F(x_r) - F(x_{r-1}).$$

$$\Longrightarrow f(\xi_r)\delta_r = F(x_r) - F(x_{r-1})$$

$$\longrightarrow \sum_{r=1}^{n} f(\xi_r)\delta_r = \sum_{r=1}^{n} [F(x_r) - F(x_{r-1})]$$

$$\longrightarrow S(P\ f) = (F(x_1) - F(x_0)) + (F(x_2) - F(x_1)) + \ldots [F(x_n) - F(x_{n-1})]$$

$$S(P\ f) = F(x_n) - F(x_0) = F(b) - F(a).$$

$$\lim_{n\to\infty} S(P.f) = \lim_{n\to\infty} [F(b) - F(a)].$$

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

2) Prove that $\frac{\pi^3}{24} < \int_0^{\pi} \frac{x^2\,dx}{5+3\cos x} < \frac{\pi^3}{6}$.

$\Longrightarrow$ Given

Let us consider $g(x) = x^2$ & $f(x) = \frac{1}{5+3\cos x}$.

clearly both are R-integrable $\forall$ $x \in [0, \pi]$.

Now $f(\pi) = \frac{1}{5+3\cos x} = \frac{1}{5+3\cos\pi} = \frac{1}{5+3(-1)} = 1/2$

$f(0) = \frac{1}{5+3\cos x} = \frac{1}{5+3\cos 0} = \frac{1}{5+3(1)} = 1/8$.

Here $m = 1/8$ & $M = 1/2$

$$\frac{1}{8} \le \frac{1}{5+3\cos x} \le 1/2$$

multiplying by $g(x)$ i.e x^2

$$\frac{x^2}{8} \le \frac{x^2}{5+3\cos x} < \frac{x^2}{2}$$

Now integrating we get

$$\int_0^{\pi} \frac{x^2}{8}\,dx \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \int_0^{\pi} \frac{x^2}{2}\,dx.$$

$$\frac{1}{8}\left[\frac{x^3}{3}\right]_0^{\pi} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \frac{1}{2}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$\frac{1}{8}\left[\frac{\pi^3}{3} - 0\right] \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \frac{1}{2}\left[\frac{\pi^3}{3} - 0\right]$$

$$\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \frac{\pi^3}{6}$$ //.

3) Prove that $\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\,dx = \pi \sin^{-1} a$ by differential under integral sign.

→ Given $F(a) = \int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x} \longrightarrow (1)$.

Then by the leibnitz rule

$$F'(a) = \int_0^{\pi} \frac{\partial}{\partial a}\left(\frac{\log(1+a\cos x)}{\cos x}\right) dx.$$

$$= \int_0^{\pi} \frac{1}{\cos x}\left\{\left[\frac{1}{1+a\cos x}\right]\cdot \cos x\right] dx.$$

$$= \int_0^{\pi} \frac{1}{1+a\cos x}\,dx \qquad \left\{ \text{if } a^2 > b^2 \text{ then } \int \frac{dx}{a+b\cos x} = \frac{\cos^{-1}}{\sqrt{a^2-b^2}}\left[\frac{b+a\cos x}{a+b\cos x}\right] \right.$$

If $1^2 > a^2$

$$F'(a) = \frac{1}{\sqrt{1-a^2}}\left\{\cos^{-1}\frac{(a+1\cos x)}{1+a\cos x}\right\}_0^{\pi}$$

$$= \frac{1}{\sqrt{1-a^2}}\left\{\cos^{-1}\left(\frac{a+(-1)}{1+a(-1)} - \frac{a+1}{1+a}\right)\right\}$$

$$= \frac{1}{\sqrt{1-a^2}}\left\{\cos^{-1}\left(\frac{a-1}{1-a}\right) - \cos^{-1}\left(\frac{a+1}{a+1}\right)\right\}$$

$$= \frac{1}{\sqrt{1-a^2}}\left\{\cos^{-1}(-1) - \cos^{-1} 1\right\}$$

$$= \frac{1}{\sqrt{1-a^2}}\left\{\pi - 0\right\}$$

$$F'(a) = \frac{\pi}{\sqrt{1-a^2}} \longrightarrow (2)$$

Integrating equation ② w.r.t parameter 'a' we get

$$\int F'(a)\, da = \int \frac{\pi}{\sqrt{1-a^2}}\, da$$

$F(a)\, da = \pi \sin^{-1} a + C \rightarrow$ ③

$F(a) da = \pi \sin^{-1} a + C$

Put $a = 0$ eqn ① becomes $F(a) = 0$.

equation ② becomes

$0 = \pi \sin^{-1}(0) + C$

$0 = \pi \cdot 0 + C$

$C = 0$

Equation ③ becomes

$F(a) = \pi \sin^{-1} a$.

$$\int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x}\, dx = \pi \sin^{-1} a.$$

R.L.E. SOCIETY'S

G. I. Bagewadi. Arts. Science. Commerce college, Nipani

DEPARTMENT OF MATHEMATICS

PRATICE TEST : II

Class: B.Sc. V sem

Date:

Roll No: 57

9/10

Answer the following question's:

1) state prove that langranges interpolation formula for unequal intervals

Statement: let $y = f(x)$ be f^n which takes the values

$y_0 = f(x_0),\ y_1 = f(x_1)\ ---\ y_n = f(x_n)$

corresponding to the $x = x_0\ x_1\ x_2\ --\ x_n$

$$f(x) = \frac{(x-x_1)(x-x_2)--(x-x_n)}{(x_0-x_1)(x_0-x_2)--(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)(x--(x-x_n)}{(x_1-x_0)(x_1-x_2)--(x_1-x_n)} f(x_1) + --$$

$$----- + \frac{(x-x_0)(x-x_1)--(x-x_n)}{(x_n-x_0)(x_n-x_1)--(x_n-x_{n-1})} f(x_n)$$

is called langranges interpolation formula for unequal intervals

Proof: let $y = f(x)$ be a polynomial fun^n of degree n in x which takes the value $f(x_0)\ f(x_1)\ f(x_2)\ --\ f(x_n)$ for $(n+1)$ unequally spaced value of x. Now the polynomial can be written as

$y = f(x) = a_0(x-x_1)(x-x_2)--(x-x_n) + a_1(x-x_0)(x-x_2) --- (x-x_n) + ---$

$--- + a_n(x-x_0)(x-x_1) --- (x-x_{n-1}) \longrightarrow ①$

Now we need to find the value of $a_0\ a_1\ a_2\ --\ a_n$ as follow

o for that $x = x_0$ in equation ① we get

$\Rightarrow y = f(x_0) = a_0(x_0-x_1)(x_0-x_2)(x_0-x_3) --- (x_0+x_n) + 0 + 0 -$

$\Rightarrow y = f(x_0) = a_0(x_0-x_1)(x_0-x_2)(x_0-x_3) ---- (x_0-x_n)$

$a_0 = f(x_0) / (x_0-x_1)(x_0-x_2)(x_0-x_3) --- (x_0-x_n) \rightarrow ②$

Now put $x = x_1$ in equation ① we get

$\Rightarrow y = f(x_1) = a_1(x_1-x_0)(x_1-x_2)(x_2-x_3) -- (x_1-x_n) + 0 + 0 --$

$\Rightarrow a_1 = f(x_1) / (x_1-x_0)(x_1-x_2) --- (x_1-x_n) \longrightarrow ③$

On continuing this process for $x = x_n$ in eq^n ① we get

$\Rightarrow f(x_n) = a_n(x_n-x_0)(x_n-x_1)(x_n-x_2) -- (x_n-x_{n-1}) + 0 + 0 -$

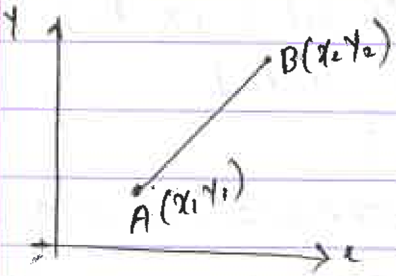
$\Rightarrow a_n = f(x_n) / (x_n-x_0)(x_n-x_1) ---- (x_n-x_{n-1}) \longrightarrow ③$

Sub. the value of $a_0\ a_1\ a_2 \ldots a_n$ from eqn ② ③ ④ in eqn ① we get

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1)$$

$$+ \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

Prove that Geodisics of plane is a straight line



let $A(x_1\ y_1)$ & $B(x_2\ y_2)$ be any two point on the xy plane and $y = f(x)$ be curve going those two point. The arc length of curve joining A & B given

$$S = \int_{x_1}^{x_2} ds = \int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx$$

Now Geodics on xy-plane is the curve $y = y(x)$ for which S is minimum

$\Rightarrow F = \sqrt{1+(y')^2}$ which is independent of the x & y

corresponding Euler's equation is

$$y'' \frac{\partial f}{\partial y'} = 0 \Rightarrow y'' = 0$$

on integrating we get $y' = m$

$$\boxed{y = mx + c}$$

Where m & c are constant

The constant m and c be determined by using the condn that the line passing through A & B

Thus Geodesics on a plane is straight line

R.L.E. SOCIETY'S

G. I. Bagewadi Arts, science and Commerce college, Nipani

DEPARTMENT OF MATHEMATICS

PRATICE TEST - I

For the year 2019-20

Class: B.Sc V sem

Date:

Roll no: 57

6/10

Answer the following questions

1) state and prove fundamental theorem of integral calculus

Statement:

If $f \in R[a\ b]$ & F is primitive of f then $\int_a^b f(x)dx = F(b) - F(a)$

Proof:→ Now $f \in R[a\ b]$ and hence $\int^b f(x)dx$ exists

given that $F(x)$ is primitive of $f(x)$ $\forall$ $x \in R[a\ b]$

$\Rightarrow F'(x) = f(x) \quad \forall x \in [a\ b] \longrightarrow ①$

$\Rightarrow F(x)$ is differentiable $\forall$ $x \in [a\ b]$

$\Rightarrow F(x)$ is differentiable in each such interval $[x_{r-1}\ x_r]$ $\forall$ $r = 1, 2, 3 \dots n$.

$\Rightarrow F(x)$ is continuous on I_r & differentiable on I_r & hence by langranges mean value theorem $f(\xi_r) \in [x_{r-1}\ x_r]$ such that $f'(\xi_r) = \dfrac{F(x_r) - F(x_{r-1})}{x_r - x_{r-1}}$

$\Rightarrow F'(\xi_r)(x_r - x_{r-1}) = F(x_r) - F(x_{r-1})$

$\Rightarrow F(\xi_r)\delta_r = F(x_r) - F(x_{r-1})$

$\Rightarrow \sum_{r=1}^{n} f(\xi_r)\delta_r = \sum_{r=1}^{n}[F(x_r) - F(x_{r-1})]$

$\Rightarrow S(P.f) = (F(x_1) - F(x_0)) + (F(x_2) - F(x_1)) + \dots - [F(x_n) - F(x_{n-1})]$

$S(P.f) = F(x_n) - F(x_0) = F(b) - F(a)$

$\lim_{n\to\infty} S(P.F) = \lim_{n\to\infty}[F(b) - F(a)]$

$\int_a^b F(x)dx = F(b) - F(a)$

1

2) Prove that $\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2\,dx}{5+3\cos x} \le \frac{\pi^3}{6}$

$\Rightarrow$ Given let us consider $g(x) = x^2$ & $f(x) = \frac{1}{5+3\cos x}\,dx$
clearly both are integrable $\forall\, x \in [0\ \pi]$

Now $f(\pi) = \frac{1}{5+3\cos x} = \frac{1}{5+3\cos\pi} = \frac{1}{5+3(-1)} = \frac{1}{2}$

$f(0) = \frac{1}{5+3\cos x} = \frac{1}{5+3\cos 0} = \frac{1}{5+3} = \frac{1}{8}$

Here $m = 1/8$ $\quad M = 1/2$

$$\frac{1}{8} \le \frac{1}{5+3\cos x} \le \frac{1}{2}$$

Multiplying by $g(x)$ ie x^2

$$\frac{x^2}{8} \le \frac{x^2}{5+3\cos x} \le \frac{x^2}{2}$$

Now integrating we get

$$\int_0^{\pi} \frac{x^2}{8}\,dx \le \int_0^{\pi} \frac{x^3}{5+3\cos x}\,dx \le \int_0^{\pi} \frac{x^2}{2}\,dx$$

$$\frac{1}{8}\left[\frac{x^3}{3}\right]_0^{\pi} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \frac{1}{2}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$\frac{1}{8}\left[\frac{\pi^3}{3} - 0\right] \le \int_0^{\pi} \frac{x^3}{5+3\cos x}\,dx \le \left[\frac{\pi^3}{3} - 0\right]$$

$$\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\,dx \le \frac{\pi^3}{6}$$

K.L.E Society's

G.I. Bagewadi Art, Science, Commerce

College Nippani

Department of Mathematics

Practice test I/II

For the year 2019-20

Class B.Sc V Sem Date ---

Roll no 112

4/10

PAGE NO.:
DATE: / /

2 Marks questions 5th sem

State ~~and prove~~ Cauchy's theorem

Statement: If the function $f(z)$ is analytic in the region bounded by a closed curve C and also on C then $\int_C f(z)\,dz$

State Liouvell's theorem

Statement: If $f(z)$ is analytic $\forall z$ in the complex plane & is bounded then $f(z)$ is constant

Solve $\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2\sin(y+2x)}$?

Consider $\frac{dx}{1} = \frac{dy}{-2}$

$\Rightarrow 2dx + dy = 0$

$2x + y = a$ —— (1)

Consider $\frac{dx}{1} = \frac{dz}{3x^2\sin(y+2x)}$

$3x^2 \sin a\, dx = dz$

$3x^2 \sin a\, dx = dz$

$\sin a\, d(x^3) = dz$

$\sin a(x^3) = z + b$

$x^3\sin(2x+y) - z = b$

Base: Let $(X, \mathcal{T})$ be any ~~any~~ topological space then the subfamily B of $\mathcal{T}$ is called as base for $\mathcal{T}$ iff every open set in $\mathcal{T}$ is expressed as union of member of B

Subbase: In topology a subbase for a topological space X with topology T is a sub collection B of T that generates T, in the sense that T is the smallest topology containing B

PAGE NO.:

DATE: / /

5 Mark questions

1) i) $A \subset B \Rightarrow \overline{A} \subset \overline{B}$

ii) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

i) Proof: Let $(X, \mathcal{J})$ be a topological Space

Let $A \subset B$ and $B \subset \overline{B}$

$\Rightarrow A \subset \overline{B}$

$\Rightarrow \overline{B}$ is a Closed Set Containing A

But $\overline{A}$ is a smallest Closed Set Containing A

$\overline{A} \subset \overline{B}$

$A \subset B \Rightarrow \overline{A} \subset \overline{B}$

ii) Let $A \subset \overline{A}$, $B \subset \overline{B}$

$\Rightarrow A \cup B \subset \overline{A} \cup \overline{B}$

$\Rightarrow \overline{A} \cup \overline{B}$ is closed Set Containing $A \cup B$

$\overline{A \cup B}$ is Smallest Closed Set Containing $A \cup B$

$\Rightarrow \overline{A \cup B} \subset \overline{A} \cup \overline{B}$ — ①

We know that

$A \subset A \cup B$ and $B \subset A \cup B$

$\overline{A} \subset \overline{A \cup B}$ and $\overline{B} \subset \overline{A \cup B}$

$\overline{A \cup B} \subset \overline{A \cup B}$ — ②

from eqⁿ ① & ②

$\overline{A \cup B} = \overline{A} \cup \overline{B}$

2) State and prove first shifting property

Statement: If Laplace transforms of $f(t)$ is $F(s)$ then

$L[e^{at} f(t)] = F(s-a)$

Proof: We have $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = F(s)$

$\therefore L[e^{at} f(t)] = \int_0^\infty e^{-st} e^{at} f(t)\,dt$

$= \int_0^\infty e^{-(s-a)t} f(t)\,dt$

$= F(s-a)$

$= F(s-a)$

K.L.E. Society's

G.I. Bagewadi Art's Science and Commurce College Nipani

Department of Mathematics

PRACTICE TEST - I

For the year 2019-20

4/10

Class :- B.Sc. V Sem Date:- 26/10/2019

Roll. No. : 02

2) Let $f(x) = \frac{1}{5+3\cos x}$ & $g(x) = x^2$

Both are R-integrable,

and $f(0) = \frac{1}{8}$, $f(\pi) = \frac{1}{2}$

$$\therefore \quad \frac{1}{8} \leq f(x) \leq \frac{1}{2}$$

$$\frac{1}{8} \leq \frac{1}{5+3\cos x} \leq \frac{1}{2}$$

multiplying by x^2 we get

$$\frac{x^2}{8} \leq \frac{x^2}{5+3\cos x} \leq \frac{x^2}{2}$$

$$\Rightarrow \int_0^\pi \frac{x^2}{8} dx \leq \int_0^\pi \frac{x^2}{5+3\cos x} \leq \int_0^\pi \frac{x^2}{2} dx$$

$$\frac{1}{8}\left[\frac{x^3}{3}\right]_0^\pi \leq \int_0^\pi \frac{x^2}{5+3\cos x} \leq \frac{1}{2}\left[\frac{x^3}{3}\right]_0^\pi$$

$$\frac{\pi^3}{24} \leq \int_0^\pi \frac{x^2}{5+3\cos x} \leq \frac{\pi^3}{6}$$

3) $F(a) = \int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\, dx$ —— ①

Then by the Leibnit's rule

$F'(a) = \int_0^{\pi} \frac{\partial}{\partial a} \frac{\log(1+a\cos x)}{\cos x}\, dx$

$F'(0) = \int_0^{\pi} \frac{1}{\cos x} \frac{1}{1+a\cos x} \cdot \cos x \, dx$

$F'(a) = \int_0^{\pi} \frac{1}{1+a\cos x}\, dx$

If $1^2 > a^2$, $F'(a) = \frac{1}{\sqrt{1-a^2}} \left\{ \cos^{-1}\left[\frac{a+\cos x}{1+a\cos x}\right] \right\}_0^{\pi}$

$= \frac{1}{\sqrt{1-a^2}} \left\{ \cos^{-1}\left[\frac{a-1}{1-a}\right] - \cos^{-1}\left[\frac{a+1}{1+a}\right] \right\}$

$\therefore F'(a) = \frac{1}{\sqrt{1-a^2}} \left\{ \cos^{-1}(-1) - \cos^{-1}(1) \right]$

$= \frac{1}{\sqrt{1-a^2}} (\pi - 0)$

$F'(a) = \frac{\pi}{\sqrt{1-a^2}}$ —— ②

Integrating eqn ② w.r.t parameter 'a' we get

$\int F'(a)\, da = \int \frac{\pi}{\sqrt{1-a^2}}\, da$

$F(a) = \pi \sin^{-1} a + c$ —— ③

Put $a=0$, $0 = \pi \sin^{-1} 0 + c$

$\Longrightarrow c = 0$

$\therefore$ eqn ③ becomes

$F(a) = \pi \sin^{-1} a$

i.e $\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\, dx = \pi \sin^{-1} a$.

K.L.E Society's

G.I. Bagewadi Art's Science and Commerce College Nipani

Department of Mathematics

PRACTICE TEST-II

For the year:...

2/10

Class: B.Sc. V Sem Date:- 26/10/2019

Roll No.: 02

1) Statement:- Let $y=f(x)$ be a function which takes the values $y_0=f(x_0)$; $y_1=f(x_1)$; ---- $y_n=f(x_n)$ corresponding to the $x=x_0, x_1, x_2 ---- x_n$ (unequally spaced value of x) then,

$$f(x)=\frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_1)\ldots(x_0-x_n)}f(x_0)+\frac{(x-x_0)(x-x_3)(x-x_3)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)}f(x_1)$$

$$+ \ldots + \frac{(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})}f(x_n)$$

is called Lagranges Interpolation formula for unequal intervals.

Proof:- Let $y=f(x)$ be a polynomial function of degree n in x which takes the values $f(x_0), f(x_1) \ldots f(x_n)$ for (n+1) unequally spaced values of x Now the polynomial can be written as follows

$$y=f(x)=a_0(x-x_1)(x-x_2)\ldots(x-x_n)+a_1(x-x_0)(x-x_2)\ldots(x-x_n)+$$
$$a_2(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)+\ldots+a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1}) \quad (1)$$

Now we need to find the value of the coefficient $a_0, a_1, a_2 \ldots a_n$ as follows, for that put $x = x_0$ in eq^n ①

$\therefore y = f(x_0) = a_0(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\ldots(x_0 - x_n) + 0$

$$\Rightarrow a_0 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\ldots(x_0 - x_n)} \quad \text{--- ②}$$

for $x = x_1$ eq^n ① becomes,

$y = f(x_1) = 0 + a_1(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\ldots(x_1 - x_n) + 0$

$$\Rightarrow a_1 = \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\ldots(x_1 - x_n)} \quad \text{--- ③}$$

On continuing this process, for $x = x_n$ eq^n ① becomes

$y = f(x_n) = 0 + 0 + a_n(x_n - x_0)(x_n - x_1)(x_n - x_2)\ldots(x_n - x_{n-1})$

$$\Rightarrow a_n = \frac{f(x_n)}{(x_n - x_0)(x_n - x_1)(x_n - x_2)\ldots(x_n - x_{n-1})} \quad \text{--- ④}$$

Substituting these values $a_0, a_1, a_2 \ldots a_n$ from eq^n ② ③ & ④ in eq^n ① we get

$$y = f(x) = \frac{(x - x_1)(x - x_2)\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots(x_0 - x_n)} f(x_0) + \frac{(x - x_0)(x - x_2)(x - x_3)\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\ldots(x_1 - x_n)} f(x_1)$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3)(x - x_4)\ldots(x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)\ldots(x_2 - x_n)} f(x_2) + \ldots$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_2)\ldots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)(x_n - x_2)\ldots(x_n - x_{n-1})} f(x_n)$$

2) This method is generaly used to improve the results obtained by previous method in minimum number of steps

Let $f(x)=0$ —① be given eqⁿ & Let x_0 be the initial approximation to the root of eqⁿ① & Let $x=x_0+h$ be the current root

$\Rightarrow f(x_1)=f(x_0+h)=0$

Then by Tailor's series we cas express $f(x_1)$ as follows

$\Rightarrow f(x_1)=f(x_0+h)=f(x_0)+hf'(x_0)+\frac{h^2}{2!}f''(x_0)+\cdots=0$

Neglecting 2nd & heigher order derivatives in the above eqⁿ then.

$\Rightarrow f(x_1)=f(x_0+h)=f(x_0)+hf'(x_0)=0$

$$\Rightarrow h=\frac{-f(x_0)}{f'(x_0)}$$ where $f'(x_0)=0$.

Let us suppose that x_1 is not our desired root then we find the next approximation root of the given eqⁿ as x_2

i.e $x_2=x_1+h=x_0+2h$

If x_2 is exact root of eqⁿ①

$\Rightarrow f(x_2)=0=f(x_1+h)$ then again by Tailor's series expansion

$f(x_2)=f(x_1+h)=f(x_1)+hf'(x_1)+\frac{h^2}{2!}f''(x_1)+\cdots$

Neglecting 2nd & higher order derivative in the above eqⁿ then

$f(x_2)=f(x_1+h)=f(x_1)+hf'(x_1)+0=0$

$$\Rightarrow h = -\frac{f(x_1)}{f'(x_1)}$$

∴ The better approximate value x_2 becomes

$$x_2 = x_1 + h = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \& \quad f'(x_1) \neq 0$$

IIIrly starting with x_2 we get the next approximation x_3 as follows

$$x_3 = x_2 + h = x_2 - \frac{f(x_2)}{f'(x_2)} \quad \& \quad f'(x_2) \neq 0$$

On continuing this process in generally we get the $(n+1)^{th}$ approximation root as follows

$$x_{n+1} = x_n + h = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0 \ \forall n \geq 0 \quad \text{—(A)}$$

eqn (A) called Newton-Rapson formula.

K.L.E Society's

G. I. Bagewadi Art's Science and Commerce College Nippani

Department of Mathematics

PRACTICE TEST - I

For the year

Class = BSC VI Sem

Roll. No. = 76

Date : 26/10/2019

3/10

2) Let, $f(x) = \frac{1}{5+3\cos 5x}$ & $g(x) = x^2$

Both are R- integrable

and $f(0) = \frac{1}{8}$, $f(\pi) = \frac{1}{2}$

$\therefore \frac{1}{8} \leq f(x) \leq \frac{1}{2}$

$\frac{1}{8} \leq \frac{1}{5+3\cos 5x} < \frac{1}{2}$

multiply by x^2 we get

$\frac{x^2}{8} \leq \frac{x^2}{5+3\cos 5x} \leq \frac{x^2}{2}$

$\Rightarrow \int_0^{\pi} \frac{x^2}{8} dx \leq \int_0^{\pi} \frac{x^2}{5+3\cos 5x} \leq \int_0^{\pi} \frac{x^2}{2} dx$

$\Rightarrow \frac{1}{8}\left[\frac{x^3}{3}\right]_0^{\pi} \leq \int_0^{\pi} \frac{x^2}{5+3\cos 5x} \leq \frac{1}{2}\left[\frac{x^3}{3}\right]_0^{\pi}$

$\Rightarrow \frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos 5x} \leq \frac{\pi^3}{6}$

3) $F(a) = \int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\, dx \longrightarrow (1)$

Then by the Leibnitz's rule.

$$F'(a) = \int_0^{\pi} \frac{\partial}{\partial x} \frac{\log(1+a\cos x)}{\cos x}\, dx$$

$$F'(a) = \int_0^{\pi} \frac{1}{\cos x} \cdot \frac{1}{1+a\cos x} \cdot \cos x \, dx$$

$$F'(a) = \int_0^{\pi} \frac{1}{1+a\cos x} \cdot dx$$

If $1^2 > a^2$, $F'(a) = \frac{1}{\sqrt{1-a^2}} \left\{ \cos^{-1}\left[\frac{a+\cos x}{1+a\cos x}\right] \right\}_0^{\pi}$

$$= \frac{1}{\sqrt{1-a^2}} \left\{ \cos^{-1}\left[\frac{a-1}{1-a}\right] - \cos^{-1}\left[\frac{a+1}{1+a}\right] \right\}$$

$$\therefore F'(a) = \frac{1}{\sqrt{1-a^2}} \left\{ \cos^{-1}(-1) - \cos^{-1}(1) \right\}$$

$$= \frac{1}{\sqrt{1-a^2}} (\pi - 0)$$

$$F'(a) = \frac{\pi}{\sqrt{1-a^2}} \longrightarrow (2)$$

Integrating eqn (2) w.r.t Parameter 'a' we get -

$$\int F'(a)\, da = \int \frac{\pi}{\sqrt{1-a^2}}\, da.$$

$$F(a) = \pi \sin^{-1} a + C \longrightarrow (3)$$

Put $a=0$, $0 = \pi \sin^{-1} 0 + C$

$$\Rightarrow C = 0$$

$\therefore$ eqn (3) becomes -

$$F(a) = \pi \sin^{-1} a.$$

$$\therefore \int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\, dx = \pi \sin^{-1} a.$$

KLE Society's

G. I. Bagewadi Arts, Science and Commerce College, Nippani

Department of Mathematics

PRACTICE TEST-II

For the year:

Class: BSc V Sem

Date: 26/10/2019

Roll. No: 74

3/10

1) **Statement:-** Let, $y = f(x)$ be a function which takes the values $y_0 = f(x_0); y_1 = f(x_1); \dots y_n = f(x_n)$ corresponding to the $x = x_0, x_1, x_2 \dots x_n$ (unequally spaced value of x) then

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)} f(x_1) + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} f(x_n)$$

is called Langrange's Interpolation formula for unequal intervals.

Proof:- Let, $y = f(x)$ be a polynomial function of degree n in x which takes the values $f(x_0), f(x_1) \dots f(x_n)$ for $(n+1)$ unequally spaced values of x. Now the polynomial can be written as follows

$$y = f(x) = a_0(x-x_1)(x-x_2)\dots(x-x_n) + a_1(x-x_0)(x-x_2)\dots(x-x_n) + a_2(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n) + \dots + a_n(x-x_0)(x-x_1)\dots(x-x_{n-1}) \longrightarrow (1)$$

Now we need to find the value of co-effecient $a_0, a_1, a_2, \ldots a_n$ as follows. for that put $x = x_0$ in eq^n ①.

i.e $y = f(x_0) = a_0(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots(x_0-x_n)+0$

$$\Rightarrow a_0 = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots(x_0-x_n)} \longrightarrow ②$$

for $x = x_1$ in eq^n ① becomes -

$y = f(x_1) = 0 + a_1(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)+0$

$$\Rightarrow a_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)} \longrightarrow ③$$

On continuing this process for $x = x_n$ eq^n ① become

$y = f(x_n) = 0 + 0 + a_n(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})$

$$\Rightarrow a_n = \frac{f(x_n)}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})} \longrightarrow ④$$

Substituting these values $a_0, a_1, a_2, \ldots a_n$ for eq^n ②, ③ & ④ in eq^n ① we get -

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)} f(x_1) +$$

$$- - - - - - - - - - - - - - - - - -$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})} f(x_n).$$

This method is generaly used to improve the results obtained by previous method in minim-um number of steps.

Let $f(x)=0 \longrightarrow (1)$ be given eqn & Let x_0 be the initial approximation to the root of eqn (1) & Let, $x_1 = x_0 + h$ be the current root.

$\Rightarrow f(x_1) = f(x_0+h) = 0$.

Then by Tailor's Series we can express $f(x_1)$ as follows

$\Rightarrow f(x_1) = f(x_0+h) = f(x_0) + h.f'(x_0) + \frac{h^2}{2!} f''(x_0) + \ldots = 0$.

Negalecting 2nd & higher order derivatives in the above eqn then

$\Rightarrow f(x_1) = f(x_0+h) = f(x_0) + h f'(x_0) = 0$

$\Rightarrow h = \frac{-f(x_0)}{f'(x_0)}$ where, $f'(x_0) = 0$.

Let us suppose that x_1 is not our desired root then we find the next approximation root of the given eqn as x_2

i.e $x_2 = x_1 + h = x_0 + 2h$

If x_2 is exact root of eqn (1)

$\Rightarrow f(x_2) = 0 = f(x_1+h)$ then again by Tailer's Series expansion

$f(x_2) = f(x_1+h) = f(x_1) + h f'(x_1) + \frac{h^2}{2!} f''(x_2) + \ldots$

Negalecting 2nd & higher order derivative in the above eqn then

$f(x_2) = f(x_1+h) - f(x_1) + h f'(x_1) + 0 = 0$

$\Rightarrow h = -\frac{f(x_1)}{f'(x_0)}$

∴ The better approximate value of x_2 becomes

$$x_2 = x_1 + h = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \& \quad f'(x_1) \neq 0$$

IIly Starting with x_2 we get the next approximation x_3 as follows

$$x_3 = x_2 + h = x_2 - \frac{f(x_2)}{f'(x_2)} \quad \& \quad f'(x_2) \neq 0$$

On continuing this process in generally we get the (n+1)th approximation root as follows -

$$x_{n+1} = x_n + h = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0 \ \forall n \geq 0 \rightarrow (A)$$

Eqn (A) called Newton-Rapson formula.

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G.I. Bagewadi Arts, Science and Commerce College
Nipani

DEPARTMENT OF MATHEMATICS

For the year. 2019-20

Roll. No. :- 04 Class :- B.Sc VI Sem

Date :- 15/10/2020

6/10

* Answer the following question

1) State and Prove necessary condition for the function $f(z)$ to be analytic.

⟹ Statement :-

If the function $f(z)$ is analytic in the domain D then the conditions $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are satisfied.

Proof :-

Let $\omega = f(z) = u(x,y) + iv(x,y)$ be analytic in the domain D.

By defination, $f'(z) = \frac{dw}{dz}$ exist $\forall$ in the domain D.

$\therefore f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exist along any path we choose for $\Delta z \to 0$.

where $\Delta z = \Delta x + i\Delta y$

$$f'(z) = \lim_{(\Delta x, \Delta y) \to (0,0)} \frac{[u(x+\Delta x, y+\Delta y) + iv(x+\Delta x, y+\Delta y)] - [u(x,y) + iv(x,y)]}{\Delta x + i\Delta y} \longrightarrow (1)$$

As the derivative exist this limit is unique irrespective of the path in which $\Delta z \to 0$.

i) Let $\Delta z \to 0$ along the x-axis

Along the x-axis $y = 0$

$\therefore \Delta z = \Delta x$ and $\Delta y \to 0$.

$\therefore$ from (1)

$$f'(z) = \lim_{\Delta x \to 0} \frac{[u(x+\Delta x, 0) + iv(x+\Delta x, 0)] - [u(x,0) - iv(x,0)]}{\Delta x}$$

$$f'(z) = \lim_{\Delta x \to 0} \frac{u(x+\Delta x, 0) - u(x,0)}{\Delta x} + i \lim_{\Delta x \to 0} \frac{v(x+\Delta x, 0) - v(x,0)}{\Delta x}$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \longrightarrow (2)$$

ii) Let $\Delta z \to 0$ along the y-axis

along the y-axis $x = 0$

$\therefore \Delta z = i\Delta y$ & $\Delta x \to 0$.

$\therefore$ from ①.

$$f'(z) = \lim_{\Delta iy \to 0} \frac{[u(0, y+\Delta y) + iv(0, y+\Delta y)] - [u(0, y) + iv(0, y)]}{\Delta iy}$$

$$= \lim_{\Delta y \to 0} \frac{u(0, y+\Delta y) - u(0, y)}{i\Delta y} + i \lim_{\Delta y \to 0} \frac{v(0, y+\Delta y) - v(0, y)}{i\Delta y}$$

$$= \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$= \frac{-i\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$= \frac{\partial v}{\partial y} - \frac{i\partial u}{\partial y} \longrightarrow ③$$

$\therefore$ From ② & ③

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - \frac{i\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\text{or } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Thus if $f(z) = u + iv$ is analytic then cauchy's Reimann equation $u_x = v_y$ & $u_y = -v_x$ are satisfied.

2) State and Prove cauchy's residue theorem.

Statement :- Let $f(z)$ be analytic within and on closed contour c except at finite no. of poles $z_1, z_2 --- z_n$ inside c then $\int_c f(z)dz = 2\pi i(R_1 + R_2 + --- R_n)$ $= 2\pi i$ (Sum of residues at these poles inside c).

Proof :-

By hypothesis $z_1, z_2, z_3 ---- z_n$ poles of $f(z)$ inside c. Therefore function $f(z)$ is not analytic at these point in side c. Hence construct small circle $\gamma_1, \gamma_2, \gamma_3, ------ \gamma_n$ around those Points then $f(z)$ is analytic in the region bounded by closed curve c. $\gamma_1, \gamma_2, \gamma_3 ------ \gamma_n$.

By cauchy's theorem for multi connected region we have

$$\int_c f(z)dz = \gamma_1 \int f(z)dz + \gamma_2 \int f(z)dz + ---- + \gamma_n \int f(z)dz \longrightarrow (1)$$

By defination of residue of $f(z)$ we have

$R_1 = \frac{1}{2\pi i} \gamma_1 \int f(z)dz$ where γ_1 is circle around the pole z_1 & R_1 is respectively

$$\therefore \gamma_1 \int f(z)dz = 2\pi i R_1$$

Simillarly

$$\gamma_2 \int f(z)dz = 2\pi i R_2, \quad \gamma_3 \int f(z)dz = 2\pi i R_3, -------$$

$$\gamma_n \int f(z)dz = 2\pi i R_n$$

Then equation (1) becomes,

$$\int_c f(z)dz = 2\pi i R_1 + 2\pi i R_2 + 2\pi i R_3 + ------ + 2\pi i R_n$$
$$= 2\pi i (R_1 + R_2 + R_3 + ----- + R_n)$$
$= 2\pi i$ (Sum of residues at these poles inside c)

Thus if $f(z)$ be analytic within and on closed contour c except at finite no. of poles $z_1, z_2, z_3 --- z_n$ inside c then

$\int_c f(z)dz = 2\pi i (R_1 + R_2 + R_3 + ---- + R_n) = 2\pi i$ (Sum of residues at these Poles inside c).

where $R_1, R_2, R_3 ---- R_n$ are residues at poles $z_1, z_2, z_3, -----z_n$ respectively.

3) Prove that $R = \{a+b\sqrt{2} \,/ a.b \in \mathbb{Q}\}$ is a ring on set of real number R

→ given $R = \{a + b\sqrt{2} \,/ a, b \in \mathbb{Q}\}$

I) To prove $(R +)$ is abelian.

i) closure law:-

Let $x_1 = a_1 + b_1\sqrt{2}$ $x_2 = a_2 + b_2\sqrt{2}$

where a_1, a_2, b_1 & b_2 are $\mathbb{Q}$ $\therefore x_1 x_2 \in R$

then $x_1 + x_2 \in R$

$\therefore$ Closure law holds

ii) Associative law:-

Let $x_1 = a_1 + b_1\sqrt{2}$ & $x_2 = a_2 + b_2\sqrt{2}$ & $x_3 = a_3 + b_3\sqrt{2}$

then $x_1 + (x_2 + x_3) = (a_1 + b_1\sqrt{2}) + [(a_2 + b_2\sqrt{2}) + (a_3 + b_3\sqrt{2})]$

$= a_1 + b_1\sqrt{2} + [(a_2 + a_3) + (b_2 + b_3)\sqrt{2}]$

$= a_1 + (a_2 + a_3) + [b_1 + (b_2 + b_3)]\sqrt{2}$

$= [(a_1 + a_2) + a_3] + [(b_1 + b_2) + b_3]\sqrt{2}$

$= (a_1 + a_2) + (b_1 + b_2)\sqrt{2} + a_3 + b_3\sqrt{2}$

$= [(a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2}) + (a_3 + b_3\sqrt{2})]$

$x_1 + (x_2 + x_3) = (x_1 + x_2) + x_3$

$\therefore$ Associative law holds

iii) Identity law:-

For $a + b\sqrt{2} \in R$ $\exists$ $0 + 0\sqrt{2}$ such that

$(a + b\sqrt{2}) + (0 + 0\sqrt{2}) = (a + 0) + (b + 0)\sqrt{2} = a + b\sqrt{2}$

$(0 + 0\sqrt{2}) + (a + b\sqrt{2}) = (0 + a) + (0 + b)\sqrt{2} = a + b\sqrt{2}$

$\therefore 0 + 0\sqrt{2}$ is acts as an identity element in R

$\therefore$ Identity law holds

iv) Inverse law :-

For $a+b\sqrt{2} \in R$ $\exists$ $-(a+b\sqrt{2}) = -a-b\sqrt{2}$ such that

$a+b\sqrt{2} + (-a-b\sqrt{2}) = (a-a)+(b-b)\sqrt{2} = 0+0\sqrt{2}$

$(-a-b\sqrt{2}) + (a+b\sqrt{2}) = (-a+a)+(-b+b)\sqrt{2} = 0+0\sqrt{2}$

$\therefore$ Inverse law holds.

v) Commutative law :-

Let $x_1 = a_1+b_1\sqrt{2}$ $\quad x_2 = a_2+b_2\sqrt{2}$

$x_1, x_2 \in R$ where a_1, a_2, b_1 & $b_2 \in Z$.

Now consider $x_1+x_2 = a_1+b_1\sqrt{2} + a_2+b_2\sqrt{2}$

$= (a_1+a_2) + (b_1+b_2)\sqrt{2}$

$= (a_2+a_1) + (b_2+b_1)\sqrt{2}$

$= a_2+b_2\sqrt{2} + a_1+b_1\sqrt{2}$

$x_1+x_2 = x_2+x_1$

$\therefore$ commutative law holds.

$\therefore$ $(R\ +)$ is an abelian group.

II) $(R\ \circ)$ is semigroup to prove

i) Closure law :-

Let $x_1 = a_1+b_1\sqrt{2}$ & $x_2 = a_2+b_2\sqrt{2}$

Now consider $x_1 \cdot x_2 = (a_1+b_1\sqrt{2})(a_2+b_2\sqrt{2}) = a_1a_2 + b_1b_2\sqrt{2}$
$+a_2b_1\sqrt{2} + 2b_1b_2$

$= (a_1a_2 + 2b_1b_2) + (a_1b_2+a_2b_1)\sqrt{2}$

$\therefore x_1 x_2 \in R$.

$\therefore$ closure law holds

ii) Associative law :-

Let $x_1 = a_1+b_1\sqrt{2}$, $x_2 = a_2+b_2\sqrt{2}$ & $x_3 = a_3+b_3\sqrt{2}$.

Now consider $x_1[x_2 x_3]$

$= (a_1+b_1\sqrt{2})[(a_2+b_2\sqrt{2})(a_3+b_3\sqrt{2})]$

$= (a_1+b_1\sqrt{2})[a_2a_3 + a_3b_2\sqrt{2} + a_2b_3\sqrt{2} + 2b_2b_3]$

$= [a_1a_2a_3 + a_1a_3b_2\sqrt{2} + a_1a_2b_3\sqrt{2} + 2a_1b_3b_2 + b_1a_2a_3\sqrt{2} + 2b_1b_2a_3$
$+ 2b_1a_2b_3 + b_1b_2b_3 2\sqrt{2}]$
$= [a_1a_2a_3 + 2a_1b_3b_2 + 2b_1b_2a_3 + 2b_1a_2b_3] + [a_1a_3b_2 + a_1a_2b_3 + b_1a_2a_3$
$+ 2b_1b_2b_3]\sqrt{2} \longrightarrow (1)$

Now

$[x_1 x_2]x_3 = [(a_1+b_1\sqrt{2})(a_2+b_2\sqrt{2})](a_3+b_3\sqrt{2})$
$= [a_1a_2 + a_1b_2\sqrt{2} + b_1a_2\sqrt{2} + 2b_1b_2](a_3+b_3\sqrt{2})$
$= [a_1a_2a_3 + a_1b_2a_3\sqrt{2} + b_1a_2a_3\sqrt{2} + 2b_1b_2a_3 + a_1a_2b_3\sqrt{2} +$
$2a_1b_2b_3 + 2b_1a_2b_3 + 2b_1b_2b_3\sqrt{2}]$
$= [a_1a_2a_3 + 2b_1b_2a_3 + 2a_1b_2b_3 + 2b_1a_2b_3] + [a_1b_2a_3 + b_1a_2a_3 +$
$a_1a_2b_3 + 2b_1b_2b_3]\sqrt{2} \longrightarrow (2)$

from (1) & (2)

$[x_1 x_2]x_3 = [x_2 x_3]x_1$

$\therefore$ Associative law holds

$\therefore (R \cdot)$ is semigroup

III To prove distrubutive law:-

Let $x_1 = a_1+b_1\sqrt{2}$ $x_2 = a_2+b_2\sqrt{2}$ $x_3 = a_3+b_3\sqrt{2}$

Let us consider $x_1 x_2 x_3 \in R$

then $x_1(x_2+x_3) = (a_1+b_1\sqrt{2})[(a_2+b_2\sqrt{2}) + (a_3+b_3\sqrt{2})]$
$= [a_1+b_1\sqrt{2}]\{(a_2+a_3) + (b_2+b_3)\sqrt{2}\}$
$= a_1(a_2+a_3) + a_1(b_2+b_3)\sqrt{2} + b_1(a_2+a_3)\sqrt{2}$
$+ b_1(b_2+b_3)2$
$= a_1a_2 + a_1a_3 + a_1b_2\sqrt{2} + a_1a_3\sqrt{2} + b_1a_1\sqrt{2} + a_3b_1\sqrt{2} + 2b_1b_2$
$+ 2b_1b_3$
$= (a_1a_2 + a_1a_3 + 2b_1b_2 + 2b_1b_3) + (a_1b_2 + a_1b_3 + b_1a_2 + a_3b_1)\sqrt{2} \rightarrow (1)$

Now consider

$x_1x_2 + x_1x_3$
$= [(a_1+b_1\sqrt{2})(a_2+b_2\sqrt{2})] + [(a_1+b_1\sqrt{2})(a_3+b_3\sqrt{2})]$
$= [(a_1a_2 + a_1b_2\sqrt{2} + b_1a_2\sqrt{2} + 2b_1b_2)] + [a_1a_3 + a_1b_3\sqrt{2} +$
$b_1a_3\sqrt{2} + 2b_1b_3]$

$= [(a_1 a_2 + 2b_1 b_2) + (a_1 b_2 + b_1 a_2)\sqrt{2}] + [a_1 a_3 + 2b_1 b_3 + (b_3 a_1 + b_1 a_3)\sqrt{2}]$

$= (a_1 a_2 + a_1 a_3 + 2b_1 b_2 + 2b_1 b_3) + (a_1 b_2 + b_1 a_2 + b_3 a_1 + b_1 a_3)\sqrt{2}$ → (2)

From (1) & (2)

$x_1(x_2 + x_3) = x_1 x_2 + x_1 x_3$

∴ Distaubutive law holds.

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G. I. Bagewadi Arts, Science and Commerce college Nipani

DEPARTMENT OF MATHEMATICS

For the Year 2019-20

Roll No. :- 04 Class :- BSc VIth sem.

Date :- 15/10/2020

4/10

* Answer the following questions:-

1) Find the necessary condition of integrability of $Pdx + Qdy + Rdz = 0$

⇒ An equation of the form $Pdx + Qdy + Rdz = 0$ where P, Q & R functions of x, y, z is called total differential equation or single differential equation.

Condition of integrability.

To find the condition of integrability of total differential equation:-

Total differential equation is $Pdx + Qdy + Rdz = 0 \rightarrow (1)$

Let the solution of total differential equation be $u = a \rightarrow (2)$

But the total differential is given by $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \rightarrow (3)$

Since (1) & (3) are same equation hence their coefficients are proportional

i.e $\mu P = \frac{\partial u}{\partial x}$, $\mu Q = \frac{\partial u}{\partial y}$, $\mu R = \frac{\partial u}{\partial z}$

Of the above three equation.

Differentiating first partially w.r.t y & second w.r.t x we get equation (4). Differentiating second partially w.r.t z & third w.r.t y we get equation (5) & differentiating partially w.r.t x & third w.r.t z we get equation (6) as follows.

$$P\frac{\partial \mu}{\partial y} + \mu\frac{\partial P}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} = Q\frac{\partial \mu}{\partial x} + \mu\frac{\partial Q}{\partial x} \longrightarrow (4)$$

$$Q\frac{\partial \mu}{\partial z} + \mu\frac{\partial Q}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} = R\frac{\partial \mu}{\partial y} + \mu\frac{\partial R}{\partial y} \longrightarrow (5)$$

$$R\frac{\partial \mu}{\partial x} + \mu\frac{\partial R}{\partial x} = \frac{\partial^2 u}{\partial z \partial x} = P\frac{\partial \mu}{\partial z} + \mu\frac{\partial P}{\partial z} \longrightarrow (6)$$

Rearranging equation (4), (5) & (6) we get

$$\mu\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=Q\frac{\partial \mu}{\partial x}-P\frac{\partial \mu}{\partial y} \longrightarrow (7)$$

$$\mu\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)=R\frac{\partial \mu}{\partial y}-Q\frac{\partial \mu}{\partial z} \longrightarrow (8)$$

$$\mu\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)=P\frac{\partial \mu}{\partial z}-R\frac{\partial \mu}{\partial x} \longrightarrow (9)$$

multiplying equation (7) by R, equation 8 by P equation 9 by Q and adding we get

$$P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0$$

This is the required condition of the integrability which can also be written in the determinant from as follows.

$$\begin{vmatrix} P & Q & R \\ P & Q & R \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix}=0.$$

2) In a topological space $(X, \mathcal{J})$ a subset of A of X is open iff A' is closed

→ Let $(X, \mathcal{J})$ be a topological space and A be any subset of X then clousure of set A is denoted by $\bar{A}$ and is defined as intersection of all the closed set containg A.

we know that $A \subset \bar{A}$ by the defination of closure of set A. & $\bar{A}$ is closed set being a intersection of closed sets.

8) Find $L\left[e^t (\sin 3t \cdot \cos 4t)\right]$

$\Longrightarrow$ Given $L\left[e^t (\sin 3t \cos 4t)\right]$

$$= L\left[e^t \left[\tfrac{1}{2}\left(\sin(3t+4t) + \sin(3t-4t)\right)\right]\right]$$

$$= L\left[\frac{e^t}{2}\left[\sin(7t) + \sin(-t)\right]\right]$$

$$= \tfrac{1}{2} L\left[e^t \sin(7t) - e^t \sin(t)\right]$$

$$= \tfrac{1}{2}\left[L\left[e^t(\sin 7t)\right] - L\left[e^t(\sin t)\right]\right]$$

$$= \tfrac{1}{2}\left[\frac{7}{(s-1)^2+49} - \frac{1}{(s-1)^2+1}\right]$$

$$= \tfrac{1}{2}\left[\frac{7\left[(s^2+1^2-2s+1)\right] - \left[s^2+1-2s+49\right]}{\left((s-1)^2+49\right)\left((s-1)^2+1\right)}\right]$$

$$= \frac{1}{2}\left[\frac{7s^2+14-14s - s^2-50+2s}{(s^2+1-2s+49)(s^2+1-2s+1)}\right]$$

$$= \tfrac{1}{2}\left[\frac{6s^2-36-12s}{(s^2-2s+50)(s^2-2s+2)}\right]$$

$$= \frac{3s^2-18-6s}{(s^2-2s+50)(s^2-2s+2)}$$

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G.I. Bagewadi Arts Science and Commurce
College Nipani
Department of Mathematics
PRACTICE TEST - I
For the year : 2019-20

4/10

Class : B. Sc. VI Sem   Date : 15/10/2020
Roll. No. : 18

1) Neccessary condition for f(z) to be analytic:

If the function f(z) is analytic in the domain D then conditions $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are satisfied.

Proof:- Let $w = f(z) = u(x,y) + iv(x,y)$ be analytic in the domain D.

By definition $f'(z) = \frac{dw}{dz}$ exist $\forall$ z in the domain D.

i.e $f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exist along any path

we choose for $\Delta z \to 0$, $\Delta z = \Delta x + i\Delta y$

$\therefore f'(z) = \lim_{(\Delta x, \Delta y) \to 0} \frac{\{u(x+\Delta x, y+\Delta y) + iv(x+\Delta x, y+\Delta y)\} - \{u(x,y) + iv(x,y)\}}{\Delta x + i\Delta y}$ — (1)

As the derivative exist, this limit is unique irrespective of path in which $\Delta z \to 0$

i) Let $\Delta z \to 0$ along the X-axis,

along X-axis, $y = 0 \Rightarrow \Delta y = 0$

$\therefore \Delta z = \Delta x$,

∴ eqⁿ ① becomes

$$f'(z) = \lim_{\Delta x \to 0} \frac{\{u(x+\Delta x,0) + iv(x+\Delta x,0)\} - \{u(x,0)+iv(x,0)\}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x,0) - u(x,0)}{\Delta x} + i \lim_{\Delta x \to 0} \frac{v(x+\Delta x,0) - v(x,0)}{\Delta x}$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad \text{——} ②$$

ii) Let $\Delta z \longrightarrow 0$ along the Y-axis, and along Y-axis, $x = 0 \implies \Delta x = 0$

∴ $\Delta z = i\Delta y$

∴ eqⁿ ① becomes

$$f'(z) = \lim_{\Delta y \to 0} \frac{\{u(0,y+\Delta y) + iv(0,y+\Delta y)\} - \{u(0,y)+iv(0,y)\}}{i\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(0,y+\Delta y) - u(0,y)}{i\Delta y} + i \lim_{\Delta y \to 0} \frac{v(0,y+\Delta y) - v(0,y)}{i\Delta y}$$

$$f'(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$f'(z) = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \quad \text{——} ③$$

∴ from eqⁿ ② & ③

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

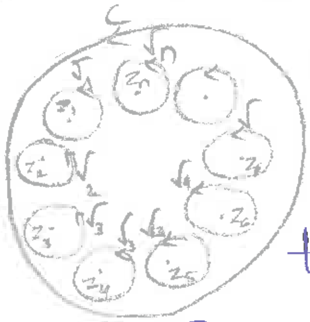
On comparing both the sides,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

2) <u>Statement</u> :- Let $f(z)$ be analytic within & on closed contour C except at finite no. of poles $z_1, z_2, z_3 \ldots z_n$ inside C then $\int_C f(z)dz = 2\pi i(R_1 + R_2 + R_3 + \ldots + R_n)$

Where, $R_1, R_2, R_3 \ldots R_n$ are residue at poles $z_1 z_2 z_3 \ldots z_n$ respectively.

<u>Proof</u> :- By hypothesis $z_1 z_2 z_3 \ldots z_n$ poles of $f(z)$ inside C. Therefour function $f(z)$ is not analytic at these



points inside C. Hence construct small circles $\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_n$ around these points then $f(z)$ is analytic in the region bounded by closed curves $C, \gamma_1, \gamma_2, \gamma_3 \ldots \gamma_n$

By Cauchy's theorem for multiconnected region we have

$$\int_C f(z)dz = \int_{\gamma_1} f(z)dz + \int_{\gamma_2} f(z)dz + \ldots + \int_{\gamma_n} f(z)dz \quad \text{—(1)}$$

By definition of residue of $f(z)$ we have

$R_1 = \frac{1}{2\pi i}\int_{\gamma_1} f(z)dz$ where γ_1 is circle around the pole z_1 & R_1 is residue

$$\therefore \int_{\gamma_1} f(z)dz = 2\pi i R_1$$

Similarly, $\int_{\gamma_2} f(z)dz = 2\pi i R_2$, $\int_{\gamma_3} f(z)dz = 2\pi i R_3$ ……

…… $\int_{\gamma_n} f(z)dz = 2\pi i R_n$

Then eq^n (1) becomes

$$\int_C f(z)dz = 2\pi i R_1 + 2\pi i R_2 + 2\pi i R_3 + \ldots + 2\pi i R_n$$

$$\int_C f(z)\,dz = 2\pi i\,(R_1 + R_2 + R_3 + \cdots R_3)$$

$\int_C f(z)\,dz = 2\pi i$ (sum of residues at these poles inside C)

Thus if $f(z)$ be analytic within & on closed contour C except at finite no. of poles $z_1 z_2 z_3 \ldots z_n$ inside C then

$\int_C f(z)\,dz = 2\pi i\,(R_1 + R_2 + R_3 + \cdots + R_n) = 2\pi i$ (sum of residue at these poles inside c)

K.L.E. SOCIETY'S

G.I. Bagewadi Arts Science and Commerce college
Nipani

DEPARTMENT OF MATHEMATICS

~~PRACICE TEST - II~~

~~For the year 2019-20~~

Class : B.Sc. VI sem Date

Roll No 57

9/20

Answer the following question:→

1) Find the necessary condition of integrability of

$P\,dx + Q\,dy + R\,dz = 0$

⇒ An eqⁿ of the form $P\,dx + Q\,dy + R\,dz = 0$ where P, Q, & R are fⁿ of x, y, z, is called total differential eqⁿ or

Condⁿ of integrability:

Total differential eqⁿ is $P\,dx + Q\,dy + R\,dz = 0 \longrightarrow (1)$

let solⁿ of total differential eqⁿ be $u = a \longrightarrow (2)$

let the total differential is given by $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \quad \text{---} (3)$

since (1) & (3) are same eqⁿ hence their co-efficient are same ie $\mu P = \partial u/\partial x$ $\mu Q = \partial u/\partial y$ $\mu R = \partial u/\partial z$

of the above three eqⁿ. Diff. Ist. p. w.r.t. y and II w.r.t. x, II w.r.t. z and III w.r.t. x. we get following eqⁿ (4) (5), (6)

$$\Rightarrow P\frac{\partial \mu}{\partial y} + \mu\frac{\partial P}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} = Q\frac{\partial \mu}{\partial x} + \mu\frac{\partial Q}{\partial x} \longrightarrow (4)$$

$$\Rightarrow Q\frac{\partial \mu}{\partial z} + \mu\frac{\partial Q}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} = R\frac{\partial \mu}{\partial y} + \mu\frac{\partial R}{\partial y} \longrightarrow (5)$$

$$\Rightarrow R.\frac{\partial \mu}{\partial x} + \mu\frac{\partial R}{\partial x} = \frac{\partial^2 u}{\partial x \partial z} = P\frac{\partial \mu}{\partial z} + \mu\frac{\partial P}{\partial z} \longrightarrow (6)$$

Rearranging eqⁿ (4) (5) (6) we get.

$$\Rightarrow \mu\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = Q\frac{\partial \mu}{\partial x} - P\frac{\partial \mu}{\partial y} \rightarrow (7)$$

$$\Rightarrow \mu\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = R\frac{\partial \mu}{\partial y} - Q\frac{\partial \mu}{\partial z} \rightarrow (8)$$

$$\Rightarrow \mu\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) = P\frac{\partial \mu}{\partial z} - R\frac{\partial \mu}{\partial x} \rightarrow (9)$$

Multiplying eqⁿ (7) by R and (8) by P and (9) by Q and adding we get

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

This is the required condⁿ of integrability. $\begin{vmatrix} P & Q & R \\ P & Q & R \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix} = 0$

e) Find $L[e^t(\sin 3t \cdot \cos 4t)]$

→ Given $L[e^t(\sin 3t \cdot \cos 4t)] = L\left[e^t\left[\frac{1}{2}(\sin(3t+4t) + \sin(3t-4t))\right]\right]$

$$= L\left[\frac{e^t}{2}[\sin(7t) - \sin(-t)]\right]$$

$$= \frac{1}{2} L[e^t \sin(7t) - e^t \sin(t)]$$

$$= \frac{1}{2}[L(e^t \sin(7t)) - e^t [e^t \sin(t)]]$$

$$= \frac{1}{2}\left[\frac{7}{(s-1)^2+49} - \frac{1}{(s-1)^2+1}\right]$$

$$= \frac{1}{2}\left[\frac{7[(s^2+1^2-2s+1)] - [s^2+1-2s+49]}{((s-1)^2+49)((s-1)^2+1)}\right]$$

$$= \frac{1}{2}\left[\frac{7s^2+14-14s-s^2-50+2s}{(s^2+1-2s+49)(s^2+1-2s+1)}\right]$$

$$= \frac{1}{2}\left[\frac{6s^2-36-12s}{(s^2-2s+50)(s^2-2s+2)}\right]$$

$$= \frac{3s^2-18-6s}{(s^2-2s+50)(s^2-2s+2)}$$

2) In a topological space $(X, \mathfrak{I})$ a subset of A of X is open iff A' is closed

⇒ let $(X, \mathfrak{I})$ be a topological space and A be any subset of X then clousure of set A is denoted by $\bar{A}$ and is defined as intersection of all the closed set containing A

we know that $A \subset \bar{A}$ by the defination of closure set A and $\bar{A}$ is closed set being a intersection of closed sets

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G. I. Bagewadi. Arts Science and Commerce, college Nipani

DEPARTMENT OF MATHEMATICS

PRACTICE TEST - I

For the year 2019-20

class : B.Sc. VI sem   Date:

Roll No: 57

5/10

Answer the following question:

1) State ~~Ae~~ and prove necessary cond^n for the fun^n $f(z)$ to be analytic

Statement:

If the function $f(z)$ is analytic in the domain in the D then the cond^n $\partial u/\partial x = \partial v/\partial y$ and $\partial u/\partial y = -\partial v/\partial x$ are satisfied

Proof: let $\omega = f(z) = u(x\,y) + iv(x\,y)$ be analytic in the Domain 'D'

By def^n $f'(z) = d\omega/dz$ exist $\forall$ in the domain D

$\therefore f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exist along any path we choose from $\Delta z \to 0$ where $\Delta z = \Delta x + i\Delta y$

$$F'(z) = \lim_{(\Delta x\,\Delta y)\to(0\,0)} \frac{[U(x+\Delta x, y+\Delta y) + iV(x+\Delta x\; y+\Delta y)] - [U(x\,y) + iV(x\,y)]}{\Delta x + i\Delta y} \quad (1)$$

As the derivative exist this limit is unique irrespective of the path in which $\Delta z \to 0$

i) let $\Delta z \to 0$ along the x-axis, along x-axis $y=0$

$\therefore \Delta z = \Delta x$ and $\Delta y \to 0$

$$\text{from (1) } F'(z) = \lim_{\Delta x \to 0} \frac{[u(x+\Delta x, 0) + iv(x+\Delta x, 0)] - [U(x\,0) - iV(x,0)]}{\Delta x}$$

$$F'(z) = \lim_{\Delta x \to 0} \frac{[u(x+\Delta x\,0) - u(x\,0)]}{\Delta x} + i \lim_{\Delta x \to 0} \frac{[v(x+\Delta x, 0) - V(x\,0)]}{\Delta x}$$

$$F'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \longrightarrow (2)$$

(ii) let $\Delta z \to 0$ along the y-axis, along the y-axis $x=0$

$\Delta z = i\Delta y$. & $\Delta x \Rightarrow 0$

$$\text{from } f'(z) = \lim_{\Delta y \to 0} \frac{U(0\;y+\Delta y) - U(0\,y)}{i\Delta y} + \lim_{\Delta y \to 0} \frac{V(0\;y+\Delta y) - V(0\,y)}{i\Delta y}$$

$$f'(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \longrightarrow (3)$$

from (2) & (3)

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

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$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ \& } \frac{\partial v}{\partial x} = \frac{-\partial v}{\partial y} \text{ or } \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

Thus if $f(z) = u+iv$ is analytic then cauchy's R.eq^n $U_x = V_y$ & $U_y = -V_x$ are satisfied

2) State and prove cauchy's residue theorem.

Statement: let $f(z)$ be analytic within and on closed counter c except at finite no. of poles $z_1\ z_2 -- z_n$ inside c then

$\int_c f(z)\,dz = 2\pi i\,(R_1 + R_2 + -- R_n) = 2\pi i$ (sum of residue at thes pole-c)

Proof:- By hypothesis $z_1\ z_2\ z_3 -- z_n$ pole of $f(z)$ inside c ∴ fun^n $f(z)$ is not analytic at these point inside c. hence construct small circle $\gamma_1\ \gamma_2\ \gamma_3 -- \gamma_n$ around those points then $f(z)$ is analytic in the region bounded by closed curve c $\gamma_1\ \gamma_2\ \gamma_3 -- \gamma_n$. By cauchy's theorem for multi connected region we have $\int_c f(z)\,dz = \gamma_1 \int f(z)\,dz + \gamma_2 \int f(z)\,dz + -- + \gamma_n \int f(z)\,dz \rightarrow ①$

By def^n we have $R_1 = \frac{1}{2\pi i}\ \gamma_1 \int f(z)\,dz$ where γ_1 is circle around the pole z_1 & R_1 respectively

$$\therefore \gamma_1 \int f(z)\,dz = 2\pi i R_1$$

Similary $\gamma_2 \int f(z)\,dz = 2\pi i R_2$ $\quad \gamma_3 \int f(z)\,dz = 2\pi i R_3$ ----

$$\gamma_n \int f(z)\,dz = 2\pi i R_n$$

Then eq^{n}① becomes:

$$\int_c f(z)\,dz = 2\pi i R_1 + 2\pi i R_2 + -- + 2\pi i R_n$$
$$= 2\pi i\,(R_1 + R_2 + -- + R_n)$$
$$= 2\pi i \text{ (sum of residues at these pole inside c)}$$

Thus if $f(z)$ be analytic ~~at~~ within and on closed inside c then

$\int_c f(z)\,dz = 2\pi i\,(R_1 + R_2 + -- + R_n)$ $R_1\ R_2 -- R_n$ residues

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G. I. Bagewadi Arts Science and Commerce College Nipani

Department of Mathematics

PRACTICE TEST - II

For the year: 2019-20

9/10

Class: B.Sc. VI Sem   Date: 15/10/2020

Roll. No: 18

2) Let, $L[e^t(\sin 3t \cdot \cos 4t)]$

$= L[e^t\{\sin 7t - \sin t\}]$

$= L[e^t \sin 7t - e^t \sin t]$

$= L[e^t \sin 7t] - L[e^t \sin t]$

$= \frac{7}{(S-1)^2+49} - \frac{1}{(S-1)^2+1}$

$\therefore L[e^t(\sin 3t \cos 4t)] = \frac{7}{(S-1)^2+49} - \frac{1}{(S-1)^2+1}$

3) The total differential eq^n is $Pdx + Qdy + Rdz = 0$ —(1)

Let the solution of total differential eq^n be, $u = c$ —(2)

But the total differential is given by

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \quad —(3)$$

Since (1) & (3) are same equation hence their coefficients are proportional i.e $\mu P = \frac{\partial u}{\partial x}$, $\mu Q = \frac{\partial u}{\partial y}$, $\mu R = \frac{\partial u}{\partial z}$.

Of the above three equations,
Differentiating first partially w.r.t y & second w.r.t x we get eqⁿ ④, Differentiating second partially w.r.t. z & the third w.r.t. y we get eqⁿ ⑤ & differentiating third w.r.t. x & the first w.r.t. z we get eqⁿ ⑥ as follows.

$$P\frac{\partial \mu}{\partial y} + \mu\frac{\partial P}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} = Q\frac{\partial \mu}{\partial x} + \mu\frac{\partial Q}{\partial x} \quad —④$$

$$Q\frac{\partial \mu}{\partial z} + \mu\frac{\partial Q}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} = R\frac{\partial \mu}{\partial y} + \mu\frac{\partial R}{\partial y} \quad —⑤$$

$$R\frac{\partial \mu}{\partial x} + \mu\frac{\partial R}{\partial x} = \frac{\partial^2 u}{\partial z \partial x} = P\frac{\partial \mu}{\partial z} + \mu\frac{\partial P}{\partial z} \quad —⑥$$

Rearranging the terms equations ④, ⑤ & ⑥ become

$$\mu\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = Q\frac{\partial \mu}{\partial x} - P\frac{\partial \mu}{\partial y} \quad —⑦$$

$$\mu\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = R\frac{\partial \mu}{\partial y} - Q\frac{\partial \mu}{\partial z} \quad —⑧$$

$$\mu\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) = P\frac{\partial \mu}{\partial z} - R\frac{\partial \mu}{\partial x} \quad —⑨$$

Multiplying eqⁿ ⑦ by 'R', eqⁿ ⑧ by 'P' & eqⁿ ⑨ by 'Q' and adding we get

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

This is required condition of integrability which can also be written in the determinant form as follows

$$\begin{vmatrix} P & Q & R \\ P & Q & R \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix} = 0$$

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G. I. Bagewadi Arts Science and Commerce
College. Nippani

Department of Mathematics
PRACTICE TEST - II
For the year :

Class : BSC VI Sem.

Roll. NO : 57

Date :

5/10

1] Let, $L[e^t(\sin 3t . \cos 4t)]$

$= L[e^t\{\sin 7t - \sin t\}]$

$= L[e^t \sin 7t - e^t \sin t]$

$= L[e^t \sin 7t] - L[e^t \sin t]$

$= \frac{7}{(s-1)^2+49} - \frac{1}{(s-1)^2+1}$

$\therefore L[e^t(\sin 3t \cos 4t)] = \frac{7}{(s-1)^2+49} - \frac{1}{(s-1)^2+1}$

5] The total differential eq^n is $Pdx + Qdy + Rdz = 0 \rightarrow (1)$

Let, the Solution of total differential eq^n be $u = 0 \rightarrow (2)$

But the total differential is given by -

$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \longrightarrow (3)$

Since (1) & (3) are Same equation hence their Co-efficients are Proportional.

i.e $\mu P = \frac{\partial u}{\partial x}$, $\mu Q = \frac{\partial u}{\partial y}$, $\mu R = \frac{\partial u}{\partial z}$

of the above three equations

Differentiating first partially w.r.t y & second w.r.t x we get eq^n ④. Differentiating second partially w.r.t z & the third w.r.t y we get eq^n ⑤ & differentiating third w.r.t x & the first w.r.t z we get eq^n ⑥ as follows-

$$P.\frac{\partial \mu}{\partial y} + \mu\frac{\partial P}{\partial y} = \frac{\partial^2 \mu}{\partial x.\partial y} = Q.\frac{\partial \mu}{\partial x} + \mu.\frac{\partial Q}{\partial x} \rightarrow (4)$$

$$Q.\frac{\partial \mu}{\partial z} + \mu\frac{\partial Q}{\partial z} = \frac{\partial^2 \mu}{\partial y.\partial z} = R.\frac{\partial \mu}{\partial y} + \mu.\frac{\partial R}{\partial y} \rightarrow (5)$$

$$R.\frac{\partial \mu}{\partial x} + \mu.\frac{\partial R}{\partial x} = \frac{\partial^2 \mu}{\partial z.\partial x} = P\frac{\partial \mu}{\partial z} + \mu.\frac{\partial P}{\partial z} \rightarrow (6)$$

Rearranging the terms equations ④, ⑤ & ⑥ become-

$$\mu\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = Q.\left(\frac{\partial \mu}{\partial x}\right) - P.\frac{\partial \mu}{\partial y} \rightarrow (7)$$

$$\mu\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = R\frac{\partial \mu}{\partial y} - Q.\frac{\partial \mu}{\partial z} \rightarrow (8)$$

$$\mu\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) = P.\frac{\partial \mu}{\partial z} - R.\frac{\partial \mu}{\partial x} \rightarrow (9)$$

Multiplying eq^n ⑦ by R, eq^n ⑧ by P & eq^n ⑨ by Q and adding we get-

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

This is required condition of integrability which can also be written in the determinant form as follows-

$$\begin{vmatrix} P & Q & R \\ P & Q & R \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix} = 0$$

KLE Society's
G. I. Bagewadi Arts, Science and Commerce College. Nippani.
Department of Mathematics
PRACTISE TEST - I
For the year : 2019-20

9/10

Class : BSc VI Sem   Date:
Roll. No : 59

1] **Necessary condition for f(z) to be analytic :**

If the function f(z) is analytic in the domain D this conditions $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are satisfied.

Proof :- Let, $w = f(z) = u(x,y) + iv(x,y)$ be analytic in the domain D.

By definition $f'(z) = \frac{dw}{dz}$ exist $\forall z$ in the domain D.

i.e $f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exist along any path we choose for $\Delta z \to 0$, $\Delta z = \Delta x + i\Delta y$

$$\therefore f'(z) = \lim_{(\Delta x, \Delta y \to 0)} \frac{\{u(x+\Delta x, y+\Delta y) + iv(x+\Delta x, y+\Delta y)\} - \{u(x,y) + iv(x,y)\}}{\Delta x + i\Delta y} \longrightarrow (1)$$

As the derivative exist, this limit is unique irrespective of path in which $\Delta z \to 0$

i] Let, $\Delta z \to 0$ along the x-axis

along x-axis, $y = 0 \Rightarrow \Delta y = 0$

$\therefore \Delta z = \Delta x$.

$\therefore$ Eqn ① becomes -

$$f'(z) = \lim_{\Delta x \to 0} \frac{\{u(x+\Delta x, 0) + iv(x+\Delta x, 0)\} - \{u(x,0) + iv(x,0)\}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x, 0) - u(x,0)}{\Delta x} + i \lim_{\Delta x \to 0} \frac{v(x+\Delta x, 0) - v(x}{\Delta x}$$

$$f'(z) = \frac{\partial u}{\partial x} + i \cdot \frac{\partial v}{\partial x} \longrightarrow ②$$

ii) Let, $\Delta z \to 0$ along the y-axis and along y-a

$x = 0 \Rightarrow \Delta x = 0$.

$\therefore \Delta z = i\Delta y$

$\therefore$ Eqn ① becomes

$$f'(z) = \lim_{\Delta y \to 0} \frac{\{u(0, y+\Delta y) + iv(0, y+\Delta y)\} - \{u(0,y) + iv(0,y)\}}{i\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(0, y+\Delta y, - u(0,y))}{\Delta y} + \lim_{\Delta y \to 0} \frac{v(0, y+\Delta y) - v(0,}{i\Delta y}$$

$$f'(z) = \frac{1}{i} \cdot \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$f'(z) = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y} \longrightarrow ③$$

$\therefore$ from eqn ② & ③ -

$$\frac{\partial u}{\partial x} + i \cdot \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}$$

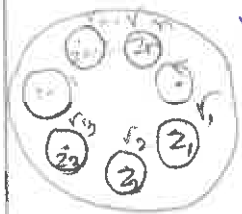
On comparing both the sides

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Statement :- Let, $f(z)$ be analytic with in & on closed contour C except at finite no. of Poles $z_1, z_2, z_3 ------ z_n$ inside C then $\int_C f(z)dz = 2\pi i (R_1+R_2+R_3+ ------+R_n)$.

where, $R_1, R_2, R_3 ------ R_n$ are residue at poles $z_1, z_2, z_3 ------ z_n$ respectively.

<u>**Proof :-**</u> By hypothesis $z_1, z_2, z_3 ------ z_n$ Poles of $f(z)$ inside C. Therefore function $f(z)$ is not analytic at these points inside C.



Hence construct small circle $\gamma_1, \gamma_2, \gamma_3 ... \gamma_n$ around these points then $f(z)$ is analytic in the region bounded by closed curves $C, \gamma_1, \gamma_2, \gamma_3, ------ \gamma_n$.

By cauchy's theorem for multiconnected region we have

$$\int_C f(z)dz = \int_{\gamma_1} f(z)dz + \int_{\gamma_2} f(z)dz + ------ + \int_{\gamma_n} f(z)dz \longrightarrow (1)$$

By definition of residue of $f(z)$ we have -

$R_1 = \frac{1}{2\pi i}\int_{\gamma_1} f(z)dz$ where γ_1 is circle around the pole z_1 & R_1 is residue.

$$\therefore \int_{\gamma_1} f(z)dz = 2\pi i R_1$$

llly $\int_{\gamma_2} f(z)dz = 2\pi i R_2$, $\int_{\gamma_3} f(z)dz = 2\pi i R_3$ -----

$$----- \int_{\gamma_n} f(z)dz = 2\pi i R_n$$

Then eqn (1) becomes

$$\int_C f(z)dz = 2\pi i R_1 + 2\pi i R_2 + 2\pi i R_3 + ------ + 2\pi i R_n.$$

$$\int_C f(z)\,dz = 2\pi i\,(R_1 + R_2 + R_3 + \cdots R_n)$$

$$\int_C f(z)\,dz = 2\pi i\,(\text{sum of residue of these poles inside } C).$$

Thus, if $f(z)$ be analytic within & on closed Contour C except at finite no. of poles z_1, z_2, z_3 ----- z_n inside C then

$$\int_C f(z)\,dz = 2\pi i\,(R_1 + R_2 + R_3 + \cdots + R_n)$$

$$= 2\pi i\,(\text{sum of residue at these poles inside } C).$$

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G I Bagewadi Arts, Science, Commerce College, Nippani

Department of Mathematics

Practice test - II

for the year 2019-20

Roll No:- 07 Marks obtained:-

Class:- B.Sc VI Sem

Date:-

8/10

Pratibha

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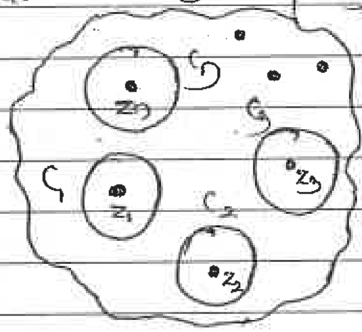
1) State and Prove Cauchy's residue theorem

Statement: If $f(z)$ is regular except at a finite number of poles $z_1, z_2, z_3, \ldots, z_n$ within a closed contour C and continuous on the boundary of C, then prove that

$$\int_C f(z)\,dz = 2\pi i\,(R_1 + R_2 + R_3 + \cdots + R_n)$$
$$= 2\pi i\,\{\text{sum of the residues at the poles within } C\}$$

Proof:

Let $C_1, C_2, C_3, \ldots, C_n$ be the circle with centre at $z_1, z_2, z_3, \ldots, z_n$ respectively & radius small that all the circles lies entirely within C and not overlap. Then $f(z)$ is analytic in the region lying bet C and the circles. Then by Cauchy's theorem for multiple connected region



$$\text{The } \int_C f(z)\,dz = \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz + \cdots + \int_{C_n} f(z)\,dz \quad \text{--- (1)}$$

but Residue at infinite is given by the definition is

$$R_1 = \frac{1}{2\pi i}\int_{C_1} f(z)\,dz$$

$$2\pi i R_1 = \int_{C_1} f(z)\,dz$$

$$\int_{C_1} f(z)\,dz = 2\pi i R_1 \quad \text{--- (2)}$$

and $R_2 = \frac{1}{2\pi i}\int_{C_1} f(z)\,dz$

$$\int_{C_2} f(z)\,dz = 2\pi i R_2 \quad \text{--- (3)}$$

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Continuing this way, we get

$$\int_C f(z)\,dz = 2\pi i R_3 \quad \text{--- (4)}$$

- - - - - -

$$\int_{C_n} f(z)\,dz = 2\pi i R_n \quad \text{--- (5)}$$

Substitute eqns (2), (3), (4) and (5) in eqn (1) we get

$$\int_C f(z)\,dz = 2\pi i R_1 + 2\pi i R_2 + 2\pi i R_3 + \dots + 2\pi i R_n$$
$$= 2\pi i (R_1 + R_2 + R_3 + \dots + R_n)$$

$$\int_C f(z)\,dz = 2\pi i \Sigma R$$

which completes the proof the theorem

2) State Prove the necessary condition for $f(z)$ to be analytic function

Necessary Condition: If the function $f(z)$ is analytic in the domain D then the condition $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ then the conditions are satisfied

A necessary condition for function $f(\omega) = f(z) = u(x,y) + i(x,y)$ be analytic in a domain

D is that u and v satisfy the eqns $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ which are called Cauchy Riemann eqns

Proof: Let $\omega = f(z) = u(x,y) + iv(x,y)$ be analytic in the domain D

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By def^n $f'(z) = \frac{dw}{dz}$ exist $\forall z$ in domain D

$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exist along where $\Delta z = \Delta x + i\Delta y$

$$f'(z) = \lim_{(\Delta x, \Delta y)} \frac{[u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y)] - (u(x,y) + iv(x,y)]}{\Delta x + i\Delta y} \rightarrow (1)$$

As the derivative exist this limit is unique irrespetive of path in which $\Delta z \to 0$

i) Let $\Delta z \to 0$ along the x-axis
Along x-axis $y=0$ $\therefore \Delta z = \Delta x$ & $\Delta y = 0$

$\therefore$ from (1)

$$f'(z) = \lim_{\Delta x \to 0} \frac{u(x+\Delta x\ 0) + iv(x+\Delta x\ 0) - u(x,0) - iv(x,0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x, 0) - u(x,0)}{\Delta x} + i\lim_{\Delta x \to 0} \frac{v(x+\Delta x, 0) - v(x, 0)}{\Delta x}$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \text{ —— (2)}$$

ii) Let $\Delta z \to 0$ along the y-axis
along y-axis $x = 0$

$\Delta z = i\Delta y$, $\Delta x = 0$
$\therefore$ from (1)

$$f'(z) = \lim_{\Delta y \to 0} \frac{[u(0, y+\Delta y) + iv(0, y+\Delta y) - u(0,y) + iv(0,y)]}{i\Delta y}$$

$$\lim_{\Delta y \to 0} \frac{u(0, y+\Delta y) - v(0,y)}{i\Delta y} + \lim_{\Delta y \to 0} \frac{v(0, y+\Delta y) - v(0,y)}{\Delta y}$$

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$\therefore f'(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$

$= \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ — ③

from ② and ③

$\frac{\partial u}{\partial x} + \frac{i\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

or $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

thus if $f(z) = u+iv$ analytic then C-R eq^n $u_x = v_y$ & $v_y = -u_x$ are satisfied

B.L.E Society's

G. I. Bagewadi Arts, Science, Commerce College

Nippani

Department of Mathematics

Practice test – I

For the year :- 2019-20

Roll no :- 07 Marks obtained :-

Class :- B.Sc VI Sem Date :-

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Find the necessary condition of integrability of the eq^n $Pdx + Qdy + Rdz = 0$

The total differential equation is $P.dx + Q.dy + Rdz = 0$ —① Let the sol^n of total differential eq^n be $u = c$ —② But the total differential is given by $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$ —③

Since ① and ③ are same equation hence their co-efficient are proportional

i.e $\mu P = \frac{\partial u}{\partial x}$, $\mu Q = \frac{\partial u}{\partial y}$, $\mu R = \frac{\partial u}{\partial z}$

of the above three eq^{ns}

Differentiating 1st p. w.r.t. y and second w.r.t. x we get eq^n ④

Differentiating IInd p. w.r.t. z and third w.r.t. y we get eq^n ⑤ & differentiating IIIrd w.r.t. x and the first w.r.t. z we get eq^n ⑥ as following

$$P\frac{\partial \mu}{\partial y} + \mu\frac{\partial P}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} = Q\frac{\partial \mu}{\partial x} + \mu\frac{\partial Q}{\partial x} \quad —④$$

$$Q\frac{\partial \mu}{\partial z} + \mu\frac{\partial Q}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} = R\frac{\partial \mu}{\partial y} + \mu\frac{\partial R}{\partial y} \quad —⑤$$

$$R\frac{\partial \mu}{\partial z} + \mu\frac{\partial R}{\partial x} = \frac{\partial^2 u}{\partial z \partial x} = P\frac{\partial \mu}{\partial z} + \mu\frac{\partial P}{\partial z} \quad —⑥$$

Rearranging the terms eq^{ns} ④, ⑤ and ⑥ becomes

$$\mu\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = Q\frac{\partial \mu}{\partial x} - P\frac{\partial \mu}{\partial y} \quad —⑦$$

$$\mu\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = R\frac{\partial \mu}{\partial y} - Q\frac{\partial \mu}{\partial z} \quad —⑧$$

$$\mu\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) = P\frac{\partial \mu}{\partial z} - R\frac{\partial \mu}{\partial x} \quad —⑨$$

$7 \times R$ & $8 \times P$ $9 \times Q$ and adding we get

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

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$$\begin{vmatrix} P & Q & R \\ P & Q & R \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix} = 0$$

2) The ring whose elements are $a+b\sqrt{2}$ where a & b are integers

Sol^n: we have already proved that

$\{a+b\sqrt{2} : a,b \in Z\}$ is a ring

Since $(a_1+b_1\sqrt{2})(a_2+b_2\sqrt{2}) = (a_2+b_2\sqrt{2})(a_1+b_1\sqrt{2})$

it is a commutative ring

It is also a ring with unity the unit elt being $1+0\sqrt{2}$, which belongs to the set

$AB = (a_1+b_1\sqrt{2})(a_2+b_2\sqrt{2})$

$= (a_1a_2 + 2b_1b_2) + (a_1b_2 + b_1a_2)\sqrt{2}$

$AB = 0$ if both $a_1a_2 + b_1b_2 = 0$ —①

$a_1b_2 + b_1a_2 = 0$ —②

Hence $a_2 = 0$, $b_1 = 0$ then

$a_1 + b_1\sqrt{2}$ i.e. $A = 0$

$AB = 0$ if $A = 0$ or if $B = 0$

Hence it is a ring without zero divisors

KLE's G. I. Bagewadi Arts, Science and Commerce College, Nipani

2019-20

Department of Commerce

Assignments

Topic : Types of Reconstruction

Name : Nikunj Potadar

Class : B. Com IV Sem

Subject : Corporate Accounting - II

Reg. No : C1830233

Reconstruction –

Reconstruction means the reorganisation of the affairs of the company when its financial position is not satisfactory.

Types of Reconstruction –

- Internal Reconstruction
- External Reconstruction.

- Internal Reconstruction –

Internal Reconstruction means reorganisation of the affairs of the company by revaluation of its assets, ascertaining correct amount of liabilities or the writing off the accumulated losses by reducing the share capital of the company with or without varying the rights of the shareholders but without liquidation of the existing company.

The internal reconstruction involves sometimes, the alteration of share capital.

- **External Reconstruction –**

External Reconstruction involves winding up of an existing company and transfering its business assets and liabilities to a newly formed company consisting sustantially the same members.

Distinction Between Internal Reconstruction and External Reconstruction –

Internal Reconstruction	External Reconstruction
(1) Internal Reconstruction means the reconstruction of a company by way of capital Reduction.	(1) External reconstruction means the reconstruction of a company by way of acquisition of its business.
(2) In this case there is neither winding up of a company nor formation of any new company.	(2) In this case the existing company is wound up and in its place a new company is formed.

(3) Internal reconstruction therefore, involves less procedural formalities.	(3) External reconstruction, therefore, tedious formalities of winding up and formation of a new company.
(4) In the case of internal reconstruction, the debentureholders, creditors etc. may continue in the company even after reconstruction.	(4) In the case of external reconstruction, if the debentureholders, creditors etc. do not want to continue in the company, their accounts will have to be settled.
(5) Internal Reconstruction is a slow process since it requires the approval of all creditors, shareholders etc and the confirmation of the court	(5) External reconstruction can be brought about by the approval of the shareholders without much delay in carrying out the reconstruction of the company.

KLE's G. I. Bagewadi College, Nipani

Department of Commerce

2019 - 20

Assignment

Topic : Dematerialisation Process

Name : Shubham Puthane

Class : B.Com VI Sem

Reg No : C1730280

Subject : Indian Financial Services

Dematerialisation Process.

An Investor having securities in physical from must get them detematerialised if the intends to sell them This requires the investor to fill a Demat request From (DRF) which is available with every DP and submit the same along with the physical certificates. Every security has an ISIN [Internationaly securities Identification Number] If there is more than one security then the equal number of DRF's has to be filled in. The whole process goes on as mentioned above.

Acount opening with DPs And Dematerialisation.
Providing the bank acount details at the time of acount opening.

It is mandatory for an investor to provide his bank acount details at the time of opening a demat acount This is done to safeguard investors own interests There are two. major resons for this The Interest and dividend warrants cant be encashed by any anauthorized person as the bank acount number is mentioned on it

Dematerialisation process.

↓

1) Investors. surrenders the physical certifiates to the DP. for dematerialisation.

↓

2) DP informs the Depository abouw the request

↓

3) DP. Submits the certificates to the register of the Issuer company.

↓

4) Register communicates with the Depository to confirm the request

↓

5) Dematerialisation of the certificates is one by the register

↓

6) Accounts are updated by the Registrar and the depository is informed about the competion of dematerialisation

↓

7) Accounts are updates by the Depository and DP is informed about the same

↓

8) Demat account of the investor is updated by DP.

BSc II sem

05/10

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Name :- Aditya K Siddannavar

R.no :- 01

Department of Botany

Reg No - S1919406

Q.2 ② Nitrification :-

The metabolic process by which the ammonia is oxidised to nitrates is called nitrification. by nitrification of soil increses.

$NH_3 + 3O_2 \longrightarrow NO_2^- + 2H^+ + 2H_2O$

nitrate

2

① Transamination :-

The transfer of amino group from one compound to other is called Transamination.

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I

01) Biological nitrogen fixation is
The conversion of molecule nitrogen into NO_2 compounds by microbes enzymes used nitrogas.

$N_2 + 3H_2 \longrightarrow 2NH_3$

ii) symbotic N_2 fixation is ~~it~~ found in roots of legumenous ~~plants~~. Plants.

$N_2 + O_2 \xrightarrow[\text{energy}]{\text{nit}} 2NO$

$2NO + O_2 \xrightarrow{\text{~~oxidation~~}} 2NO_2$ (nitrogen peroxide)

$2NO_2 + H_2O \longrightarrow HNO_2 + HNO_3$ (nitrogenous)

Ca / K salt + $HNO_3 \longrightarrow$ Ca / KNO_3
(alkaline radicals)

2

Name:- Reayanna S. Chikodi
R.No:- 21

S1919521

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Q-1) Explain in breife biological Nitrogen Fixation?

Q-2) Define the term.
① Transamination.
② Nitrification.
③ Atmosphiaric Nitrogen fixation.

Q-2)
② Nitrification:- ~~Sie~~ -
The metabolic process by with the amonia is oxidiced to nitrace is called Nitrification. by nitrification of soil involves.

2

$$NH_3 + 3O_2 \rightarrow NO_2 + 2H^+ + 2H_2O$$

mixture.

① Transonimation:-
The transffer of ~~one~~ amonia group of the compound to other is called transmination.

③

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3) Nitrogen Fixation :-

The Conversion of molecule nitrogen into NO_2 Compounds by nitroces Enzymes

$$N_2 + 3H_2 \longrightarrow 2NH_3$$

1) Symbotic N_2 fixation!

if found in roots of layers & plants

$$N_2 + O_2 \xrightarrow[\text{energy}]{\text{nil}} 2NO.$$

$$2NO + O_2 \xrightarrow{\text{oxidation}} 2NO_2 \text{ (nitrogen Peroxide)}$$

$$2NO_2 + H_2O \longrightarrow HNO_2 + HNO_3 \text{ (Nitrogenous)}$$

$$\text{or/t salt} + HNO_3 \longrightarrow Co$$

3

Name :- Rutuja M. Nare.
Roll No :- 218.
Std :- BSc. 2 sem.
Reg- S1919533

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6/10

Q 1 Explain in Brief. biological nitrogen fixation.
⇒ Biological nitrogen fixation :-
The Symbiotic relationship betn Rhizobium & the roots of legumes Such Sweet pea, lentils. The assosiation is visible as nodules on roots. The reduction of nitrogen to ammonia by living orgenism is called "biological nitrogen fixation".

The enzyme it neede for this reaction, nitrogenase is present exclusively in prokaryants & those microbes are called

2

Q 2. Define the term :-
1) Transamination :-
The ~~mata~~ process through which ammonia is converted into ammino acid & transported through all the parts of the plant. known as Transamination.
Rean :-

2

2) Nitrification :- The metabolic process by which ammonia is oxidesed in nitrogen called Nitrification.
Reaction :-

$NH_2 + 3O_2^2 \longrightarrow 2NO_2 + H_2O + 2^+H$

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③ Atmospheric nitrogen fixation:-
The Atmospheric nitrogen fixation is a process converting atmospheric nitrogen into ammonia is called as Atmospheric nitrogen fixation.

Name :- kiran s walaki
R. No :- 203
class :- BSc II
Reg No - S1919450

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Q.1 Explain in brief biological nitrogen fixation?
→ convertion of molecule nitrogens into N_2 compound by microbes. enzyme used : nitrogenesis
$N_2 + 3H_2 \longrightarrow 2NH_3$.

i) Symbiotic N_2 fixation?
It founds in root's of negumenous Plants.

2

2) Define the term
① Transmination.
② nitrification
③ atmospheric nitrogen fixation.

1) Transmination :- The transfor of amino group from one compound to another compound is called transmination.

2) Nitrification :- ~~and~~ The metabolic process in which amino ~~acids~~ is oxidised into nitrate is called as nitrification

$NH_3 + 3O_2 \longrightarrow NO_2^- + 2H + 2H_2O$

2

3) Atmospheric nitrogen fixation :-

$N_2 + O_2 \longrightarrow 2NO$

Prajwal. 1. Jothiwale
136
BSc I[3S]
Reg - 51913499

05
10

Q.1] Explain ~~the~~ in brief biological nitrogen fixation.

Q.2] Define the term
1] Transmination
2] Nitrification
3] Atmospheric nitrogen fixation.

Answers

1]
2] Nitrification
It is the metabolic process by which ammonia is oxidised into nitrates is called nitrification

$$NH_3 + 3O_2 \longrightarrow NO_3^- + 2H^+ + 2H_2O$$
nitrates.

1] Transmination:-
The transfer of amino group from one compound to another compound is called Transmination

~~NH~~$_2$

I] Biological nitrogen fixation.

-) conversion of molecule nitrogen into NO_2 compounds by microbes enzymes used nitrogenase

$$N_2 + 3H_2 \longrightarrow 2NH_3$$

i] Symboitic N_2 fixation
It found in roots of legumenous plants.

3)

$$N_2 + O_2 \xrightarrow[\text{energy}]{\text{high}} 2NO$$

Tushar Patil
Roll:-32.
Regno-S1919607

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(1)

Q 1 explain the biological Nitrogen fixation

→ It response the enzymes.
[molecular nitrogen into nitrogenous compound]
Bacteria. rizobium, azetobacter, basilla is a nitrogen fixer.

~~N_2 →~~

~~$N_2 + 3H_2 \xrightarrow{\text{nitrogenous}} 2NH_3$~~

Q.2.

(1) Transmination :- The transfer of amonea group from one compound to other compound is called Transmination.

(2) Nitrification :- The metabolic Process by which ameno acid has oxidised to nitrates is called as Nitrification.

(3) Atmosphoric nitrogen fixation :-

It response the sunlight.

$N_2 + O_2 \xrightarrow{\text{High energy}} 2NO$ (Nitric acid)

Roll No = 91 51717777

BSC V sem

7/10

write a note on air Pollution.

The air Pollution means the The Pure air is Polluted with human Activities or Chemical Activites is Called as air Pollution.

The Chemical activities are CO_2 dele ~~ted trees~~ human but ~~Observed is~~ obsorsen is not found because ~~human~~ man is ~~tute~~ Cut the trees to his ~~ore~~ own use So tree are the best obseption of CO_2. So in the air the CO_2 Percentage is increasing So this Purpose the air Pollution is occring.

indoor air Pollution and Poor urban air quality are listed as two of the world's worst toxic Polluction Problems in the 2008 Blacksmith institute world's worst Polluted Place report. ist. According word ~~worst Polluted Places re~~ Health organization report, air Pollution in 2012 Caused the deaths of around 7 million People worldwild, an estimate roughly echoed by one from the International Energy Agency.

The air Pollution it may Cause diseases, allergies and even death to humans it may also Cause harm to other living organisums Such as animals and food Crops and may damge the natural or built Cnvironment. Both human activity on natural Processes Can generate air Pollution.

B.Sc IV Sem

06/10

Roll. no: 64
BSc. IV sem.
Date: 20/1/2020
Reg - S1819404

Q.1] Medicinal uses.

a] Aloe vera:
- Diabetes treatment
- Hair and its treatment
- Aloevera gel is used in many cosmetics
- It is used in the hydration of skin
-

b] Tinospora cordifolia
Tulsi

c] Ocimum sanctum.
Tulsi
- It is used in cough & cold
- It has many medicinal importance for this plant
- It is

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Q.2. Withania somnifera

Ashwagandha.

- It is used in a nervous dis spinal cord / nerve treatment
- It is used as to reduce Anxiety / Anxiety.
- It stimulate and increase sleeping time.
- It stimulates the brain of mental health.
- It is used as fertility supplement.

Q:3.

a) Sarpagandha.

b) Turmeric.

It is used in Cancer Treatment
It is also used in cosmetics
It is used to colouring & flavouring many dishes.

R.NO - 18　　Reg no - S1819523
class - BSc II yr
division - A
date - 20-1-2020

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Medicinal use of plants.

Q.1 a) Aloe vera

* Aloevera used in cosmatics
* Aloevera it miostsurises the skin, & it is also used making shampoo
* It is also used in treatment of diabetes
* It is also used in hair & skin to remove dandraf & miostursing skin
* It is used to cure skin diasease.

2

b) Tinospora Cardifolia

*It is used in making medicinal products
*It is used in making of cosmatics
* It is used to moisturising skin

Q. 2

b) Turmeric
botanical name :
family : ~~Zingi~~ Gingiberacea.

1

a) Sarphgandha
botanical name :
family :

Roll No :— 9 class Test
Class :— B.Sc. IVth Sem
Date :→ 20/01/2020

09/10 Reg no 51819590

Q.1 Write the medicinal use of following plants

a) Aloe vera

→ It belongs to the Aspodelaceae.

Medicinal use :→

* It is used to cure the skin disease.
* It is also used to cure the diabeties, which control the sugar level in the blood.
* It moist the hairs and controll the pH of hair. It is used to in shampo's & conditioner
* It is effectively used in the infections, swelling, increases the inmunity.
* It improves the hair growth
- Oral health.

b) Tinospora cordifolia :→

It is commonly called as Geloy
• It belongs to the family menispermaceae

Medicinal uses :→

Skin disease :→ Geloy is taken with the ghee in the empty stomach in the morning it is useful to cure the skin diseases.

* It effectively used to cure the fever ~~like~~ especially chronic fever.
* Gastro - intestinal diseases :→ It is used to cure the hepatices, worm disease in the intestine.
* It is mainly effective for the cardiac system Used to cure the heart disease.
* It also controlls the blood pressure.
* Stress management -

c] Oscimum sanctum :→

• It is commonly called as Tulsi

Medicinal uses :→

* Tulsi is used as herbal ~~tree~~ tea.
* The oil extracted from the Isarpoorg tulsi is used ~~as~~ for toiletry.
* It is mainly used to cure the malaria
* It also act as blood purifier.
* This also used to cure the fever, cough, diaria.
* It is also used to cure skin disease.
* Tulsi is also used to worship the God.

Q.3 Give the botanical name & family

a) Serpgandha
Rouldfia Ser serpentina, family – Apocynacea

b) Turmeric
Curcuma longa
Family – Zingiberaceae

Q.2 Explain Withani somnifera

- It belongs to the family — Solanaceae
- Commonly called as Ashwagandha, it is sanskrit word, Ashwa – Hourse & gandha – smell.

Medicinal Uses :→

* It effectively used as sleep inducer.
* Central nervous system.
* It is used to cure one side paralysis.
* Memmory sharper.
* Stress management
* Reduces the backbone pain.
* It used to ~~cure~~ reduce the effect of medicines used to cure the cancer.
* It controll the fat & sugar level in the blood.

07/10

Roll No - 14
Class - BSc 1st 'A'
Reg No - S1819455

Q. 1. Explain in brief biological Nitrogen fixation
Q. 2. Define the term
① Transmination
② Nitrification
③ Atmospharic nitrogen fixation.

Q. 1. Biological nitrogen fixation is occurs in 2 steps.
* symbiotic nitrogen fixation
* non symbiotic nitrogen fixation
Nitrogen fixation - transfer or conversion of molecular nitrogen into nitrogenous usable form through agency of living organisms is known as Biological nitrogen fixation.

The Biological nitrogen fixation constitutes 90% of free nitrogen on earth
Biological / some biological nitrogen fixers are
1) Azatobacter 2) Rhizobium
Two main important biological nitrogen fixers are
1) symbiotic nitrgen fixation
2) non symbiotic nitrogen fixation

2

1) Symbiotic Nitrogen fixation :-
Symbiotic N_2 fixers are capable of to fix the nitrogen in root of the leguminous plant
* This was demonstrated by the french chemist Boussingoult in 1837

* Since the symbiotic N_2 nighter the plant are

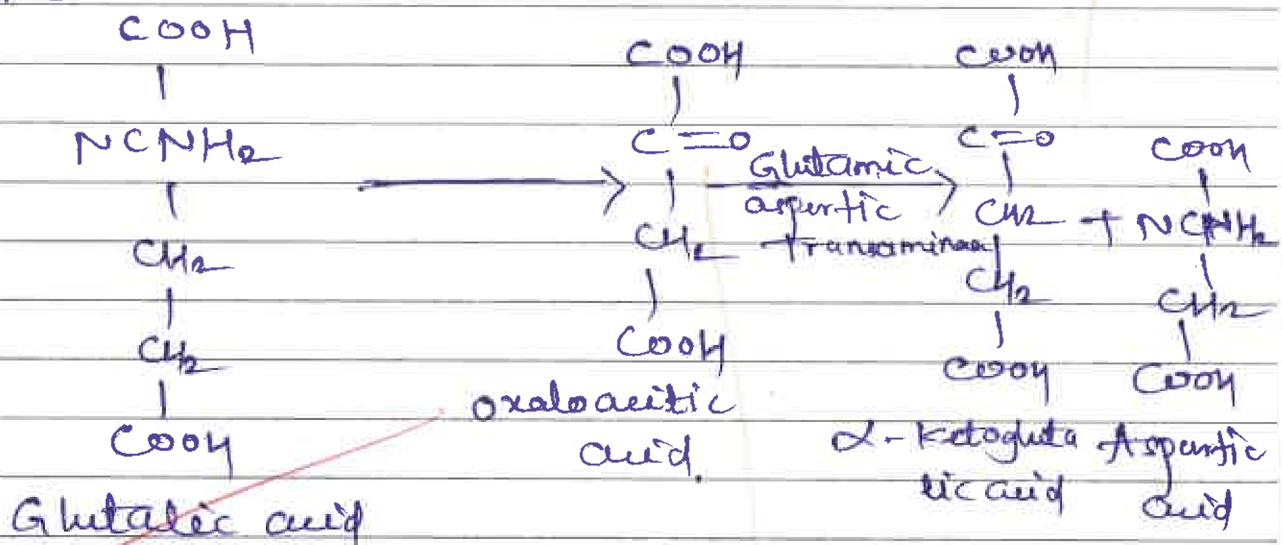
Bateria fix the N2 that is called symbiotic N_2 fixation.

2) Non symbiotic fixation —
Non symbiotic are not capable to fix N_2
It occurs in biological N_2 fixation
It occurs in the root of Nodules

Q. 2. Define the term :—

1) Transamination :— Transfer of the amino group from one of compound to the another compound is known as the Transamination.

Example :—
The Glutamic acid is transfered into oxaloacetic acid to form α-ketoglutalic acid and aspartic acid in presence of Glutamic aspartic transamminase.
i.e



2

③ Atmospheric Nitrogen fixation —
The Conversion of molecular nitrogen into nitrogenous compound by lightning and the cosmic radiations is known as the Atmospheric Nitrogen fixation.

2

Atmospheric N2 fixation involves served steps :—

$N_2 + O_2 \xrightarrow[\text{energy}]{\text{high}} NO_2$ (Nitric oxide)

$NO_2 + O_2 \xrightarrow{\text{oxidation}} 2NO_2$ nitrogene peroxide

$2NO_2 + H_2O \longrightarrow HNO_2 + HNO_3$.
(Nitrous | Nitric acid)

$C / KNO_3 \longrightarrow C/KNO_3$
(Alkali radical)

(2) In this process I n which Nitrate is reduced to form Nitrite and then to Ammonia and substituent to a gaseous Nitrogen known as Nitrification

Sub: Botany
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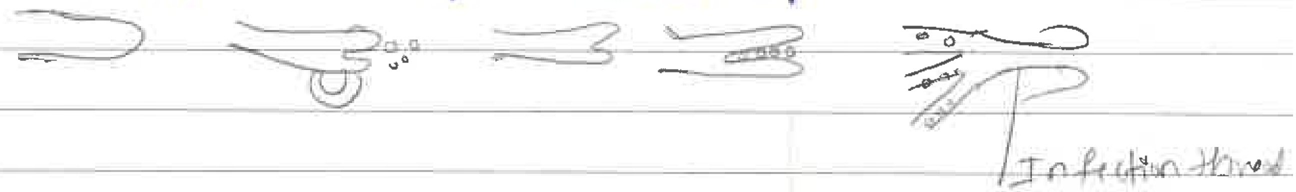
1) Explain in brief biological nitrogen fixation.

→ The conversion of molecular nitrogen into nitrogenous from or molecular nitrogen into nitrogen compound by microbes is called biological nitrogen fixation.

$$N_2 + 3H_2 \xrightarrow{nitrogenase} 2NH_3$$

Ex: Rhizobium

- In biological nitrogen fixation 2 type are them
  - i) Symboitic nitrogen fixation. ex: leguminous plants
  - ii) Asymboitic nitrogen fixation.
- In symboitic nitrogen fixation IAA is released.
- Examples of biological nitrogen fixers:-
  - i) Bacillus
  - ii) Rhizobium
- Nodulation:- Process of formation of nodules.



2)

i) Transamination:-

The process of tranfer of aminogroups from one compound to another compound.

One glutamic acid is formed, the formation of other amino acids is possible by the tranfer of amino groups to the other skeleton.

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② Nitrification:-

The metabolic process by which ammonia is oxidised to nitrates is called nitrification. By nitrification fertility of soil increases.

③ Atmospheric nitrogen fixation:-

The process of conversion of nitrogen molecule into nitrogen compound by atmospheric processess.

Ex:- By consemidization, By lightning.

$N_2 + O_2 \rightarrow 2NO$ (nitrogen oxide)

$2NO + O_2 \rightarrow 2NO_3$ (nitrogen peroxide)

$2NO_3 + H_2O \rightarrow HNO_2 + HNO_3$ (nitratite, nitrate)

Sub: Botany
R. No: 03
Reg - S1919511

6/10

1. Explain in brief biological nitrogen fixation.
→ convertion of nitrogen into nitrogen compound is called nitrogen fixation.

Nitrogen is essential microelement. Approximate amount of nitrogen in the hole plant is 1-3 %.

$N_2 + 3H_2 \longrightarrow 2NH_3$

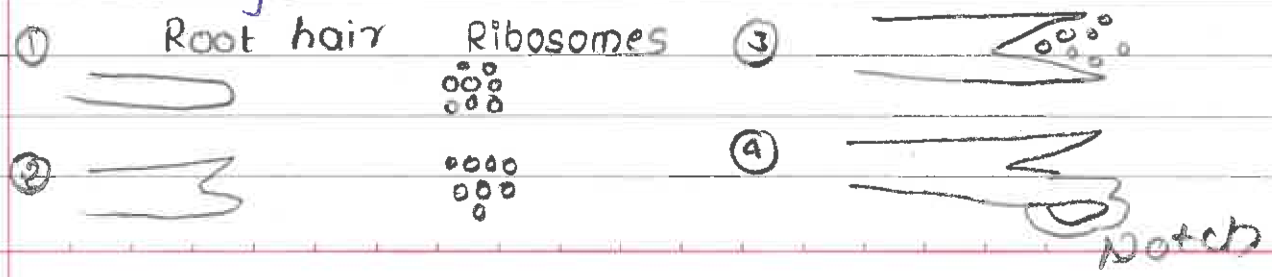
Types of nitrogen fixation-
(i) Bilogical nitrogen fixation.
(ii) Industrial
(iii) Atmospheric nitrogen fixation

(i) Biological nitrogen fixation. → convertion of Air molecular nitrogen into nitrogen compound by microbes is called biological nitrogen fixation.

$N_2 + 2H_2 \xrightarrow{\text{nitrogenase}} 2NH_3$.

* in Biological nitrogen fixation 2 types
(i) Symbiotic nitrogen fixation
(ii) Asymbiotic nitrogen fixation.

(i) symbiotic nitrogen fixation
Nodule formation.



⑤

3

(i) Transamination - The process of transfer of aminogroup from one compound to another compou-nd known as transamination.

(ii) Nitrification :- It is metabolic process by which ammonia is oxidised to nitrates is called nitrification.

(iii) Atmospheric nitrogen fixation - The process of convention nitrogen molecule into nitrogen compound by atmospheric process.
Ex - By lightining.

$$N_2 + O_2 \longrightarrow 2NO.$$

2

R. 100 - 221

BSc - II Sem

8/10

2019-20

Reg - S171705

Write a note on Air pollution

→ The contaminants present in the air is called Air pollution.

The contaminants like dust particles gaseous molecules which may cause harmful effect on the environment & human beings is called Air pollution.

There are 2 types of Air pollutants

1. Primary pollutants
2. secondary pollutants

1) primary pollutants → These are emitted from source such as carbon monoxide emited from motar vehicles

2) Secondary pollutants → They not emited directly into the atmosphere
Ex:- dust

Effects of air pollution

* The Green house gases such as CO_2, CO, NO_2, SO_2 which increases it's level & effect on atmosphere and human living things
* CFC's are completely band because it cause damage to ozone
* The motar vehicle which emits CO_2 gas which become harmful to environment
* Horons are responsible for air pollution
* Due to air pollution many breathing & respiratory problems cause such as pnemonia.

* It may cause headache, asthma, higher diseases

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Date:

control measures

* drive less & walk more. If we walk more instead of using motor vehicles we can also save money & control air pollution.

* Now a days CFC's are completely baned instead of CFC we can use HFC (Hydro floro carbons).

* Keep our environment & surrounding clear.

* Growing of more plants that is Aforestation which is also one of the control measure of air pollution.

Causes of air pollution?

Reg No-S1717735 (6/10) Rall no-93

1) Write a note on Air pallution-
Contamination of air due to human activity is called air pallution.
harm air pallution occurs because of human activity. Now day Increasing the rate of air pollution because the Factorys are producing such a gas that why air will be polluted. gases converted into air than air will be polluted.

Air pallution occurs & they are harmfull to the human being & also plants & animals. In atmosphere there many zones are present & ozone layer is very important layer present in stratosphere. ozone is the main layer which protects the earth. due to the human activity ozone layer is destroyed. is causes the skin diseas. & ultravoilet rays Come to the earth. these are ultravoilet rays are harmfull to the earth or organisms. ozone is protective layer because of air palution ozon is effected. It Consist holes of on the layers. through this hole ultravoilet rays are reach to the earth. These are harmfull to the human. due to the industrial gases use of plastics. air pallution is occurs.

* pollutants - Carbon dioxide, Nitrous acid, dust, & Carbon monoxide.

~~* pollu~~

* Causes of Air pollution -
* The Industrial release of harmful gases
* Burning of plastics.
* Burning of Fossil petrol & desil.

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Air Pollution

now a day air pollution is the bigest Problem in the word main reason for air Pollution is (CO_3) carbon dioxide is the main reason for Air pollution who to prevent the Air Pollution used to electric car's and Solar charging Car then if you used the old wheeal then it will be change is because they Engin will be old and they Produced more amount of CO_3 then secound number of air pollution will be indestrial factor they are now a day highest amount of CO_3 is produced and the CO_3 is damag the ozon layer it's Air Pollution for main reason for man and then fire will be cock the forest and hale forest will be destred then they CO_3 will be Produced more amount then the colling estrume are Eg frezz Ac they types of divice will be Produced CO_3 and they will be distroed the envoroment and vehicals are produce more amount of carbon dioxide and if you produce the CO_3 then if re cycle it's youesed to tree they it will give to oxygen in if you cut the tree and who it's cycale will complete then CO_3 amount is incroging in Enviroment and it's effect on green house effect the Earth tempretuer was inorriging then rain will be not come in perfet tyme then it's harmfull effect on People they if you Prevent the air poluation then it will be help to keep enveroment or nature will any type of Pollution will be reson is man if you keep the natre will then it will be help you then it will be distroed it

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